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AN OPTIMAL SOLUTION TO FUZZY ASSIGNMENT PROBLEM USING ONES SUFFIX AND IMPROVED ONES SUFFIX METHOD

Mathematics

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ABSTRACT

Assignment problem is a special type of linear programming problem in which our objective is to assign n number of jobs to n number of persons at a minimum cost(maximum profit).In this paper ones suffix and improved ones suffix methods are used to find the optimal solution to Triangular Intuitionistic fuzzy assignment problem. These methods require least iteration to get the optimality. Defuzzification Measure is used to convert Triangular Intuitionistic fuzzy assignment problem to crisp assignment problem. Numerical examples illustrate the efficiency of these methods.

KEYWORDS:

Fuzzy assignment problem, Triangular intuitionistic fuzzy number, Ones suffix method, Improved ones suffix method.

INTRODUCTION:

Assignment problem is one of the combinatorial optimization problem which was investigated by G.Monge in 1784. Assignment problem plays an important role in an assigning of persons to jobs, jobs to machines, operators to machines, problems to research team etc. Assignment problem is a special type of linear programming problem in which our objective is to assign n number of jobs to n number of machines at a minimum cost. Well known method Hungarian Method was developed and published by Harold Khun in 1955. James Munkres reviewed the algorithm in 1957 and observed that it is strongly a polynomial. Hence the algorithm is known as Khun-Munkres or Munkres assignment algorithm.

K. Atanassov[1] introduced the concept of Intuitionistic fuzzy sets. An intuitionistic fuzzy set is characterized by membership degree and non membership degree. Intuitionistic fuzzy set theoryhas been studied and applied in different areas including decision making. K.Thangavelu et.al[14] proposed a Hungarian approach of solving Intuitionistic fuzzy assignment problem using a new defuzzification measure. Gaurav Kumar and Rakesh Kumar Balaji[4] presented the solution of interval valued Intuitionistic fuzzy assignment problem using similarity measure and score function. Shiny Jose and Sunny Kuriakose[10] discussed an algorithm for solving an assignment problem using Intuitionistic fuzzy concept.

Hadi Basirzadeh[5] proposed ones assignment for solving assignment problem ,where we obtain a reduced matrix which has at least one 1 in each row and column. Optimal solution is obtained by assigning ones to each row and column. Fegade.M.R.et.al [3] proposed zero suffix method and Robust ranking methodology to solve fuzzy transportation problem. Sudhagar .V.J and Navaneetha Kumar.V[13] used zero suffix method to determine the optimal compromise solution for a multi objective two stage transportation problem. Jayaraman.P& Jahirhussian.R[6] applied improved zero suffix to fuzzy optimal transportation problem Rupsha Roy and Rukmani Rathore[7] applied zero suffix method for finding optimal solution for assignment problem. Sudha.S and Vanisri.D [11] proposed improved zero suffix method for finding an optimal solution for assignment problem.

This paper is organized as follows: In section 2, some basic definitions of fuzzy number and intuitionistic fuzzy number are given. In section 3, we construct the mathematical model for the problem. In section 4, the methodology for the problem is given. In section 5, solution algorithm for the problem is discussed. In section 6, numerical examples are given to show the efficiency of the algorithm and finally we ,conclude the problem.

2. PRELIMINARIES:

Definition 2.1: Fuzzy Set

A fuzzy set is characteristized by a membership function mapping element of a domain, Space of the universe of discourse X to unit interval [0,1].i.e $A=\{(x,\mu A(x);x\in X\}$. Here μA:X→[0,1] is a mapping called the degree of membership function of the fuzzy set A and $\mu A(x)$ is called the membership value of $x\in X$ in a fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

Definition 2.2: Intuitionistic fuzzy set

Let a set X be fixed. An intuitionistic fuzzy set A^* in X is an object having the form $A^*=\{(x,\mu A^*(x),\nu_A^*(x)):x\epsilon X\}$,where $\mu A^*(x)$:X→[0,1] and $\nu A^*(x)$:X→[0,1] define the degree of membership and non-membership respectively,of the element x X to the set A^*,which is a subset of X,for every element of $x\epsilon X$, $0<\mu A^*(x)+\nu A^*(x)<1$.

