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CALCULATION OF REDUCED TRANSITION PROBABILITIES B(E2) FOR ROTATIONAL EXCITED GROUND BAND STATES EVEN-EVEN NUCLEI IN LANTHANIDE AND ACTINIDE SERIES

Physics

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ABSTRACT

The measurement of B(E2) values of excited states in nuclei is one of the most active areas of nuclear structure physics. In this work the asymmetric rotor model of Davydov- Filippov (DF) has been employed to study the reduced transition probabilities in the rotational excited ground state band even-even nuclei of lanthanide and actinide series which comprises of 57 nuclides. These E2-transitions raging up to 12^+ spin states transitions have been studied in detailed in the spectra of nuclei whose mass number ranges as 150<=A<=190 and A>=228, and for those the first excited state 2^+ and the second excited state $2^{+'}$ gamma band energies are available. In data calculations experimental and theoretical methods have been used. The best input parameters for the above approaches have been determined. These input parameters includes the energy involved in the transition, the total internal conversion coefficient, the half-life of rotational excited states, the intrinsic quadrupole moment Qo and the nuclear asymmetric parameter γ. The reduced transition probability B(E2) for the above nuclei have been calculated by using the most recent available experimental data. Comparison of results of theoretical calculations with the corresponding experimental data shows a very good agreement, including high angular momentum states. Both the empirical and experimental B(E2) values usually increase with spin for low-lying states within a rotational ground state band. This work has determined particularly the B(E2) values for rotational excited states of which no report has been done so far. So this work has incorporated many nuclides and transitions for which neither experimental nor theoretical values are available.

KEYWORDS:

asymmetric rotor model, reduced transition probabilities, internal conversion coefficient, half-life.

1. INTRODUCTION

The internal structure of the atomic nucleus varies greatly and often suddenly with the number of constituent nucleons (protons and neutrons). These changes in structure are associated with corresponding changes in the nuclear excitation spectrum and in the decay properties of the nuclear excited states. The predominant undertaking of the field of nuclear structure physics is to extract from observed properties of the ground and excited states of the nucleus an under-standing of the physical structure of these states and to develop a comprehensive theoretical de-scription of the nuclear system (Caprio, 2003).

In principle, all the properties of a nucleus are contained in the nuclear Hamiltonian. Un-fortunately, however, we do not know the nuclear Hamiltonian, and even if we did know it, the mathematical difficulties of obtaining its eigenfunctions and eigenvalues would be inseparable. Even if these could be overcome, the amount of information we would obtain would be unman-ageable with the technology in hand (Hodgson, Gadioli & Gadioli, 1997).

The two principle strains of nuclear structure theory were embodied in the shell model and the collective model pioneered by Bohr and Mottelson (Bohr & Mottelson, 1998). In nuclei far from closed shells where the shell model is either intractable or unreliable, one normally takes recourse in other theoretical frameworks. One of the significant and most fruitful of these ap-proaches can be called geometrical or collective models, which bypass the shell model by taking a more macroscopic approach of assigning a specific shape to the nucleus and examining the rota-tions and vibrations of such a shape.

An interesting feature of nuclear structure is nuclear deformation. It has been known for a long time, that heavy nuclei with many valence particles of both kinds tend to take on a static prolate axially symmetric quadrupole deformation in their ground state. Regular rotational exci-tation bands are beautiful evidence of this fact. Excitations and behavior of deformed even-even nuclei can be very successfully described using nuclear collective models.

Further from closed shells, the accumulating p-n interaction strength leads to additional configuration mixing and deviations from spherical symmetry even in the ground state, and so we now turn to consider nuclei with stable and permanent deformations. The lowest applicable shape component is a quadrupole distortion. There can also be octupole and hexadecapole shapes in the nuclear deformation.

Nuclear shape is usually specified in terms of the two nuclear deformation parameters β and γ. The β parameter represents the extent of quadrupole deformation, while γ gives the de-gree of axial asymmetry. Nuclear triaxiality is associated with the breaking of axial symmetry of the quadrupole deformation (Nadirbekov, 2016).

The nuclear deformation parameters β and γ of the collective model are basic description of the nuclear equilibrium shape and structure, while values for these variables have been dis-cussed for many nuclei (Singh, 2011). It has been shown by the study of even-even nuclei in the regions A <=150 and A >=190 that the properties of their excited levels can be accounted for by considering the rotational motion of non-axial (non-axially symmetric) nuclei or nuclear vibra-tions.

