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MATHEMATICAL REPRESENTATION FOR ZYGOTIC COMBINATION OF AUTOTERAPLOID, AUTOHEXAPLOID AND AUTO OCTAPLOID OF RANDOM MATING GAMETIC POPULATION UNDER EQUILIBRIUM CONDITION.

Mathematics

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ABSTRACT

In this paper we express zygotic combination on the basis of binomial expansion of auto tetraploid, autohexaploid and autooctaploid of large random mating gametic population under equilibrium condition.

KEYWORDS

Zygote, autotetraploid, outohexaploid, atutooctaploid, equilibrium

1. Introduction

The law of segregation is a law of inheritance proposed by mendel in 1866(i). It states "Each organism is formed of a bundle of character. Each character is controlled by a pair of factors (genes). During gamete formation, the two factors of a character separate and entre different gametes" in 1908 Hardy and Weinberg (ii) independently discovered the principle concerned with the frequency of alleles in a population. It states that at equilibrium both alleles and genotype frequencies remain constant. That is the alleles frequencies do not change from generation to generation. Also the equilibrium genotypic frequencies are given by the square of the allelic frequencies.

2. Computation :- Consider an infinite (large) random mating population of autotetraploid, autohexaploid and auto octaploid with two alleles a_1 and a_2 The two ploid gametes produced by the population are a_1a_1, a_1a_2 $(=a_2a_1)$, a_2a_2 which will be abbreviated as a_{11}, a_{12} $(=a_{21})$, a_{22} respectively. The 3-ploids gametes produced by the population are a_{111}, a_{112}, a_{122} and a_{222}. The 4-ploids gametes produced by the population are a_{1111}, a_{1112}, a_{1122}, a_{1222} and a_{2222}. We assume that zygotes with the same alleles have the same viabilities (frequency) after selection where a_{11}, a_{22}, a_{111}, a_{222}, a_{1111}, a_{2222} is homogametes and a_{12}, a_{112}, a_{122}, a_{1112}, a_{1122}, a_{1222} is heterogametes.

3. Mendilian Segregation :- It is represented mathematically by the binomial expansion of $(a+b)^n$ where a is the probability that an event will occur and b is the probability that it will not occur. on the basis of binomial expansion segregation of a single pair of alleles a_1,a_2 in a monohybrid cross be represented as

$$(a_1+a_2)^2 = {}^2c_0a_1^2+{}^2c_1a_1a_2+{}^2c_2a_2^2$$
$$= {}^2c_0a_1a_1+{}^2c_1a_1a_2+{}^2c_2a_2a_2$$
$$= {}^2c_0a_{11}+{}^2c_1a_{12}+{}^2c_2a_{22}$$

4. Generalisation of segregation behavior :- In a population the frequencies of two alleles belonging to a particular gene are not always the same and therefore, in a random mating population the ratio of 1:2:1 is not always available. Let p and q are the frequency of alleles a_1 and a_2 respectively such that p+q=1

4.1 Segregation of single pair of alleles

Eggs	Sperm	
	$a_1(p)$	$a_2(q)$
$a_1(p)$	$w_1a_{11}(p^2)$	$w_2a_{12}(pq)$
$a_2(q)$	$w_2a_{21}(qp)$	$w_3a_{22}(q^2)$

The equilibrium values of the frequencies p, q, of a_1 and a_2 be $w = w_1p^2+2w_2pq+w_3q^2$

4.2 Zygotic combination in diploid population be

$= p^2+2pq+q^2$
$= (p+q)^2$

4.3 Segregation behavior of 2-ploid gametes.

	a_{11} (p)	$a_{12}(q)$	$a_{22}(r)$
a_{11} (p)	$w_4a_{11}(p^2)$	$w_5\{(a_{11}/2)+(a_{12}/2)\}$ (pq)	$W_6\{(a_{11}/6)+(2a_{12}/3)+(a_{22}/6)\}$ (pr)
a_{12} (q)	$w_5\{(a_{11}/2)+(a_{12}/2)\}$ (qp)	$W_6\{(a_{11}/6)+(2a_{12}/3)+(a_{22}/6)\}(q^2)$	$W_7\{(a_{12}/2)+(a_{22}/2)\}$(qr)
a_{22} (r)	$W_6\{(a_{11}/6)+(2a_{12}/3)+(a_{22}/6)\}$(rp)	$W_7\{(a_{12}/2)+(a_{22}/2)\}$ (rq)	$w_8a_{22}(r^2)$