3. Construction of Mathematical Model

3.1 The general assignment problem

Suppose there are n people and n jobs. Each job must be done by exactly one person, also each person can do, at most, one job. The problem is to assign the people to the jobs so as to minimize the total cost of completing all of the jobs. The general assignment problem can be mathematically stated as follows:

$$\text{Minimize } Z=\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} = 1 \text{ for } i = 1,2,\ldots, n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for } j = 1,2,\ldots, n$$

$$x_{ij} = 0 \text{ or } 1$$

3.2 Fuzzy assignment problem

$$\text{Minimize } Z = \sum_{i=1}^{n}\sum_{j=1}^{n} \widetilde{c}_{ij}\, x_{ij}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{ij} = 1 \text{ for } i = 1,2,\ldots, n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for } j = 1,2,\ldots, n$$

$$x_{ij} = 0 \text{ or } 1$$

4. Methodology:

4.1. Triangular fuzzy number:

A fuzzy number $A=(a_1,a_2,a_3)$ is defined to be a triangular fuzzy number if its membership

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0, & otherwise \end{cases}$$

4.2. Triangular Intuitionistic fuzzy number:

A^* is a triangular Intuitionistic fuzzy number with parameters $a_1*\le a_1 \le a_2 \le a_3 \le a_3^*$ and denoted by A^*=$(a_1,a_2, a_3)(a_1,a_2,a_3)$ having membership and non-membership function as follows.

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1, & x = a_2 \\ \frac{x-a_2}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0, & x > a_3 \end{cases}$$

$$v_A(x) = \begin{cases} 0, & x < a_1^* \\ \frac{x-a_1}{a_2-a_1}, & a_1^* \le x \le a_2 \\ 1, & x = a_2 \\ \frac{x-a_2}{a_3-a_2}, & a_2 \le x \le a_3^* \\ 0, & x > a_3^* \end{cases}$$

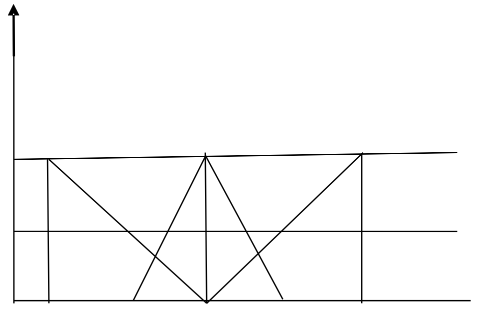

Fig.1.Membership and non-membership function of TIFN

4.3.Defuzzification:

We define a average function $D(A^*)= \frac{(a_1+4a_2+a_3)+(a_1^*+4a_2+a_3^*)}{12}$ to defuzzify a given Triangular Intuitionistic fuzzy number.

5. Solution Procedure:

5.1. Ones Suffix Method:

Step 1: Consider triangular intuitionistic fuzzy assignment problem.

Step 2: Defuzzify the fuzzy numbers to crisp numbers.

Step 3: Divide each row with the minimum cost and divide each column with the minimum cost so as to have at least one 1's in each row and column.

Step 4: Find the suffix values of all the 1's in the reduced cost matrix by the following formula

$$S = \frac{\textit{Add the cost of nearest adjacent sides of ones}}{\textit{Number of costs added}}$$

Step 5: Choose the maximum of S, if it has one maximum value then assign that job to machine if if has more than one maximum value then also assign the jobs to the machines(if the ones' donot lie in the same column or row) .If the ones lie in the same row or column then assign the job to machine whose cost is minimum. Now crate a new assignment table by deleting tat corresponding row and column

Step 6: Repeat Step 3 – Step 5 until all the jobs are assigned to machines.

5.2.Improved one's suffix Method:

Step 1: Consider triangular intuitionistic fuzzy assignment problem.
Step 2: Defuzzify the fuzzy number to crisp numbers.
Step 3: Divide each row with the minimum cost and divide each column with the minimum cost so as to have at least one 1's in each row and column.

Step 4: Find the suffix values of all the 1's in the reduced cost matrix by the following formula

$$S = \frac{\textit{sum of non zero costs in the ith row and jth column}}{\textit{Number of zeros in the ith row and jth column.}}$$

Step 5: Choose the maximum of S, if it has one maximum value then assign that job to machine if if has more than one maximum value then also assign the jobs to the machines(if the ones' donot lie in the same column or row) .If the one's lie in the same row or column then assign the job to machine whose cost is minimum. Now create a new assignment table by deleting that corresponding row and column

Step 6: Repeat Step 3 – Step 5 until all the jobs are assigned to machines.