A new collective theory of the behavior of nuclei has been developed by Davydov and Filippov (DF) taking into account possible violations of the axial symmetry of the nucleus. This violation affects the rotational spectrum of the axial even nucleus, and some new rotational states with total angular momenta of 2, 3, 4,… appear. A deformed nucleus has a rotational degree of freedom. For even-even nuclei, the 0+ state is always the ground state. The next states with a rotational degree of freedom are $I^\pi = 2^+, 4^+, 6^+, 8^+$,… on symmetry grounds (Pearson, 2008).

Even-even nuclei are known to have 0+ ground states and several low-energy integer spin states. The transition strengths between these levels are sufficiently strong and well established to support the view that most nuclei are collective (Allmond, 2007). When the full and true structure of these nuclei is not known, the structure is clearly dominated by low spin degrees of freedom. Models of adiabatic rotations are by far the most useful in establishing a reference frame in which we can discuss and compare data.

In this study the systematic study of the properties of nuclear rotational excited levels in the deformed even-even nuclei will be considered in the framework of Asymmetric Rotor Model of Davydov and Filippov. Although the assumption of rigid tri-axial shapes with fixed shape parameters β and γ can be considered as an approximation to the actual nuclear wave functions, but this has been turned out to be very useful and is well supported by new data obtained from heavy ion experiments (Davydov & Filippov, 1958).

The aim of the present work is to apply the Davydov-Fillipov model for calculating the values of the reduced electric quadrupole transition probability B(E2) for the rotational excited ground band states even-even nuclei of lanthanide and actinide series. We have employed the model dependent intrinsic quadrupole moment Q_0 in plotting our systematics which reflects the asymmetric spirit of the model in true sense. The striking success of empirical relation over the other existing approaches in describing the B(E2) values of gamma-ray cascades has contradicted the axial symmetry in the nucleus at lower spins.

2. METHODS

2.1. Experimental Method

The experimental transition probabilities in units of $e^2.b^2$ is related to the experimental mean-life by the expression (Raman, Nestor & Tikkaneny, 2001; Varshney, 1982)

$$B(E2:I+2\rightarrow I)exp = \frac{0.08162}{E\gamma 5(1+\alpha T)\tau} \quad (2.1)$$

where Eγ is the energy involved in the transition and is expressed in MeV, τ is the mean-life of the excited state in Ps, and αT are the total internal conversion coefficients calculated using Bricc online software (Australian National University, 2011).

The total internal conversion coefficient has been calculated using online software known as Bricc v2.35 (conversion coefficient calculator), specifically RpIcc was used (Australian Na-tional University, 2011). The internal conversion coefficient is the ratio of the electron emission rate to the gamma emission rate. They are known to depend on four parameters: (1) the charge of the decaying nucleus, (2) the energy of the nuclear transition, (3) the atomic subshell out of which the orbital electron is ejected and finally, (4) the multi-polarity and parity of the nuclear transition.

Mean-life of states can be calculated from half-life using the known relations T1/2 =ln2/λ and τ=1/λ (Marton & Marton, 1963), where λ is the decay constant. From these relations we ob-tained τ = T1/2/ln2 or T1/2 = ln2* τ

(2.2)

Using these relations, we can rewrite the experimental transition probability as

$$B(E2:I+2\rightarrow I)exp = \frac{0.0566}{E\gamma 5(1+\alpha T)T1/2} \quad (2.3)$$

This experimental formula has been derived to incorporate the half-life rather than the mean-life of rotational excited states to compute the values of B(E2).

1.1. Theoretical Method

Davydov and Filippov axially asymmetric model has been found quite suitable (Allmond, 2007; Davydov, 1958; Varshney, 2009) in explaining the rotational levels of the deformed even-even nuclei, the large electric quadrupole moments and the transition probabilities.

The electric quadrupole transition probabilities for transitions inside the ground rotational band between two states of spin I and I' are described by the following empirical formula (Varshney, 1982):

$$B(E2:I\rightarrow I')emp = \frac{5e^2Q_0^2}{32\Pi}(2I00|I'0)^2\{1+\frac{3-2\sin^2(3\gamma)}{[9-8\sin^2(3\gamma)]^{1/2}}\} \quad (2.4)$$

Where $(2I00|I'0)$ are Clebsch-Gordon coefficients in the notation (2Jmm'|J'm').

