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Application of Structural Properties of Planar Graphs without Small Cycles to the Edge Choosability

Science

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ABSTRACT In this paper, we investigate structural properties of planar graphs without small cycles.

INTRODUCTION

Edge choosability is a more general version of the edge coloring problem. This generalesation supplies a fundamental link between the classical chromatic index of a graph and its other invariants. It has been conjectured that every graph G (Δ(G)+1) is edge choosable. Vizing has proposed this conjecture.

This paper consist three parts, first is introductory. In second part, we state preliminary definitions and result which are needed for final result. Third part is conclusion.

NECESSARY AND RELEVANT RESULTS

Lemma 1. If G is a graph then $\sum_{v\in V(G)} d(v) = 2m,$ where m is the number of edges in E(G).

Lemma 2. If G is a planar graph with n vertices, m edges, f faces and k components then *n-m+f=k+1.*

Lemma 3. If G is a triangle-free connected planar graph with n vertices, m edges and f faces then *m ≤2n-4.*

Lemma 4. The sum of the degrees of all faces of a plane graph G is twice the number of edges in G. That is $\sum_{f\in F(G)} \lambda(f) = 2m,$ where m is the number of edges in E(G).

Lemma 5. Let G be a triangle-free plane graph with Δ (G)≥5. If d(u)+d(v)≥Δ (G)+3 for every edge uv ∈ E(G) , then Δ (G)=5 and G contains a 4-face incident with two 3-vertices and two 5-vertices.

Lemma 6. Let G be a plane connected graph with Δ (G)≥6. If G does not contain 4-cycles, then there is an edge uv ∈E(G) such that d(u)+d(v)≤Δ (G)+2.

Lemma 7. Let G be a planar graph Δ (G)=5. If G does not contain 4-cycles, then there is an edge uv ∈E(G) , such that d(u)+d(v)≤8.

Theorem 1. For any simple graph G,
Δ (G)≤ χ'(G)≤ Δ (G)+ 1.

Theorem 2. Every connected graph G with Δ (G)≤2 is 3-edge-choosable.

Normal weight function. Let G be a graph. A weight function w on V(G)∪F(G) defined by w(v)=d(v)-6 for every vertex v∈V(G) and w(f)=λ(v)-6 for every face f∈F(G) is called the normal weight function. Then we have

$$\sum_{v\in V(G)\cup F(G)} w(x) = -12.$$

CONCLUSION

We conclude that theorem 1 holds for every graph with Δ (G)≤4 and for all planar graphs without 3-cycles and 4-cycles, 6-cycles with Δ (G) ≥6.

The method of discharging weights does not work for proving theorem 1 for planar graphs without graphs 4-cycles and 6-cycles when Δ (G)=5.

REFERENCE

[1.] L. Zang and B. Wu(2004). Edge Choosability of Planar Graphs without Small Cycles. Discrete Math., p289-293.

[2.] W. Weifan and K.W. Lih(2001). Structural properties of planar graphs without 6- Cycle. Combin., Probab. And Comput., p267-276.

[3.] W. Weifan and K.W. Lih(1999). The 4-choosability of planar graphs without 5-cycles.