INTERNATIONAL JOURNAL OF SCIENTIFIC RESEARCH

SQUARE SUM PRIME LABELING OF SOME SNAKE GRAPHS

Mathematics

Sunoj B S Department of Mathematics, Government Polytechnic College, Attingal, Kerala, India.

Mathew Varkey T K Department of Mathematics, T K M College of Engineering, Kollam, Kerala, India.

ABSTRACT

Square Sum prime labeling of a graph is the labeling of the vertices with {0,1,2-------,p-1} and the edges with sum of the squares of the labels of the incident vertices. The greatest common incidence number of a vertex (gcin) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the gcin of each vertex of degree greater than one is one, then the graph admits square sum prime labeling. Here we identify some snake graphs for square sum prime labeling.

KEYWORDS

Graph labeling, square sum, greatest common incidence number, prime labeling.

INTRODUCTION

All graphs in this paper are simple, finite, connected and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1] and [2]. The square sum labeling was defined by V Ajitha, S Arumugan and K A Germina in [5]. In this paper we introduced square sum prime labeling using the concept greatest common incidence number of a vertex. We proved that some snake graphs admit square sum prime labeling.

Definition Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

MAIN RESULTS

Definition 2.1 Let G = (V, E) be a graph with p vertices and q edges. Define a bijection

f: V(G) → {0,1,2,3,----------------,p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping f_{sqsp}^* : E(G) → set of natural numbers N by $f_{sqsp}^*(uv) = \{f(u)\}^2 + \{f(v)\}^2$. The induced function f_{sqsp}^* is said to be a square sum prime labeling, if the ***gcin*** of each vertex of degree at least 2, is 1.

Definition 2.2 A graph which admits square sum prime labeling is called a square sum prime graph.

Theorem 2.1 Triangular Snake T_n admits square sum prime labeling, when (n+2) is not a multiple of 5.

Proof: Let G = T_n and let v_1, v_2,----------------,v_{2n-1} are the vertices of G

Here |V(G)| = 2n-1 and |E(G)| = 3n-3

Define a function f : V → {0,1,2,3,--------------,2n-2 }

by $f(v_i)$ = i-1 , i = 1,2,------,2n-1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sqsp}^* is defined as follows

$f_{sqsp}^*(v_i\, v_{i+1}) = 2i^2 - 2i + 1, \quad 1 \le i \le 2n-2$

$f_{sqsp}^*(v_{2i-1}\, v_{2i+1}) = 8i^2 - 8i + 4, \quad 1 \le i \le n-1$

Clearly f_{sqsp}^* is an injection.

gcin of (v_{i+1}) = gcd of $\{f_{sqsp}^*(v_i\, v_{i+1}),\ f_{sqsp}^*(v_{i+1}\, v_{i+2})\}$

= gcd of $\{2i^2+2i+1, 2i^2-2i+1\}$

= gcd of $\{4i, 2i^2-2i+1\}$,

= gcd of $\{i, 2i^2-2i+1\}$

= 1, $1 \le i \le 2n-3$

gcin of (v_1) = 1

gcin of (v_{2n-1}) = gcd of $\{f_{sqsp}^*(v_{2n-2}\, v_{2n-1}),\ f_{sqsp}^*(v_{2n-3}\, v_{2n-1})\}$

= gcd of $\{8n^2-20n+13, 8n^2-24n+20\}$

= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence T_n , admits square sum prime labeling. ■

Theorem 2.2 Quadrilateral Snake Q_n admits square sum prime labeling, when n is not a multiple of 5.

Proof: Let G = Q_n and let v_1,v_2,----------------,v_{3n-2} are the vertices of G

Here |V(G)| = 3n-2 and |E(G)| = 4n-4

Define a function f : V → {0,1,2,3,--------------,3n-3 }

by $f(v_i)$ = i-1 , i = 1,2,------,3n-2

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\, v_{i+1})$ = $2i^2 - 2i + 1$, $1 \le i \le 3n-3$

$f^*_{sqsp}(v_{3i-2}\, v_{3i+1})$= $18i^2 - 18i + 9$, $1 \le i \le n-1$

Clearly f^*_{sqsp} is an injection.

gcin of (v_{i+1}) = 1, $1 \le i \le 3n-4$

gcin of (v_1) = 1

gcin of (v_{3n-2}) = gcd of {$f^*_{sqsp}(v_{3n-2}\, v_{3n-3})$, $f^*_{sqsp}(v_{3n-2}\, v_{3n-5})$ }

= gcd of { $18n^2$ -42n+25, $18n^2$ -54n+45}

= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence Q_n , admits square sum prime labeling. ■

Theorem 2.3 Alternate Triangular Snake T_n admits square sum prime labeling, when n is even and not a multiple of 10 and the triangle starts from first vertex.