For transition between two states of ground rotational band, the Clebsch-Gordon coefficients have the form $(2I0|I'0)^2 = \frac{3}{2}\frac{(I+1)(I+2)}{(2I+3)(2I+5)}$ in decay or de-excitation from I+2→I. Hence

$$B(E2:I+2\rightarrow I)emp = \frac{15e^2Q_0^2}{64\Pi}\frac{(I+1)(I+2)}{(2I+3)(2I+5)}\{1+\frac{3-2\sin^2(3\gamma)}{[9-8\sin^2(3\gamma)]^{1/2}}\} \quad (2.5)$$

In this empirical formula we have used the intrinsic quadrupole moment given by

$$Qo = \frac{3}{\sqrt{5\pi}}ZR_0^2\beta(1+0.16\beta) \text{ to second order in } \beta. \quad (2.6)$$

If the deformation can be characterized by a single parameter β, then Qo is approximately given by (Heyde, 1999)

$$Qo = \frac{3}{\sqrt{5\pi}}ZR_0^2\beta, \text{ (where } R_0^2 = 0.0144A^{2/3}\text{b)} \quad (2.7)$$

The intrinsic quadrupole moment in b can also be calculated using the relation

$$Qo = \left[\frac{16\Pi}{5}\frac{B(E2)\uparrow}{e^2}\right]^{1/2} \quad (2.8)$$

Where B(E2)↑is the reduced electric quadrupole moment transition probability between the 0^+ ground state and the first 2^+ state in even-even nuclides. The B(E2)↑values are basic experimental quantities that do not depend on nuclear models.

In the frame work of the rigid tri-axial rotor model (Varshney, 1982), deformation param-eters β and γ are extraxted from both level energies and E2 transition rates in even-even nuclei. The rigid tri-axial rotor model considers the nucleus as a rigid rotor with rigid tri-axial asymmetry as specified by β and γ.

In this research we have employed the most widely used method to calculate the asym-metry parameter γ which is used in empirical calculation. The asymmetry parameter is evaluated from the ratio of two band head energies E2+'/E2+ (Singh et al, 2013; Varshney et al, 2011),

where E2+'=

$$\frac{9}{sin^2(3\gamma)}\left[1+\sqrt{1-\frac{8sin^2(3\gamma)}{9}}\right]\frac{\hbar^2}{4B\beta^2} \text{ and } E2^+ = \frac{9}{sin^2(3\gamma)}\left[1-\sqrt{1-\frac{8sin^2(3\gamma)}{9}}\right]\frac{\hbar^2}{4B\beta^2} \quad (2.9)$$

So that we can simplify the E2+'/E2+=

$$\frac{\left[1+\sqrt{1-\frac{8sin^2(3\gamma)}{9}}\right]}{\left[1-\sqrt{1-\frac{8sin^2(3\gamma)}{9}}\right]} = \frac{1+X}{1-X}, \text{ where } X = \sqrt{1-\frac{8sin^2(3\gamma)}{9}}$$

Further simplifying the ratio we can write for the asymmetric parameter as

$$\gamma = \frac{1}{3}\sin^{-1}\left\{\frac{9}{8}\left[1-\frac{(m-1)^2}{(m+1)^2}\right]\right\}^{1/2} \quad (2.10)$$

Where m is the energy ratio E2+'/E2+.

3. RESULT

In this section the B(E2) values computed have been tabulated and figured. Table 1 give B(E2) values for transition from rotational level 2^+ to 0^+ ground state rotational band while Table 2 give values for transition between rotational states of 0^+, 2^+, 4^+, 6^+, 8^+, 10^+ and 12^+ for only se-lected nuclides where reference data is available to compare with.

The experimental values either extracted from the half-life data using the derived relation (Equation 2.3) or taken from references (Raman, Nestor & Tikkaneny, 2001) and (Varshney, 1982) are presented along with the calculated values obtained from the DF model.

Table 1: Reduced Transition Probabilities B(E2), Intrinsic Quad rupole Moment (Qo), Nuclear De-formation Parameters and Total Internal Conversion Coefficient (ICC) for $2^+\rightarrow 0+$ Transition.