Where p,q,r, are the frequencies of gametes a_{11},a_{12},a_{22} respectively such that p+q+r = 1

4.4 Allele frequency relation for autoteraploid

$w_p = [\,(p^2)w_4+(pq)w_5+\{(q^2/6)+(pr/3)w_6]a_{11}$
$w_q = [\{(2q^2/3)+(4pr/3)\}w_6+pqw_5+qrw_7]a_{12}$
$w_r = [\{(q^2/6)+(pr/3)\}w_6+qrw_7+r^2w_8]a_{22}$

4.5 The equilibrium values of the frequencies p,q,r, of gametes a_{11},a_{12},a_{22} respectively be

$w = w_4p^2+w_52pq+w_6q^2+w_72qr+w_62pr+w_8r^2$

4.6 Zygotic combination in 2 – ploid gamete population be

$= p^2+2pq+q^2+2qr+2pr+r^2$
$= (p+q+r)^2$
$= 1$

4.7 Segregation behavior of 3-ploid gametes :-

	$a_{111}(p)$	$a_{112}(q)$	$a_{122}(r)$	$a_{222}(s)$
a_{111} (p)	$w_9a_{111}(p^2)$	$w_{10}[(a_{111}/2)+(a_{112}/2)]$ (pq)	$w_{11}[(a_{111}/5)+(3a_{112}/5)(1a_{122}/5)]$ (pr)	$w_{12}[(a_{111}/20)+(9a_{112}/20)+(9a_{122}/20)+(1a_{222}/20)]$(ps)
a_{112} (q)	$w_{10}[(a_{111}/2)+(a_{112}/2)]$ (qp)	$w_{11}[(a_{111}/5)+(3a_{112}/5)+(1a_{122}/5)]$ (q^2)	$w_{12}[(a_{111}/20)+(9a_{112}/20)+(9a_{122}/20)+(1a_{222}/20)]$(ps)	$w_{13}[(a_{112}/5)+(3a_{122}/5)(1a_{222}/5)]$ (qs)
a_{122} (r)	$w_{11}[(a_{111}/5)+(3a_{112}/5)(1a_{122}/5)]$ (q^2)	$w_{12}[(a_{111}/20)+(9a_{112}/20)+(9a_{122}/20)+(1a_{222}/20)]$(ps)	$w_{13}[(a_{112}/5)(3a_{122}/5)(1a_{222}/5)]$ (r^2)	$w_{14}[(a_{122}/2)+(a_{222}/2)]$ (rs)
a_{222} (s)	$w_{12}[(a_{111}/20)+(9a_{112}/20)+(9a_{122}/20)+(1a_{222}/20)]$(ps)	$w_{13}[(a_{112}/5)+(3a_{122}/5)+(1a_{222}/5)]$ (sq)	$w_{14}[(a_{122}/2)+(a_{222}/2)]$ (sr)	$w_{15}a_{222}(s^2)$

Where p,q,r,s are the frequencies of gametes a_{111},a_{112},a_{122}, and a_{222} respectively such that p+q+r+s = 1

4.8 Allele frequency relation for autohexaploid

$w_p = [w_9p^2+w_{10}pq+w_{11}\{(2pr/5)+(q^2/5)\}+w_{12}\{(ps/10)+(qr/10)\}]a_{111}$

$w_q = [w_{10}pq+w_{11}\{(6pr/5)+(3q^2/5)\}+w_{12}\{(9ps/10)+(9qr/10)\}+w_{13}\{(2sq/5)+(r^2/5)\}]a_{112}$