6.1. Numerical Example:[Ones Suffix method]

Consider the following Triangular intuitionistic fuzzy cost minimization problem.

	M1	M2	M3	M4	M5
J1	(4,7,14) (3,7,24)	(12,14,26) (11,14,28)	(8,15,26) (7,15,30)	(5,9,12) (4,9,16)	(9,12,16) (8,12,26)
J2	(6,18,25) (5,18,28)	(13,19,28) (12,19,36)	(20,25,33) (19,25,38)	(7,10,16) (6,10,20)	(6,10,15) (5,10,25)
J3	(20,25,38) (19,25,42)	(2,11,14) (1,11,19)	(6,12,15) (5,12,20)	(4,9,16) (3,9,20)	(3,6,14) (2,6,24)
J4	(5,11,28) (4,11,30)	(4,9,16) (3,9,20)	(12,14,26) (11,14,30)	(8,10,15) (7,10,20)	(6,8,12) (5,8,15)
J5	(7,9,15) (6,9,20)	(3,5,10) (2,5,15)	(4,7,12) (3,7,16)	(7,10,16) (6,10,20)	(8,12,17) (7,12,20)

	M2	M4	M5
J2	1.64	1[1.3]	1[1]
J3	1.13	1.25	1[1.1]
J4	1[1.2]	1.22	1[1.1]

Solution:

Using Defuzzification Techniques, we get

R((4,7,14)(3,7,24)) = $\frac{(a_1+4a_2+a_3)+(a_1^*+4a_2+a_3^*)}{12} = 8$

8. Proceeding further for remaining values we get

	M1	M2	M3	M4	M5
J1	8	16	16	9	13
J2	17	20	26	11	11
J3	27	10	12	10	8
J4	13	10	16	11	9
J5	10	6	8	11	12

Applying ones assignment, ie, choose the minimum value from each row and divide it from the remaining values from the each row. Similarly choose the minimum value from each column and divide it from the remaining values from the each column. Each row and column should contain at least one 1's.

Now find the suffix value of each 1's and write it within [ ].The suffix value is calculated using the formula

$$S = \frac{Add\ the\ cost\ of\ nearest\ adjacent\ sides\ of\ ones}{Number\ of\ cost\ added}$$

	M2	M3	M4	M5
J2	1.82	1.77	1[1.3]	1[1]
J3	1.25	1.13	1.25	1[1.1]
J4	1.11	1.34	1.22	1[1.4]
J5	1[1.1]	1[1.4]	1.83	2

From all the above suffix, 1.8 is the maximum. Assign J1 to M1.Delete first row and first column from the above table and apply the same process.

From the above table, since the maximum suffix value 1.4 lies in forth row fifth column and fifth row third column, assign minimum cost out of the two. Minimum cost corresponds to fifth row third column. Assign J5 to M3. Delete last row and second column.

Assign J2 to M4.Delete first row and second column.

	M2	M5
J3	1.13	1[1.1]
J4	1[1.1]	1[1]

It is clear that J3 is assigned to M5 and J4 is assigned to M5.

The optimal assignment is J1 ⟶ M1,
J2 ⟶ M4, J3 ⟶ M5,
J4 ⟶ M2, J5 ⟶ M3.
Minimum assignment cost = Rs.(8+11+8+10+8) =

	M1	M2	M3	M4	M5
J1	1	2	1.5	1.13	1.63
J2	1.55	1.82	1.77	1	1
J3	3.38	1.25	1.13	1.25	1
J4	1.44	1.11	1.34	1.22	1
J5	1.67	1	1	1.83	2

	M1	M2	M3	M4	M5
J1	1[1.8]	2	1.5	1.13	1.63
J2	1.55	1.82	1.77	1[1.3]	1[1.2]
J3	3.38	1.25	1.13	1.25	1[1.1]
J4	1.44	1.11	1.34	1.22	1[1.4]
J5	1.67	1[1.3]	1[1.4]	1.83	2

Rs.45.