S/No	Nucleus	Experimental *	Present Work		Previous Work **		Qo *	Deformation Parameters		ICC
			Experimental	Empirical	Experimental	Empirical		β *	γ	
1.	$^{150}_{62}Sm$	0.270 (30)	0.271	0.252	0.274 (6)	0.265	3.684	0.1931	20.42	0.0408
2.	$^{152}_{62}Sm$	0.692 (6)	0.696	0.662	0.670 (15)	0.657	5.90	0.3061	13.23	1.17
3.	$^{154}_{62}Sm$	0.872 (5)	0.854	0.850	0.922 (40)	0.911	6.620	0.3410	9.54	4.93
4.	$^{152}_{64}Gd$	0.334 (14)	0.347	0.311	0.394 (26)	0.368	4.09	0.206	19.76	0.0399
5.	$^{154}_{64}Gd$	0.778 (7)	0.768	0.743	0.770 (16)	0.760	6.25	0.3120	13.86	1.20
6.	$^{156}_{64}Gd$	0.928 (5)	0.934	0.910	0.914 (10)	0.918	6.830	0.3378	11.05	3.92
7.	$^{158}_{64}Gd$	1.004 (5)	1.010	0.980	0.994 (10)	0.948	7.104	0.3484	10.32	6.00
8.	$^{160}_{64}Gd$	1.050 (6)	1.035	1.020	1.030 (10)	1.015	7.265	0.3534	10.98	7.42

