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ANALYSIS OF SIMULATION BASED QUEUE MODEL

Mathematics

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ABSTRACT In this paper , we study the sensitivity analysis of $G_1/G_1/1$ Queueing model under Simulation Technique by transformation method , and it is called Push-Out Simulation. This term is derived from the fact that we push-out the parameter θ from the original sample performance $L(Y,\theta)$ into an ordinary probability function through a suitable transformation. Then we apply the score function simulation method to perform sensitivity analysis and optimization.

1. INTRODUCTION

Here we consider $G_1/G_1/1$ queueing model which is based on discrete events dynamic system using structural parameter. In $G_1/G_1/1$ model, the customers arrive and take services as general distribution using single server. This study is an extension of score function simulation method for sensitivity analysis and stochastic optimization. After that, we take differentiation of the auxillary sample performance with respect to $\theta=(\theta_1,\theta_2)$

The earlier researchers have studied sensitivity analysis and stochastic optimization for many queueing models in different frame-works. Glasserman [2] discussed gradient estimation via perturbation analysis. Dussault et al studied combining stochastic counterpart and stochastic approximation methods. Asmussen and Melamed considered regenerative simulation of TES progress. Sensitivity analysis of $G_1/G_1/m/B$ queue with respect to buffer size by the score function method was studied by Krimann [4]. Rubensteen [7] studied sensitivity analysis of discrete events dynamic system by the " Push-Out" method. Convergent rates for steady-state a derivative estimator was considered by Ecuyer . Marti discussed stochastic optimization method of structural design and considered efficiency of score function method for sensitivity analysis and optimization of queueing networks.

2. PRELIMINARIES

STEADY-STATE WAITING TIME

In the $G_1/G_1/1$ Queueing model, the steady-state waiting time $\psi(\theta)$ is stable as

$$\psi(\theta)=E_{\theta_1}\{L(Y_i,\theta_2)\} \quad ....(1)$$

where $L(Y_i,\theta_2)$ is sample performance depending on the parameter vector θ_2 and input sequence $Y_i=\{y_1,y_2.......y_t\}$ of independent identically distributed random vectors with common probability density function (pdf) $f(Y,\theta_1)\theta_1\in E_{\theta_1}\{L\}$ and combained vector of parameter given here by $\theta=(\theta_1,\theta_2)$ We assume f depends on the parameter vector θ_1 but not on θ_2 and L depends on θ_2 but not on θ_1

3. "PUSH-OUT" SIMULATION

We consider "Push-Out" simulation technique which shows that it typically smoothes out the sample performance function $L(Y_i,\theta_2)$ with respect to θ_2 by rendering it independent of θ_2 . To determine the push-out technique, let there exist a vector valued function $X=X(y,\theta_2)$ and the real valued function $\bar{L}(X)$ independent of θ_2 such that

$$L(Y_i,\theta_2)=\bar{L}\{X(y,\theta_2)\} \quad ..... (2)$$

Suppose that $Y-f(y,\theta_2)$ and the corresponding random vector $X=X(y,\theta_2)$ for which the pdf is $\tilde{f}(x,\theta_1,\theta_2)$ then

$$L(\theta)=\int_{x\in X}\bar{L}(x)\tilde{f}(x,\theta)dx=E_{\tilde{f}}\{\bar{L}(x)\} \quad .....(3)$$

where $E_{\tilde{f}}\{\bar{L}\&\}$ the expectation is now taken with respect to the pdf $\tilde{f}(x,\theta_1,\theta_2)$.

The derivative of $\nabla\psi(\theta)$ is similar, we use the identity,

$$\frac{d}{d\theta}\int_{a(\theta)}^{b(\theta)}g(\theta,x)dx=\int_{a(\theta)}^{b(\theta)}\frac{\partial g(\theta,x)}{\partial\theta}dx+\frac{db(\theta)}{d\theta}g(\theta,b(\theta))-\frac{da(\theta)}{d\theta}g(\theta,a(\theta)) \quad .... (4)$$

A representation of $L(Y,\theta_2)$ of the form (1) and the subsequent transformation (2) are not always available, and if available, it may be difficult to calculate $\tilde{f}(x,\theta_1,\theta_2)$.