$w_r = [w_{11}\{(2pr/5)+(q^2/5)\}+w_{12}\{(9ps/10)+(9rq/10)\}+w_{13}\{(6qs/5)+(3r^2/5)\}+w_{14}(rs)]a_{122}$

$w_s = [w_{12}\{(9ps/10)+(9qr/10)\}+w_{13}\{(2qs/5)+(r^2/5)\}+w_{14}(rs)+w_{15}s^2]a_{222}$

4.9 The equilibrium values of the frequencies p,q,r,s, of a_{111},a_{112},a_{122},a_{222} be

$w = w_9p^2+w_{10}2qp+w_{11}2pr+w_{11}q^2+w_{12}2ps+w_{12}2qr+w_{13}2sq+w_{13}r^2+w_{14}2rs+w_{15}s^2$

4.10 Zygotic combination in 3-ploid gamete population be

$= p^2+2pq+2pr+q^2+2ps+2qr+2sq+2rs+s^2+r^2 = (p+q+r+s)^2 = 1$

4.11 Segregation behavior of 4- ploid gametes.

	$a_{1111}(p)$	$a_{1112}(q)$	$a_{1122}(r)$	$a_{1222}(s)$	$a_{2222}(t)$
$a_{1111}(p)$	$W_{16}a_{1111}(p^2)$	$w_{17}[(a_{1111}/2)+(a_{1112}/2)](pq)$	$W_{18}[(9a_{1111}/34)+(8a_{1112}/17)+(9a_{1122}/34)](pr)$	$w_{19}[(5a_{1111}/34)+6a_{1112}/17)+6a_{1122}/17)+(5a_{1222}/34)](ps)$	$w_{20}[(a_{1111}/34)+(8a_{1112}/17)+(8a_{1222}/17)+a_{2222}/34)](pt)$
$a_{1112}(q)$	$w_{17}[(a_{1111}/2)+(a_{1112}/2)(qp)$	$w_{21}[(3a_{1111}/17)+(11a_{1112}/17)+(3a_{1122}/17)](q^2)$	$w_{22}[(a_{1111}/17)+(15a_{1112}/34)+(15a_{1122}/34)+(a_{1222}/17)](qr)$	$w_{23}[(a_{1111}/34)+(7a_{1112}/34)+(9a_{1122}/17)+(7a_{1222}/34)](qs)$	$w_{24}[(5a_{1112}/34)+(6a_{1122}/17)+(6a_{1222}/17)+(5a_{2222}/34)](qt)$
$a_{1122}(r)$	$w_{18}[(9a_{1111}/34)+(8a_{1112}/17)+(9a_{1122}/34)](rp)$	$w_{22}[(a_{1111}/17)+(15a_{1112}/34)+(15a_{1122}/34)+(a_{1222}/17)](rq)$	$w_{25}[(4a_{1112}/17)+(9a_{1122}/17)+(4a_{1222}/17)](r^2)$	$w_{26}[(a_{1112}/17)+(15a_{1122}/34)+(15a_{1222}/34)+(a_{2222}/17)](rs)$	$w_{27}[(9a_{1122}/34)+(8a_{1222}/17)+(9a_{2222}/34)](rt)$
$a_{1222}(s)$	$W_{19}[(5a_{1111}/34)+(6a_{1112}/17)+(6a_{1122}/17)+(5a_{1222}/34)](sp)$	$w_{23}[(a_{1112}/34)+(7a_{1112}/34)+(9a_{1122}/17)+(7a_{1222}/34)+(a_{2222}/34)](sq)$	$w_{26}[(a_{1112}/17)+(15a_{1122}/34)+(15a_{1222}/34)+(a_{2222}/17)](sr)$	$w_{28}[(3a_{1122}/17)+(11a_{1222}/17)+(3a_{2222}/17)](s^2)$	$W_{29}[(a_{1222}/2)+(a_{2222}/2)]$ (st)
$a_{2222}(t)$	$w_{20}[(a_{1111}/34)+(8a_{1112}/17)+(8a_{1222}/17)+a_{2222}/34)]$ (tp)	$w_{24}[(5a_{1112}/34)+(6a_{1122}/17)+(6a_{1222}/17)+(5a_{2222}/34)](tq)$	$w_{27}[(9a_{1122}/34)+(8a_{1222}/17)+(9a_{2222}/34)](tr)$	$W_{29}[(a_{1222}/2)+(a_{2222}/2)]$ (ts)	$W_{30}a_{2222}(t^2)$