9.	$^{154}_{66}Dy$	0.478 (13)	0.476	0.447			4.90	0.237	21.98	0.0467
10.	$^{156}_{66}Dy$	0.742 (40)	0.748	0.702	0.753		6.107	0.2929	15.42	0.85
11.	$^{158}_{66}Dy$	0.932 (5)	0.942	0.895	0.980 (70)	0.969	6.844	0.3255	12.79	2.85
12.	$^{160}_{66}Dy$	1.026 (11)	0.979	0.991	0.998 (60)	0.961	7.18	0.3387	11.89	4.68
13.	$^{162}_{66}Dy$	1.070 (11)	1.045	1.030	1.089 (30)	1.061	7.33	0.3430	11.96	6.21
14.	$^{164}_{66}Dy$	1.120 (5)	1.125	1.080	1.140	1.095	7.503	0.3481	12.30	9.00
15.	$^{160}_{68}Er$	0.876 (20)	0.865	0.830	0.840 (40)	0.795	6.63	0.304	15.07	1.27
16.	$^{162}_{68}Er$	1.002 (6)	1.175	0.960	0.976 (49)	0.933	7.097	0.3222	13.3	2.76
17.	$^{164}_{68}Er$	1.090 (6)	1.174	1.044	1.150 (70)	1.135	7.402	0.3333	12.9	4.21
18.	$^{166}_{68}Er$	1.166 (5)	1.177	1.120	1.122 (40)	1.076	7.656	0.3420	12.68	6.89
19.	$^{168}_{68}Er$	1.158 (10)	1.157	1.114	1.170 (40)	1.150	7.63	0.3381	12.36	7.16
20.	$^{170}_{68}Er$	1.164 (10)	1.176	1.124	1.185 (30)	1.160	7.65	0.3363	11.53	7.62
21.	$^{166}_{70}Yb$	1.048 (31)	1.033	1.001	1.05 (150)	1.010	7.25	0.315	13.09	2.98
22.	$^{168}_{70}Yb$	1.116 (30)	1.167	1.105	1.080 (50)	1.050	7.49	0.322	11.85	5.44
23.	$^{170}_{70}Yb$	1.158 (13)	1.141	1.130	1.040 (20)	1.010	7.63	0.3258	10.80	6.39
24.	$^{172}_{70}Yb$	1.208 (7)	1.225	1.180	1.186 (56)	1.160	7.792	0.3302	9.27	8.40
25.	$^{174}_{70}Yb$	1.188 (6)	1.174	1.170	1.148	1.130	7.727	0.3249	8.66	9.50
26.	$^{176}_{70}Yb$	1.060 (19)	1.075	1.030	1.050	1.030	7.30	0.305	10.18	7.08
27.	$^{166}_{72}Hf$	0.700 (20)	0.691	0.660	0.86		5.93	0.250	17.23	0.646
28.	$^{168}_{72}Hf$	0.860 (23)	0.850	0.815	0.836		6.57	0.275	14.79	1.57
29.	$^{170}_{72}Hf$	1.060 (12)	1.025	1.020	1.26		7.3	0.301	12.81	3.47
30.	$^{172}_{72}Hf$	0.894 (33)	0.872	0.862	0.928		6.70	0.276	11.81	4.35
31.	$^{174}_{72}Hf$	0.976 (31)	0.892	0.950	0.935 (44)	0.932	7.00	0.286	10.85	5.22
32.	$^{176}_{72}Hf$	1.054 (10)	1.070	1.025	1.215 (35)	1.20	7.28	0.2953	10.24	5.87
33.	$^{178}_{72}Hf$	0.964 (6)	0.948	0.933	0.998	0.980	6.961	0.2803	11.20	4.74
34.	$^{182}_{74}W$	0.840 (8)	0.829	0.812	1.029 (11)	1.02	6.50	0.2503	11.38	3.96
35.	$^{184}_{74}W$	0.756 (13)	0.737	0.720	0.667 (12)	0.660	6.16	0.2362	13.84	2.57
36.	$^{186}_{74}W$	0.700 (12)	0.716	0.660	0.669 (9)	0.660	5.93	0.2257	15.94	1.784
37.	$^{180}_{76}Os$	0.720 (8)	0.711	0.680			6.0	0.226	15.29	1.456
38.	$^{182}_{76}Os$	0.772 (35)	0.780	0.730	1.702		6.22	0.234	14.84	1.700
39.	$^{184}_{76}Os$	0.646 (16)	0.620	0.616	0.569		5.70	0.213	14.05	2.13
40.	$^{186}_{76}Os$	0.580 (10)	0.587	0.550	0.63 (6)	0.630	5.40	0.2000	16.51	1.271
41.	$^{188}_{76}Os$	0.510 (5)	0.492	0.460	0.568 (6)	0.580	5.06	0.1862	19.16	0.810
42.	$^{190}_{76}Os$	0.470 (6)	0.484	0.442	0.496 (4)	0.510	4.86	0.1775	22.28	0.420
43.	$^{228}_{90}Th$	1.412 (24)	1.410	1.380	1.41		8.42	0.2301	9.76	153.2
44.	$^{230}_{90}Th$	1.608 (10)	1.638	1.560	1.60 (400)	1.57	8.99	0.2441	10.40	228
45.	$^{232}_{90}Th$	1.856 (10)	1.705	1.810	1.850 (80)	1.82	9.66	0.2608	10.01	327
46.	$^{234}_{92}U$	2.132 (20)	2.021	2.087	2.030 (90)	1.98	10.35	0.2718	8.68	713
47.	$^{236}_{92}U$	2.322 (15)	2.160	2.276	2.320 (80)	2.27	10.80	0.2821	8.71	589
48.	$^{238}_{92}U$	2.418 (20)	2.498	2.370	2.380 (50)	2.33	11.02	0.2863	8.25	610
49.	$^{238}_{94}Pu$	2.522 (17)	2.438	2.475	2.520		11.26	0.2861	8.30	787
50.	$^{240}_{94}Pu$	2.604 (30)	2.645	2.550	2.530 (70)	2.47	11.44	0.2891	8.23	905
51.	$^{242}_{94}Pu$	2.680 (16)	2.728	2.630	2.68 (110)	2.63	11.61	0.2917	8.07	748
52.	$^{244}_{94}Pu$	2.736 (16)	2.766	2.684	2.770 (70)	2.71	11.73	0.2931	8.53	640
53.	$^{244}_{96}Cm$	2.934 (17)	3.791	2.882			12.14	0.2972	7.84	1050
54.	$^{246}_{96}Cm$	2.988 (19)	3.040	2.930	3.010 (90)	2.94	12.26	0.2983	8.23	1064
55.	$^{248}_{96}Cm$	2.998 (19)	3.036	2.950		2.98	12.28	0.2972	7.88	1002
56.	$^{250}_{98}Cf$	3.200 (16)	3.183	3.150			12.7	0.299	8.16	1274
57.	$^{252}_{98}Cf$	3.340 (11)	3.355	3.260			12.95	0.304	9.58	917

* Taken from adopted value of Reference Raman, S., Nestor, C. W., J. R., and Tikkanen, P. (2001).

** Taken from Reference Varshney, V. P. (1982).

Table 2: Values of Reduced Transition Probability B(E2) of Various Transitions of Present (Experimental and Empirical), and Previous (Experimental and Empirical) Data.