Let for every θ_2 , $X=X(y,\theta_2)$ be 1-1 function and have an inverse $Y=Y(x,\theta_2)$ and assumed to be continuously differentiable in each component of x, then we have ,

$$\tilde{f}(x,\theta_1,\theta_2)=f\{y(x,\theta_2),\theta_1\}\frac{\partial y(x,\theta_1)}{\partial x} \quad ... (5)$$

4.BASIC DEFINITIONS

4.1 DEFINITION

Let G be the probability measure with probability density function g(z) so that

$$dG(z)=g(z)dz$$

$$Sup\{f(z,\theta_1)\}\subset Sup\{g(z)\} \quad ,\text{then}$$

$$l(\theta)=E_g\{L(z,\theta_2)W(z,\theta_1)\} \quad ...... (6)$$

Where $W(z,\theta_1)=f(z,\theta_1)/g(z)$ is likelihood ratio discussed in Glym [ ] and g indicates that the expectation is taken with respect to dominating pdf g(z).

Under the standard regulatory condition admitting the interchangeability of the expectation and differentiation operator ∇ we have the following

$$\nabla_{\theta_1}^k l(\theta)=E_g\{L(z,\theta_2)\nabla_{\theta_1}^k W(z,\theta_1)\}, \quad k=1,2,....... \quad ...... (7)$$

$$\nabla_{\theta_2}^k l(\theta)=E_g\{L(z,\theta_2)\nabla_{\theta_2}^k W(z,\theta_1)\} \quad k=1,2,....... \quad ...... (8)$$

4.2 DEFINITION

Let g(z) be a pdf that dominates $\tilde{f}(z,\theta)$,then from (6),(7),(8) we have

$$\nabla^k l(\theta)=E_g\{\bar{L}(z)\nabla^k\hat{W}(z,\phi)\}$$

Where

$$\hat{W}(z,\phi)=\tilde{f}(z,\phi)/g(z) \quad and\ Z=g(z) \quad ...... (9)$$

Now for a given sample $\{Z_1,Z_2,......Z_n\}$ from $g(z)\nabla^k l(\theta)$ may be estimated as

$$\nabla^k l_N(\theta)=\frac{1}{N}\sum_{i=1}^{N}L(z_i)\nabla^k\hat{W}(z,\theta), \quad k=0,1,2......$$

Where $\nabla^k l_N(\theta) = l_N(\theta)$ and estimate $\nabla^k \tilde{l}_N(\theta)$ referred to as Push-Out score function

5. Mathematical Model Formulation

Let S_n and W_n be sojourn time and waiting time distribution of n^{th} customer respectively.

Let y_{1n} = service time of n^{th} customer

Let X be the steady-state random variable with steady-state cumulative density function.

$$E_x(X,\theta) = P_\theta(X \le x) = \lim_{n\to\infty} P_\theta(X_n \le x)$$

and $f_x(X_1)$ is pdf of random variable such that $X \in (S,W)$ and

$$f_x(X,\theta) = \frac{\partial F_X(X,\theta)}{\partial x}$$

Estimation of $F_X(X,\theta)$ and $f_x(X,\theta)$ for related variables such as virtual waiting time and queue length are discussed. Let ψ be the number of customers served during a busy period in a steady-state $G_1/G_1/1$ queue with First come-First served (FCFS) pattern. Let θ be the service time distribution variable $f(y,\theta)$

6. Main Results

Theorem: 1

Let $X \in \{L,\xi\}$ be the steady-state expected waiting time Cdf. $E_X(X,\theta) = P_\theta(X \le x)$ of the sample performance is estimated as

$$E_X(X,\theta) = \frac{1}{N}\sum_{j=1}^{N} f(z_i)\overline{W}(z,u)$$

Proof:

Since $X \in \{L,\xi\}$ be the random variable of the waiting time distribution such that, $X = \sum_{i=1}^{T} L_T$ Where T is the number of customers served during busy period in the steady-state. Using likelihood ratio which is discussed in Glynn [ ] we have,

$$E_X(X,\theta) = E_\theta\{X = x \;\; W_T(Z_T,\theta)\} \quad \text{.... (10)}$$