Proof: Let G =A(T_n) and let v_1,v_2,----------------,$v_{\frac{3n}{2}}$ are the vertices of G

Here |V(G)| = $\frac{3n}{2}$ and |E(G)| = 2n-1

Define a function f : V → {0,1,2,3,--------------,$\frac{3n-2}{2}$ }

by $f(v_i)$ = i-1 , i = 1,2,------, $\frac{3n}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\, v_{i+1})$ = $2i^2 - 2i + 1$, $1 \le i \le \frac{3n-2}{2}$

$f^*_{sqsp}(v_{3i-2}\, v_{3i})$ = $18i^2 - 24i + 10$, $1 \le i \le \frac{n}{2}$

Clearly f^*_{sqsp} is an injection.

gcin of (v_{i+1}) = 1, $1 \le i \le \frac{3n-4}{2}$

gcin of (v_1) = 1

gcin of ($v_{\frac{3n}{2}}$) = gcd of {$f^*_{sqsp}(v_{(\frac{3n}{2})}\, v_{(\frac{3n-2}{2})})$, $f^*_{sqsp}(v_{(\frac{3n}{2})}\, v_{(\frac{3n-4}{2})})$ }

= gcd of { $\frac{9n^2}{2}$ - 9n+5, $\frac{9n^2}{2}$ - 12n+10}

= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence A(T_n) , admits square sum prime labeling. ■

Theorem 2.4 Alternate Triangular Snake T_n admits square sum prime labeling, when n is even and the triangle starts from second vertex.

Proof: Let G =A(T_n) and let v_1,v_2,------------,$v_{(\frac{3n-2}{2})}$ are the vertices of G.

Here |V(G)| = $\frac{3n-2}{2}$ and |E(G)| = 2n-3

Define a function f : V → {0,1,2,3,-------------, $\frac{3n-4}{2}$ }

by $f(v_i)$ = i-1 , i = 1,2,------, $\frac{3n-2}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\, v_{i+1})$ = $2i^2 - 2i + 1$, $1 \le i \le \frac{3n-4}{2}$

$f^*_{sqsp}(v_{3i-1}\, v_{3i+1})$ = $18i^2 - 12i + 4$, $1 \le i \le \frac{n-2}{2}$

Clearly f^*_{sqsp} is an injection.

gcin of (v_{i+1}) = 1, $1 \le i \le \frac{3n-6}{2}$

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence A(T_n) , admits square sum prime labeling. ■

Theorem 2.5 Alternate Triangular Snake T_n admits square sum prime labeling, when n is odd and the triangle starts from first vertex.

Proof: Let G =A(T_n) and let v_1,v_2,------------,$v_{(\frac{3n-1}{2})}$ are the vertices of G

Here |V(G)| = $\frac{3n-1}{2}$ and |E(G)| = 2n-2

Define a function f : V → {0,1,2,3,-----------, $\frac{3n-3}{2}$ }

by $f(v_i)$ = i-1 , i = 1,2,------, $\frac{3n-1}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\, v_{i+1})$ = $2i^2 - 2i + 1$, $1 \le i \le \frac{3n-3}{2}$

$f^*_{sqsp}(v_{3i-2}\, v_{3i})$ = $18i^2 - 24i + 10$, $1 \le i \le \frac{n-1}{2}$

Clearly f^*_{sqsp} is an injection.

gcin of (v_1) = 1

gcin of (v_{i+1}) = 1, $1 \le i \le \frac{3n-5}{2}$

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence A(T_n) , admits square sum prime labeling. ■

Theorem 2.6 Alternate Triangular Snake T_n admits square sum prime labeling, when n is odd and n+3 is not a multiple of 10 and the triangle starts from second vertex.

Proof: Let G =A(T_n) and let v_1,v_2,------------,$v_{(\frac{3n-1}{2})}$ are the vertices of G.