S/No.	Nucleus	Transition	Present Work		Previous Work *	
			Experimental	Empirical	Experimental	Empirical
1.	$^{152}_{62}Sm$	2+→0+	0.696	0.662	0.670 (15)	0.657
		4+→2+	1.010	0.946	1.03 (1)	0.938
		6+→4+	1.181	1.042	1.16 (2)	1.00
		8+→6+	1.372	1.100	1.41 (7)	1.19
		10+→8+	1.525	1.120	1.54 (14)	1.78
		12+→10+		1.140		
2.	$^{154}_{62}Sm$	2+→0+	0.854	0.850	0.922 (40)	0.911
		4+→2+	1.199	1.215	1.18 (3)	1.30
		6+→4+	1.428	1.340	1.31 (4)	1.30
		8+→6+	1.559	1.400	1.54 (15)	1.56
		10+→8+	1.539	1.440		
		12+→10+	1.381	1.464		
3.	$^{154}_{64}Gd$	2+→0+	0.768	0.743	0.770 (16)	0.760
		4+→2+	1.204	1.061	1.18 (4)	1.086
		6+→4+	1.395	1.170	1.36 (5)	1.17
		8+→6+	1.529	1.223	1.50 (5)	1.40
		10+→8+	1.749	1.260	1.74	
		12+→10+		1.280		
4.	$^{156}_{64}Gd$	2+→0+	0.934	0.910	0.915 (10)	0.918
		4+→2+	1.316	1.293	1.29 (2)	1.32
		6+→4+	1.470	1.424	1.47 (3)	1.36
		8+→6+	1.598	1.491	1.61 (11)	1.65
		10+→8+	1.568	1.532	1.56	
		12+→10+	1.495	1.560		

5.	$^{158}_{64}$Gd	2+→0+	1.010	0.980	0.994 (10)	0.948
		4+→2+	1.470	1.400	1.37 (11)	1.361
		6+→4+		1.540		
		8+→6+	1.648	1.610	1.69 (13)	1.66
		10+→8+	1.699	1.650	1.77 (9)	1.73
		12+ →10+	1.556	1.681	1.57 (12)	1.50
6.	$^{160}_{66}$Dy	2+→0+	0.999	0.991	0.998 (60)	0.961
		4+→2+	1.483	1.420	1.49 (13)	1.40
		6+→4+	1.228	1.560	1.23 (6)	1.46
		8+→6+	1.874	1.632	1.57 (10)	1.74
		10+→8+	1.695	1.680	1.65 (15)	1.77
		12+ →10+	1.607	1.710	1.29 (79)	1.60
7.	$^{162}_{66}$Dy	2+→0+	1.045	1.030	1.089 (30)	1.061
		4+→2+	1.513	1.470	1.53 (9)	1.52
		6+→4+	1.577	1.620	1.38 (16)	1.59
		8+→6+	1.818	1.700	1.91 (8)	1.89
		10+→8+	1.835	1.741	1.79 (11)	1.93
		12+ →10+	1.705	1.772	1.48 (8)	1.47
8.	$^{164}_{66}$Dy	2+→0+	1.125	1.080	1.140	1.095
		4+→2+	1.446	1.540	1.57	1.622
		6+→4+	1.618	1.700	1.67 (6)	1.697
		8+→6+	1.602	1.771	1.69 (12)	2.02
		10+→8+	1.899	1.820	1.89 (10)	2.05
		12+ →10+	1.768	1.850	1.87 (3)	1.86
9.		2+→0+	0.865	.0830	0.840 (40)	0.795
		4+→2+	1.237	1.184	1.17 (6)	1.14
		6+→4+	1.352	1.304	1.36 (12)	1.25
		8+→6+	1.523	1.365	1.17 (28)	1.49
		10+→8+	1.508	1.400	1.09 (55)	1.43
		12+ →10+	1.480	1.430		
10.	$^{160}_{68}$Er	2+→0+	1.174	1.044	1.150 (70)	1.135
		4+→2+	1.383	1.500	1.40 (13)	1.62
		6+→4+		1.640		
		8+→6+	1.831	1.720	1.83 (13)	2.06
		10+→8+	1.903	1.770	2.97 (12)	2.05
		12+ →10+	1.411	1.800	1.20 (9)	1.84
11.	$^{164}_{68}$Er	2+→0+	1.177	1.120	1.122 (40)	1.076
		4+→2+	1.690	1.600	1.69 (11)	1.57
		6+→4+	1.465	1.760	1.60	1.66
		8+→6+	1.982	1.850	1.85 (11)	1.98
		10+→8+	2.009	1.900	2.10 (2)	1.99
		12+ →10+	2.040	1.930	1.97 (15)	1.79
12.	$^{170}_{68}$Er	2+→0+	1.176	1.124	1.185 (30)	1.160
		4+→2+		1.61	1.55 (4)	1.66
		6+→4+		1.770		
		8+→6+	2.075	1.850	2.21 (16)	2.06
		10+→8+	1.790	1.902	1.83 (12)	2.11
		12+ →10+	2.097	1.940	2.12 (10)	1.92
13.	$^{166}_{70}$Yb	2+→0+	1.033	1.001	1.060 (150)	1.010
		4+→2+	1.475	1.430	1.47 (5)	1.45
		6+→4+	1.575	1.580	1.57 (6)	1.55
		8+→6+	1.747	1.650	1.75 (2)	1.84
		10+→8+	1.650	1.700	1.65 (109)	1.84
		12+ →10+	1.451	1.724	1.46 (102)	1.64
14.	$^{166}_{72}$Hf	2+→0+	0.691	0.660	0.86	
		4+→2+	1.062	0.940	1.10 (7)	1.165
		6+→4+	1.110	1.040	1.14	1.31
		8+→6+	1.340	1.084	1.38 (56)	1.568
		10+→8+	1.370	1.113	1.42 (99)	1.44
		12+ →10+	0.840	1.133	0.84 (64)	1.26
15.	$^{168}_{72}$Hf	2+→0+	0.850	0.815	0.836	
		4+→2+	1.145	1.164	1.14 (13)	1.138
		6+→4+	1.304	1.280	1.28 (13)	1.24
		8+→6+	1.390	1.340	1.42 (15)	1.47
		10+→8+	1.421	1.380	1.50 (21)	1.42
		12+ →10+	1.784	1.400	1.59 (4)	1.27