Where

$$W_T = \prod_{j=1}^{n} W_j = \frac{f(Z_T,\theta)}{g(Z_T)} \quad \text{and}$$

$$Z_T = (Z_1, Z_2, \ldots\ldots Z_n)$$

Standard likelihood ratio Glynn [ ] estimators of $E_X(X,\theta)$ based on (1) required calculation of the estimator function. Estimation of $E_X(X,\theta)$ for multiple values of X and θ from single simulation run is as, $E_X(X,\theta) = E_\theta I_{(-\infty,0)}\,(X = x)$

Since L_u and ξ_n are sojourn and waiting time of n^{th} customer then,

$$L_u = \xi_n + Y_{1n}, \; for \; n = 1,2,3\ldots\ldots\ldots$$

Now $E_X(X,\theta)$ may be written as

Where $X = \sum_{n=1}^{y-1} L_n + \xi_T + Y_T$

$$\tilde{Y}_T = Y_T - X \;\; and \;\; \tilde{Y}_T \sim$$

$$\tilde{f}(\theta,u) \;\; for \; \tilde{f}(\theta,0,u) = f(\theta,u)$$

So the cdf $E_X(X,\theta)$ may be represented as

$$\text{Where} \quad \overline{W}_T(u) = \overline{W}_T(u)\prod_{j=1}^{T-1} W_j(u)$$

$$W_T(u) = \frac{f(Z_j,\theta)}{g(Z_j)} \quad , j = 1,2,\ldots\ldots\ldots T-1$$

Now we may estimate $E_X(X,\theta)$ as

$$E'_{X\theta}(X,\theta) = \frac{1}{N}\sum_{i=1}^{N} \bar{I}(Z_n)\,\overline{W}(z_n,u) \quad \ldots\ldots (11)$$

At Steady-state when T= 1in the above equation (11) we get the required result.

Example : Let the probability density function of waiting time distribution $f(y,v) = ve^{-vy}$ be known, then we can estimate the waiting time auxillary pdf.

Solution:

$$f(y,v) = ve^{-vy} \quad \text{then}$$

$$\tilde{f}(y,x,v) = ve^{-[v(y+x)]} \;,\; y \ge -x$$

Now dominating pdf may be selected as

$$g(y) = v_o e^{-[v_o(y+x_o)]} \;, where \;\; x < x_o \;, y \ge -x_o$$

Then

$$\tilde{f}(y,x,v) = f(y,v), \quad for \;\; n = 1,2,\ldots\ldots T-1$$

$$\text{So} \;\; \tilde{f}(y,x,v) = ve^{-[v(y+x)]} \;,\;\; for \;\; n = T$$

By similar process we can estimate the dominating pdf g(y) . The above procedure can be adapted to general Queueing networks under FCFS. Particularly to those in which the distribution of neither L_n nor ξ_n is analytically estimated.

Conclusion

In this paper we have discussed the analysis of $G_1/G_1/1$ Queueing model by Push-Out simulation transformation method.The study of Push-Out technique is the extension of score function method for sensitivity analysis and stochastic optimization. When sojourn and waiting time of any customer in the Queueing system are not estimated analytically, then using the suitable transformation and score function method, we can estimate the waiting time and sojourn time distribution of the customer for the given model in Simulation transformation technique.

References

1. Asmussen.S and Melamed.B. "Regenerating Simulation technique" of TES process. – Appl. Math. 34 (1994) pp 237- 260.
2. Glassemann.P. "Gradient Estimation via pertarbance Analysis – Kulwer , Narwell (1991)
3. Glynn.P.W. "Likelihood ratio on Gradient estimation for Stochastic problem". – Commun. ACM.33(10) (1990) . pp 75- 84.
4. Krimann. V. "Sensitivity Analysis of Queue with respect to buffer size score function method". - Stochastic Models 39(1) (1995) . pp 171 – 194.
5. Pflug. G.C. "Simulation and Optimization " –The Interface between Simulation and Optimization – Kluwer (1993)
6. Raz .T."Efficiency of score function method for Sensitivity Analysis and Optimization in Queueing Networks. – Techin .mion Haifa. Israel. (1995)
7. Rubinstein .R. Y. "Sensitivity Analysis of discrete events system by the Push-Out Simulation. – Oper.Res.39 (1992) pp 229 – 251.