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FUZZY LINEAR FRACTIONAL PROGRAMMING PROBLEM USING FOURIER MOTZKIN ELIMINATION

Mathematics

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ABSTRACT

Linear Fractional programming problem is an optimization problem in which the objective is to maximize the fraction of two linear functions subject to a set of linear constraints. A new approach called Fourier Motzkin Elimination Method is used to solve the fuzzy linear fractional programming problem (FLFPP). In this paper, FLFPP is converted into crisp LFPP. Then it is transformed into a multi objective linear programming problem and the objective functions are then changed into linear inequalities. Fourier Motzkin elimination is used to solve the system of linear inequalities. A numerical example is given to show the efficiency of the proposed method.

KEYWORDS:

Fuzzy linear fractional programming problem, multi objective linear programming problem, trapezoidal fuzzy number, Fourier Motzkin elimination.

1. INTRODUCTION

Many researchers suggested various approaches to solve the linear fractional programming problem since it has been applied to various fields such as finance, healthcare, hospitals and production planning. Geir Dahl [2] presented a paper on combinatorial properties of Fourier-Motzkin Elimination. Juraj Stacho [3] suggested Fourier Elimination method to solve linear programming problems. Kanniappan and Thangavel [4] used modified Fourier eliminationation method for solving linear programming problems. Macro Chirandini [5] used Fourier Motzkin elimination to solve Linear programming problem. Pandian and Natarajan [6] proposed Modified Fourier Elimination Method for finding an optimal solution of transportation problems with mixed constraints. Partha Pratim Dey and Surapati Pramanik [7] proposed TOPSIS approach to solve linear fractional bi-level multi objective decision making problem. O'Donnell [8] solved 3-variable 5-constranit linear programming problem using Fourier Motzkin elimination method. Sohana Jahan and Ainul Islam [9] proved complementary slackness theorem for linear fractional programming problem. K. Subramani [10] used Fourier Motzkin elimination to solve the linear programming problem. Ryan H.P. Williams [13] used Fourier Motzkin Elimination to solve Integer Programming problems. Ranking technique plays an important role in decision making. Yager's ranking method [15] is used for ranking of fuzzy numbers.

The paper is organized as follows. Section 2 gives the preliminaries of fuzzy numbers. In section 3, mathematical formulation of the FLFPP is given.. Fourier Motzkin elimination and the ranking of fuzzy numbers are given in Section 4. Numerical example is given in Section 5 and the conclusion is given in Section 6.

2. Preliminaries

Fuzzy Number: A fuzzy number is a fuzzy set $\widetilde{A}=\{x,\mu_A(x)\}$ where $\mu_A : X \to [0,1]$ and satisfies

(i) μ_A is piecewise continuous

(ii) There is at least one element $x \in X$ such that $\mu_A(x)=1$.

(iii) A is convex, that is, $\mu_A(\lambda x+(1-\lambda)y) \geq \min\{\mu_A(x),\mu_A(y)\}$ if for any $x, y \in X$ and any $\lambda \in [0,1]$

TrapezoidalFuzzy Number: A trapezoidal fuzzy number $\widetilde{A}^T=\{a_1,a_2,a_3,a_4\}$ where a_1,a_2,a_3,a_4 are real numbers and its membership is given below

$$\mu_{\widetilde{A}^T}=\begin{cases} 0, & for \quad x<a_1 \\ \left(\dfrac{x-a_1}{a_2-a_1}\right), & for\ a_1 \leq x \leq a_2 \\ 1, & for\ a_2 \leq x \leq a_3 \\ \left(\dfrac{a_4-x}{a_4-a_3}\right), & for\ a_3 \leq x \leq a_4 \\ 0, & for\ x>a_4 \end{cases}$$

3. Mathematical formulationof fuzzy problem

Maximize $\tilde{Z} = \dfrac{\tilde{c}^T x + \tilde{\alpha}}{\tilde{d}^T x + \tilde{\beta}}$

Subject to

$$\tilde{A}x \le \tilde{b}$$
$$x \ge 0 \qquad (1)$$

where x is an n-dimensional vector of decision variables, and c, d are $n \times 1$ vectors, A is an $m \times n$ constraint fuzzy matrix, b is an m- dimensional fuzzy vector, $\tilde{\alpha}$ and $\tilde{\beta}$ are scalars.

4. Solution Procedure

The problem given in (1) is converted into crisp LFPP using Yager's ranking index as below.