* Taken from adopted value of Reference Raman, S., Nestor, C. W., J. R., and Tikkanen, P. (2001).

In the figures below we would like to show the relation between the input parameters employed to determine the experimental B(E2) values.

Figure 1 illustrates the relation between the energy involved in the transition, the total internal conversion coefficient and half-life of the transitions for selected nuclides of lanthanide series, while Figure 2 demonstrates the energy involved in the transition, the total internal conversion coefficient and the half-life of the transition for selected nuclides of actinide series.

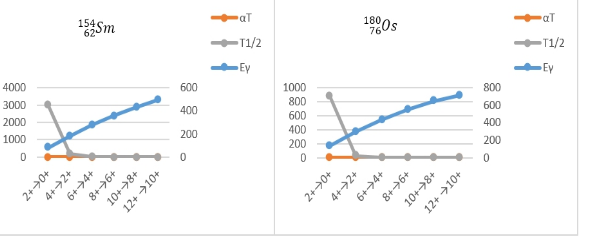

Figure 1. Comparison of Energy Involved in the Transition (Eγ), the Total Internal Conversion Coefficient (αT) and Half-Life (T1/2) of the Transitions for Lanthanide Nuclides.

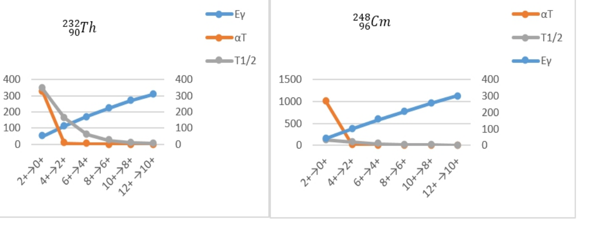

Figure 2. Comparison of Energy Involved in the Transition (Eγ), the Total Internal Conversion Coefficient (αT) and Half-Life (T1/2) of the Transitions for Actinide Nuclides.

4. DISCUSSION

In this study half-life rather than mean-life has been employed in computing the experimental reduced transition probabilities. This is because usually half-life of states will be provided in literature. We have never seen any report of B(E2) values employing half-life, so it can be considered a new approach in estimating the transition probabilities. And also this formula has given due emphasis of significant figures to make the obtained result as much accurate as possible. So the result obtained in this work could be compared to the previous works which employ mean-life instead of half-life based on the fact that it is possible to convert mean-life to half-life and vice versa.

And the other point that makes this work peculiar is that an updated experimental data of energies and half-life have been employed. This may also bring difference when compared to the calculated values of previous works. So we can say that this work may not be accurately compared to others work because the differences may arise due to the mentioned reasons. There is no experimental data reported for higher transitions than $2^+\rightarrow0^+$ transition to compare with the current computed data except the report of (Varshney, 1982).