Where p,q,r,s,t are the frequencies of gametes $a_{1111},a_{1112},a_{1122},a_{1222},a_{2222}$ respectively such that $p+q+r+s+t=1$

4.12 Allele frequency relation for autooctaploid

$w_p = [w_{16}p^2+w_{17}pq+w_{18}(9pr/17)+w_{19}(5ps/17)+w_{20}(pt/17)+w_{21}(3q^2/17)+w_{22}(2qr/17)+w_{23}(qs/17)]a_{1111}$

$w_q = [w_{17}pq+w_{18}(16pr/17)+w_{19}(12ps/17)+w_{20}(16pt/17)+w_{21}(11q^2/17)+w_{22}(15qr/17)+w_{23}(7qs/17)+w_{24}(5qt/17)+w_{25}(4r^2/17)+w_{26}(2rs/17)]a_{1112}$

$w_r = [w_{18}(9pr/17)+w_{19}(12ps/17)+w_{21}(3q^2/17)+w_{22}(15qr/17)+w_{23}(18qs/17)+w_{24}(12qt/17)+w_{25}(9r^2/17)+w_{26}(15rs/17)+w_{27}(9rt/17)+w_{28}(3s^2/17)]a_{1122}$

$w_s = [w_{19}(5ps/17)+w_{20}(16pt/17)+w_{22}(2qr/17)+w_{23}(7qs/17)+w_{24}(12qt/17)+w_{25}(4r^2/17)+w_{26}(15sr/17)+w_{27}(16rt/17)+w_{28}(11s^2/17)+w_{29}st]a_{1222}$

$w_t = [w_{20}(pt/17)+w_{23}(qs/17)+w_{24}(5qt/17)+w_{26}(2rs/17)+w_{27}(9rt/17)+w_{28}(3s^2/17)+w_{29}st+w_{30}t^2]a_{2222}$

4.13 The equilibrium values of the frequencies p,q,r,s,t of $a_{1111},a_{1112},a_{1122},a_{1222},a_{2222}$ respectively are given by the equations.

$W=w_{16}p^2+w_{21}q^2+w_{25}r^2+w_{28}s^2+w_{30}t^2+w_{17}2pq+w_{18}2pr+w_{19}2ps+w_{20}2pt+w_{22}2qr+w_{23}2qs+w_{24}2qt+w_{26}2rs+w_{27}2rt+w_{29}2st$

4.14 Zygotic Combination in 4-ploid gamete population be

$=p^2+q^2+r^2+s^2+t^{2+}+2pq+2pr+2ps+2pt+2qr+2qs+2qt+2rs+2rt+2st$

$=(p+q+r+s+t)^2$

$=1$

5. Conclusion :- (i) In zygotic combination genotypic proportion of alleles or gametes remain constant.

(ii) Mating is at random then (4.2)(4.6)(4.10) and (4.14) shows that frequencies are given by the square of the alletic frequencies.

(iii) In each trait summation of zygotic combination is equal to one.

(iv) The Knowledge of the zygotic combination on the besis of bionomial expansion of auto tetraploid, auto hexaploid and autooctaploid is essential under equilibrium condition.

(v) This is true for a minority only of animal and plant species.

6. REFERENCES :-

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(ii) Hardy, G.H (1908), Mendilian proportion a mixed population science 28;29