6.2 Numerical Example:[Improved one's suffix Method]

Consider the following Triangular intuitionistic fuzzy cost minimization problem.

	M1	M2	M3	M4	M5
J1	(4,7,14) (3,7,24)	(12,14,26) (11,14,28)	(8,15,26) (7,15,30)	(5,9,12) (4,9,16)	(9,12,16) (8,12,26)
J2	(6,18,25) (5,18,28)	(13,19,28) (12,19,36)	(20,25,33) (19,25,38)	(7,10,16) (6,10,20)	(6,10,15) (5,10,25)
J3	(20,25,38) (19,25,42)	(2,11,14) (1,11,19)	(6,12,15) (5,12,20)	(4,9,16) (3,9,20)	(3,6,14) (2,6,24)
J4	(5,11,28) (4,11,30)	(4,9,16) (3,9,20)	(12,14,26) (11,14,30)	(8,10,15) (7,10,20)	(6,8,12) (5,8,15)
J5	(7,9,15) (6,9,20)	(3,5,10) (2,5,15)	(4,7,12) (3,7,16)	(7,10,16) (6,10,20)	(8,12,17) (7,12,20)

Solution:

Using Defuzzification Techniques, we get

R((4,7,14)(3,7,24)) = $\frac{(a_1+4a_2+a_3)+(a_1^*+4a_2+a_3^*)}{12} = 8$

8. Proceeding further for remaining values we get

	M1	M2	M3	M4	M5
J1	8	16	16	9	13
J2	17	20	26	11	11
J3	27	10	12	10	8
J4	13	10	16	11	9
J5	10	6	8	11	12

Applying one's assignment, ie, choose the minimum value from each row and divide it from the remaining values from the each row. Similarly choose the minimum value from each column and divide it from the remaining values from the each column. Each row and column should contain at least one 1's.

	M1	M2	M3	M4	M5
J1	1	2	1.5	1.13	1.63
J2	1.55	1.82	1.77	1	1
J3	3.38	1.25	1.13	1.25	1
J4	1.44	1.11	1.34	1.22	1
J5	1.67	1	1	1.83	2

Find the suffix value of each element whose value is one and write the suffix value within the bracket[ ].

The suffix value is calculated using the formula

$$S = \frac{sum\ of\ non\ zero\ costs\ in\ the\ ith\ row\ and\ jth\ column}{Number\ of\ zeros\ in\ the\ ith\ row\ and\ jth\ column.}$$

	M1	M2	M3	M4	M5
J1	1[14.29]	2	1.5	1.13	1.63
J2	1.55	1.82	1.77	1[5.28]	1[2.19]
J3	3.38	1.25	1.13	1.25	1[3.54]
J4	1.44	1.11	1.34	1.22	1[2.91]
J5	1.67	1[5.84]	1 [5.62]	1.83	2

Since the maximum suffix value lies in first row and first column, assign J1 toM1 and delete first row and first column.

	M2	M3	M4	M5
J2	1.82	1.77	1[3.95]	1[1.4]
J3	1.25	1.13	1.25	1[1.88]
J4	1.11	1.34	1.22	1[1.89]

J5	1[4.01]	1[4.04]	1.83	2

Maximum suffix value corresponds to second column fourth row. Assign J5 to M3.Delete second column fourth row.

	M2	M4	M5
J2	1.64	1[2.06]	1[0.41]
J3	1.13	1.25	1[0.79]
J4	1[1.995]	1.22	1[0.31]

Assign J2 to M4.Delete first row and second column.

	M2	M5
J3	1.13	1[0.57]
J4	1[0.57]	1[0]

It is clear that J3 assigned to M5 and J4 assigned to M2.
The optimal assignment is as follows.

J1 → M1, J2 → M4, J3 → M5,
J4 → M2, J5 → M3.
Minimum assignment cost= Rs.(8+11+8+10+8) =

Minimum assignment cost= Rs.(8+11+8+10+8) = Rs.45.

7. Conclusion:

From the proposed methods, it can be concluded that Ones suffix method and Improved ones suffix method provide an optimal solution to Triangular Intuitionistic Fuzzy Assignment Problem in fewer iterations. Both methods give the same optimal solution with the same optimal assignment. These methods consume less time and are easy to understand and apply. The result is verified with LINGO 13.0 version.

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