The experimental B(E2) values are dependent inversely upon the transition gamma energy to the power of five, the total internal conversion coefficient and half-life of the rotational excited state (See Equation 2.3). It is easily observed that the total internal conversion coefficient is inversely proportional to the transition gamma energy (See Fig.1 &2). This is because the coefficient decreases as the gamma energy get increased, keeping the atomic number same. This could be easily verified using the calculated values of Bricc online software. It is also possible to see from experimental half-life of the rotational states that the half-life get decreases while the transition gamma energy increases as the nuclear spin increases.

It is easy to observe that the limiting factors of the experimental values are the parameters in the denominator since the numerator is a constant number. It has been seen that the experimental B(E2) values are increasing until the transition $6^+\rightarrow4^+$ for most cases and decreases for higher transitions. This may be ascribed to the difficulty in determining the ICC values for transition gamma energy greater than 400 KeV using the Bricc software. The transition gamma energy get increases for the higher transitions and are observed to be greater than the mentioned value especially for the rare earth series.

The ICC values are also known to depend upon the charge of decaying nucleus, the energy of the transition, the atomic subshell out of which the orbital electron is ejected and the multi-polarity and parity of the nuclear transition. So it is important to take in to consideration these all in order to have a full description of the transition magnitude.

In the case of the theoretical (empirical) calculations it can be seen from the corresponding formula that the B(E2) values are proportional to the square of the intrinsic quadrupole moment (Qo). This in turn directly proportional to the elongation deformation parameter β. So in these empirical calculations both the asymmetric parameter γ and the elongation parameter β have been considered. Actually these parameter are so important in determining the actual nuclear deformation properties.

In this empirical calculation we do observe that the first factor consisting Qo and the factor in parenthesis remain constant (See Equation 2.5) for a nuclide under consideration since the Qo and γ parameters are constant regardless of the transition for a specific nuclide. So the limiting factor for different transitions is the nuclear spin dependent part. The empirical values are seen getting increased as the nuclear spin I increases (See Table 2). The difference in this increase between successive transitions will actually decrease. So we do observe that the empirical values of B(E2) increases as transition state increase.

We also observe that for the nuclides with the same atomic number (isotopes) the asymmetric parameter increases as the mass number increases when excitation energy (2^+) get increase (for most isotopes) even though the second excitation ($2^{+'}$) also do have power to determine the value (See Table 1). So we see that the empirical value of B(E2) increases (slowly) as the nuclear spin for transition states increases. For higher transition states this increase get much slower in opposition to the decrease in experimental values.

This present study reveals by compare the percentage difference between experimental result and present experimental result obtained that it is less than 9% except for $^{162}_{68}$Er (17.27%) and for $^{244}_{96}$ (29.2%). For most nuclides the value lies below 3%. When the empirical result is compared to the experimental result we obtain less than 10% and for most case the value lies below 4%. Finally the compared result between present empirical result and the previous empirical result is found to be less than 18% except for $^{188}_{76}$ (26.1%), and for most cases it is less than 5%. This comparison show that the present study is in good agreement with the experimental data and the reported previous works.

It is possible to say that at higher spin transitions the experimentally computed B(E2) values seems exact for actinide series. This is because the excitation energy of transitions will not exceed the limiting value of Bricc software so that the computed ICC values are more exact when compared to the rare earth ones at higher spin transitions.

5. CONCLUSION

The investigation of the B(E2) values for the ground state band of the even-even deformed nuclei in the rare earth and actinide regions have been considered. This investigation reveals that the DR predictions are closer to the experimental values for higher transitions, thus giving support to the idea that the nuclear shape tends to become triaxial with higher angular momentum.

So it is possible to say that from the result obtained there is a strong agreement between the experimental and empirical values in the lower transitions than higher transitions both in the experimental versus experimental, experimental versus empirical and empirical versus empirical for the present work and the previous works considered in this study.

The B(E2) values usually increase with spin for low-lying states within a rotational band. For lower transitions, especially for $2^+ \rightarrow 0^+$, the result obtained is in good agreement with the existing experimental data and the reported works so far. So it is possible to infer the result obtained for the transitions could also be a go for result. And if this is the case the theoretically (empirically) obtained results can be used to estimate the half-life or mean-life of rotational excited states of which neither the mean-life nor half-life has been experimentally not reported yet.

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