Maximize $Z = \dfrac{c^T x + \alpha}{d^T x + \beta}$

Subject to

$$Ax \le b$$
$$x \ge 0 \qquad (2)$$

The problem given in (2) can be written as the following multi objective LPP.

Maximize $z_1 = c^T x + \alpha$

Minimize $z_2 = d^T x + \beta$

Subject to

$$Ax \le b$$
$$x \ge 0$$

Since

$z_1 \le Maximize z_1$ and $z_2 \ge Minimize z_2$

The above problem can again be written as

Maximize Z

Subject to

$$z_1 - \left(c^T x + \alpha\right) \le 0$$
$$d^T x + \beta - z_2 \le 0$$
$$Ax \le b$$
$$x \ge 0$$

4.1 Fourier MotzkinElimination

Step 1 Since the objective function is to be maximized, convert all the inequalities into ≤ type.

Step 2 To eliminate x_1, add each of the inequalities having positive coefficient of x_1 with each of the inequalities having negative coefficient of x_1 by making the numerical value of the coefficients of x_1 same. Addition of every pair of such inequalities generates a new inequality without x_1. Now we get the new system of linear inequalities without x_1.

Step 3 Repeat the same process for eliminating the variables $x_2, x_3, \ldots, z_1$ until only z_2 remains to be eliminated. Redundant inequalities can be omitted.

Step 4 Find the greatest value of z_1 and the smallest value of z_2 that satisfy all the resulting inequalities.

Step 5 Substitute the values of z_1 and z_2 in the reverse order so that the values of eliminated variables can be obtained.

4.2. Ranking of Trapezoida Fuzzy numbers

Given a convex trapezoidal fuzzy number $\tilde{C} = \{a_1, a_2, a_3, a_4\}$, the α -cut of the fuzzy number $\tilde{C}$ is given by

$$\left(C_\alpha{}^L, C_\alpha{}^U\right) = \left(\left(a_2 - a_1\right)\alpha + a_1, a_4 - \left(a_4 - a_3\right)\alpha\right)$$

The Yager's Ranking index is defined by

$$R(\tilde{C}) = \int_0^1 0.5\left(C_\alpha{}^L + C_\alpha{}^U\right)d\alpha,$$

where $\left(C_\alpha{}^L, C_\alpha{}^U\right)$ is a α -level cut of fuzzy number $\tilde{C}$.

5. Numerical Example

Maximize

$$Z = \frac{(1,2,4,5)x_1 + (1,4,6,9)x_2 + (0,1,3,4)x_3 - (1,1,1,1)}{(1,1,1,1)x_1 + (0,2,4,6)x_2 + (-1,0,2,3)x_3 + (3,4,6,7)}$$

Subject to the constraints

$(0,2,4,6)x_1 + (-1,1,3,5)x_2 + (1,1,1,1)x_3 \le (-2,1,3,6)$

$(-1,0,2,3)x_1 + (2,3,5,6)x_2 + (1,1,1,1)x_3 \le (-2,0,2,4)$

$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$

$$R(1,2,4,5) = \int_0^1 0.5((2-1)\alpha + 1 + 5 - (5-4)\alpha)d\alpha = 3$$

$$R(1,4,6,9) = \int 0.5((4-1)\alpha + 1 + 9 - (9-6)\alpha) = 5$$

Proceeding in this way, the above fuzzy linear fractional programming problem can be written as the following crisp linear fractional programming problem.

Maximize $Z = \dfrac{3x_1 + 5x_2 + 2x_3 - 1}{x_1 + 3x_2 + x_3 + 5}$

Subject to

$3x_1 + 2x_2 + x_3 \le 2$

$x_1 + 4x_2 + x_3 \le 1$

$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$

The given problem can be written as the following multi objective linear programming problem.

$Maximize z_1 = 3x_1 + 5x_2 + 2x_3 - 1$

$Minimize z_2 = x_1 + 3x_2 + x_3 + 5$

Subject to

$3x_1 + 2x_2 + x_3 \le 2$

$x_1 + 4x_2 + x_3 \le 1$

$x_1 \ge 0;\ x_2 \ge 0;\ x_3 \ge 0$

Change the objective functions into constraints and all the constraints are of ≤ type. The problem becomes as below.

Maximize Z

Subject to

$-3x_1 - 5x_2 - 2x_3 + z_1 \le -1$

$x_1 + 3x_2 + x_3 - z_2 \le -5$

$3x_1 + 2x_2 + x_3 \le 2$

$x_1 + 4x_2 + x_3 \le 1$

$-x_1 \le 0; -x_2 \le 0; -x_3 \le 0$

To eliminate x_1, add each of the inequalities having positive coefficient of x_1 with each of the inequalities having negative coefficient of x_1 by making numerical value of the coefficients of x_1 same. A new system of linear inequalities without x_1 is obtained.