Here |V(G)| = $\frac{3n-1}{2}$ and |E(G)| = 2n-2

Define a function f : V → {0,1,2,3,------------, $\frac{3n-3}{2}$ }

by $f(v_i)$ = i-1 , i = 1,2,------, $\frac{3n-1}{2}$

Here $|V(G)| = \frac{5n-3}{2}$ and $|E(G)| = \frac{7n-7}{2}$

Define a function $f : V \rightarrow \{0,1,2,3,\text{-------------}, \frac{5n-5}{2}\}$

by $f(v_i) = i-1$, $i = 1,2,\text{------}, \frac{5n-3}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\, v_{i+1}) = 2i^2 - 2i + 1,\ 1 \le i \le \frac{5n-5}{2}$

$f^*_{sqsp}(v_{5i-4}\, v_{5i-2}) = 50i^2 - 80i + 34,\ 1 \le i \le \frac{n-1}{2}$

$f^*_{sqsp}(v_{5i-2}\, v_{5i+1}) = 50i^2 - 30i + 9,\ 1 \le i \le \frac{n-1}{2}$

Clearly f^*_{sqsp} is an injection.

gcin of $(v_1) = 1$

gcin of $(v_{i+1}) = 1, \quad 1 \le i \le \frac{5n-7}{2}$

gcin of $(v_{(\frac{5n-3}{2})})$ = gcd of $\{f^*_{sqsp}(v_{(\frac{5n-3}{2})}\, v_{(\frac{5n-5}{2})})$, $f^*_{sqsp}(v_{(\frac{5n-3}{2})}\, v_{(\frac{5n-9}{2})})\}$

= gcd of $\{ \frac{50n^2-120n+74}{4}, \frac{50n^2-160n+146}{4}, \}$

= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence A{(TQ)$_n$},admits square sum prime labeling. ■

Theorem 2.8 Alternate Quadrilateral Triangular Snake A{(QT)$_n$} admits square sum prime labeling, when n is odd

Proof: Let G = A{(QT)$_n$} and let $v_1, v_2, \text{--------}, v_{(\frac{5n-3}{2})}$ are the vertices of G

Here $|V(G)| = \frac{5n-3}{2}$ and $|E(G)| = \frac{7n-7}{2}$

Define a function $f : V \rightarrow \{0,1,2,3,\text{------------}, \frac{5n-5}{2}\}$

by $f(v_i) = i-1$, $i = 1,2,\text{------}, \frac{5n-3}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\, v_{i+1}) = 2i^2 - 2i + 1, \quad 1 \le i \le \frac{5n-5}{2}$

$f^*_{sqsp}(v_{5i-4}\, v_{5i-1}) = 50i^2 - 70i + 29,\ 1 \le i \le \frac{n-1}{2}$

$f^*_{sqsp}(v_{5i-1}\, v_{5i+1}) = 50i^2 - 20i + 4,\ 1 \le i \le \frac{n-1}{2}$

Clearly f^*_{sqsp} is an injection.

gcin of $(v_1) = 1$

gcin of $(v_{i+1}) = 1, \quad 1 \le i \le \frac{5n-7}{2}$

gcin of $(v_{(\frac{5n-3}{2})})$ = gcd of $\{f^*_{sqsp}(v_{(\frac{5n-3}{2})}\, v_{(\frac{5n-5}{2})})$, $f^*_{sqsp}(v_{(\frac{5n-3}{2})}\, v_{(\frac{5n-7}{2})})\}$

= gcd of $\{ \frac{50n^2-120n+74}{4}, \frac{50n^2-140n+106}{4}, \}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\, v_{i+1}) = 2i^2 - 2i + 1,\ 1 \le i \le \frac{3n-3}{2}$

$f^*_{sqsp}(v_{3i-1}\, v_{3i+1}) = 18i^2 - 12i + 4,\ 1 \le i \le \frac{n-1}{2}$

Clearly f^*_{sqsp} is an injection.

gcin of $(v_1) = 1$

gcin of $(v_{i+1}) = 1, \quad 1 \le i \le \frac{3n-5}{2}$

gcin of $(v_{(\frac{3n-1}{2})})$ = gcd of $\{f^*_{sqsp}(v_{(\frac{3n-1}{2})}\, v_{(\frac{3n-3}{2})})$, $f^*_{sqsp}(v_{(\frac{3n-1}{2})}\, v_{(\frac{3n-5}{2})})\}$

= gcd of $\{ \frac{9n^2}{2} - 12n + \frac{17}{2}, \frac{9n^2}{2} - 15n + \frac{29}{2}\} = 1.$

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence A(T_n), admits square sum prime labeling. ■

Theorem 2.7 Alternate Triangular Quadrilateral Snake A{(TQ)$_n$} admits square sum prime labeling, when n is odd

Proof: Let G = A{(TQ)$_n$} and let $v_1, v_2, \text{-------}, v_{(\frac{5n-3}{2})}$ are the vertices of G

= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence A{(QT)$_n$},admits square sum prime labeling

CONCLUSIONS

In this paper we proved certain snake type graphs admit square sum prime labeling. Triangular snakes, quadrilateral snakes and its alternate types are covered in this paper.

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