$4x_2 + x_3 + z_1 - 3z_2 \le -16$

$-3x_2 - x_3 + z_1 \le 1$

$7x_2 + x_3 + z_1 \le 2$

$3x_2 + x_3 - z_2 \le -5$

$2x_2 + x_3 \le 2$

$4x_2 + x_3 \le 1$

$-x_2 \le 0$

$-x_3 \le 0$

To eliminate x_2, add each of the inequalities having positive coefficient of x_2 with each of the inequalities having negative coefficient of x_2 by making numerical value of the coefficients of x_2 same. A new system of linear inequalities without x_2 is obtained.

$-x_3 + 7z_1 - 9z_2 \le -44$

$-4x_3 + 10z_1 \le 13$

$z_1 - z_2 \le -4$

$x_3 + 2z_1 \le 8$

$-x_3 + 4z_1 \le 7$

$x_3 + z_1 - 3z_2 \le -16$

$x_3 + z_1 - 3z_2 \le -16 x_3 + z_1 \le 2$

$x_3 - z_2 \le -5$

$x_3 \le 1;\ -x_3 \le 0$

To eliminate x_3, add each of the inequalities having positive coefficient of x_3 with each of the inequalities having negative coefficient of x_3 by making numerical value of the coefficients of x_3 same. A new system of linear inequalities without x_3 is obtained.

$9z_1 - 9z_2 \le -36 \Rightarrow z_1 - z_2 \le -4$

$18z_1 \le 45 \Rightarrow z_1 \le 2.5$

$6z_1 \le 15 \Rightarrow z_1 \le 2.5$

$2z_1 \le 8 \Rightarrow z_1 \le 4$

$18z_1 \le 45 \Rightarrow z_1 \le 2.5$

$8z_1 - 12z_2 \le -60 \Rightarrow 2z_1 - 3z_2 \le -15$

$14z_1 - 12z_2 \le -51$

$5z_1 - 3z_2 \le -9$

$z_1 - 3z_2 \le -16$

$8z_1 - 9z_2 \le -42$

$14z_1 \le 21 \Rightarrow z_1 \le 1.5$

$3z_1 \le 10 \Rightarrow z_1 \le 3.3$

$5z_1 \le 9 \Rightarrow z_1 \le 1.8$

$z_1 \le 2$

$7z_1 - 10z_2 \le -49$

$10z_1 - 4z_2 \le -7$

$4z_1 - z_2 \le 2$

$-z_2 \le -5 \Rightarrow z_2 \ge 5$

$7z_1 - 9z_2 \le -43$

$10z_1 \le 17;\ 4z_1 \le 7$

$z_1 - z_2 \le -4$

$2z_1 - 3z_2 \le -15$

$14z_1 - 12z_2 \le -51$

$5z_1 - 3z_2 \le -9$

$z_1 - 3z_2 \le -16$

$8z_1 - 9z_2 \le -42$

$7z_1 - 10z_2 \le -49$

$10z_1 - 4z_2 \le -7$

$4z_1 - z_2 \le 2$

$z_2 \ge 5;\ z_1 \le 1.5$

The least upper bound is z_1 is 1.5.Therefore, Max $z_1 = 1.5$

On Substituting $z_1 = 1.5$ using backward substitution, we get

$z_2 \ge 5,\ z_2 \ge 5.5;\ z_2 \ge 6$

$z_2 \ge 5.5;\ z_2 \ge 5.8$

Therefore, the greatest lower bound of z_2 is 6. Therefore Min $z_2 = 6$

$x_3 \ge 0.5;\ x_3 \le 5$

$x_3 \ge -1;\ x_3 \le 0.5;\ x_3 \le 1$

Therefore, $x_1 = 0.5,\ x_2 = 0,\ x_3 = 0.5$

The optimal solution is

$x_1 = 0.5,\ x_2 = 0,\ x_3 = 0.5,\quad Z = 0.25$

6. Conclusion

We consider the fuzzy linear fractional programming problem and it is converted into crisp linear fractional programming problem using Yager's ranking method. The crisp linear fractional programming problem is written as multi objective linear fractional programming problem. The objective functions are changed into linear inequalities and then the system of linear inequalities is solved by Fourier Motzkin elimination by eliminating one variable at each iteration and the solution is obtained by the backward substitution. The solution is verified by lingo 13.0 version also.

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