



COMPARISON OF QUICK SWITCHING SYSTEM USING FUZZY BINOMIAL DISTRIBUTION AND FUZZY POISSON DISTRIBUTION

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ABSTRACT

A sampling system is a grouping of two or more sampling plans with specified rules for switching between the plans for sentencing the lots of finished products. Quick Switching System (QSS) are used to make accept or reject decisions when inspecting a series of lots. QSS is a sampling system with reference to single sampling plan that involves normal and tightened plans by incorporating a switching rule. Quick Switching System can be applied to protect from poor quality and to reduce the cost of inspection. When the fraction defectives or uncertainty attained the fuzzy logic can be applied. This paper studies the determination of Quick Switching System using both fuzzy Binomial distribution and fuzzy Poisson distribution with OC curves of various sample size and fixed acceptance number.

KEYWORDS

Quick Switching System, OC, SSP, Fuzzy set

INTRODUCTION AND REVIEW OF LITERATURE:

This article shows how to evaluate and select quick switching system (QSSs) for acceptance sampling. It introduces a new method of describing the production provided by sampling plans during periods of changing quality called transitive operating characteristic (OC) curves. It also compares QSSs with Single, Double, Chain, and Variable Sampling Plans as well as with the MIL-STD-105E (1989) switching systems.

The Quick Switching System explored in this article consists of two sampling plans along with a set of rules for switching between them. The first sampling plan, called the Normal plan, is intended for use during periods of good quality. It has a smaller sample size in order to reduce inspection costs. The second sampling plan, called the Tightened plan, is intended for use when problems are encountered. It is designed to give a high level of protection. The switching rules ensure that the correct plan is used. They are designed to be easy to use and to react quickly to changes in quality. QSSs concentrate one's inspection effort where it will do the most good. Further, for processes running at low level of inspection but that react severely to the first hint of a problem.

QSSs were originally proposed by Dodge (1967) and investigated by Romboski (1969) and Govindaraju (1991). Taylor (1992) contains tables of QSSs and a program to select and evaluate QSSs with some modifications.

The Quick Switching System to be considered consists of two single sampling plans. This QSS will be denoted QSS-SS the last two "S"s representing the two single sampling plans. One starts with the tightened plan. For each lot inspected, two decisions are made: first, whether to accept or reject the current lot; second, which of the two sampling plans to use for the next lot. To accomplish this last task, each of the sampling plans has an associated switch number.

The QSSs investigated in this article differ from those originally proposed by Dodge in that the decision to switch is made separate from the decision to accept/reject.

QUICK SWITCHING SYSTEM

Quick switching system require normal plan when the quality is good and tightened plan when the quality is bad. Dodge (1967) proposed a new sampling system consisting of pairs of normal and tightened plans with the switching rules constitute a sampling system. The application of system is as follows.

1. Adopt a pair of sampling plans, a normal plan (N) and tightened plan (T) the plan to T to be tighter 'OC' wiser than plan N
2. Use plan N for the first lot
3. For each lot inspected: if the lot is accepted, use plan N for the next lot; and if the lot is rejected, use plan T for the next lot.

Due to instantaneous switching between normal and tightened plan, the system is referred to as Quick Switching System. Using the OC function of QSS is derived by Romboski (1969) as

$$P_a = \frac{P_T}{(1-P_N)^k + P_T} \quad (1)$$

CONDITIONS OF APPLICATION

The conditions for application under which the Quick Switching System can be applied and the operation procedures are as follows:

1. The production is steady so that results on current and proceedings lots are broadly indicative of a continuing process and submitted lots are expected to be essentially of the same quality.
2. Lots are submitted substantially in the order of production.
3. Inspection is by attributes with quality defined as fraction nonconforming.

The below table gives the designation of QSS where single sampling plan is the reference plan utilizing two kinds of tightening procedure. They are:

- Acceptance number tightening
- Sample size tightening

For the above two systems following conditions are imposed.

$$\begin{aligned} \text{(i)} \quad & c_N > c_T \\ \text{(ii)} \quad & k > 1 \end{aligned}$$

When $c_N = c_T$ and $k=1$, the above two systems degenerate into single sampling plan. Romboski has limited his study to the first system.

Romboski (1969) has introduced another sampling inspection system QSS-1 ($n, k; c_0$) which is QSS-1 with single sampling plan as a reference plan (n, c_0) and (nk, c_0) $k>1$ are respectively the normal and tightened single sampling plans. The conditions for application of this system are the same as that of QSS-1 QSS($n; c_N, c_T$).

OPERATING PROCEDURE FOR QSS ($n, k; c_0$):

From a lot, take a sample of size 'n' at the normal level. Count the number of defectives 'd'.

Step 1:

(i) If $d \leq c_0$ accept the lot.

(ii) If $d > c_0$ reject the lot and go to step- 2

From the next lot, take a sample of size 'nk' at the tightened level; count the number of defectives 'd'.

Step 2:

- (i) If $d \leq c_0$ accept the lot.
- (ii) If $d > c_0$ reject the lot and go to step-2.

PRELIMINARIES AND DEFINITIONS

Parameter ‘ p ’ (probability of a success in each experiment) of the crisp binomial distribution is known exactly, but sometimes we are not able to obtain exact some uncertainty in the value ‘ p ’ and is to be estimated from a random sample or from expert opinion. The crisp Poisson distribution has one parameter, which we also assume is not known exactly.

Definition 1: The fuzzy subset \tilde{N} of real line IR , with the membership function $\mu_N: IR \rightarrow [0,1]$ is a fuzzy number if and only if (a) \tilde{N} is normal (b) \tilde{N} is fuzzy convex (c) μ_N is upper semi continuous (d) $\text{supp}(\tilde{N})$ is bounded.

Definition 2: A triangular fuzzy number \tilde{N} is fuzzy number that membership function defined by three numbers $a_1 < a_2 < a_3$ where the base of the triangle is the interval $[a_1, a_3]$ and vertex is at $x = a_2$.

Definition3: The α - cut of a fuzzy number \tilde{N} is a non-fuzzy set defined as

$$N[\alpha] = \{x \in IR; \mu_N(x) \geq \alpha\}. \text{ Hence } N[\alpha] = [N_\alpha^L, N_\alpha^U] \text{ where}$$

$$N_\alpha^L = \inf\{x \in IR; \mu_N(x) \geq \alpha\}$$

$$N_\alpha^U = \sup\{x \in IR; \mu_N(x) \geq \alpha\}$$

Definition 4: Due to the uncertainty in the l_i ’s values we substitute \tilde{l}_i , a fuzzy number, for each l_i and assume that $0 < \tilde{l}_i < 1$ all i . Then X together with the \tilde{l}_i value is a discrete fuzzy probability distribution. We write \tilde{p} for fuzzy P and we have $\tilde{P}(\{x_i\}) = \tilde{l}_i$. Let $A = \{x_1, x_2, \dots, x_i\}$ be subset of X . Then define:

$$\tilde{P}(A)[\alpha] = \frac{\sum_{i=1}^i l_i}{s} \quad (2)$$

For $0 < \alpha < 1$, where stands for the statement “ $l_i \in \tilde{k}_i[\alpha], 1 < i < n, \sum_{i=1}^i l_i = 1$ ”
This is our fuzzy arithmetic.

Definition 5: Let x be a random variable having the Poisson mass function. If $P(x)$ stands for the probability that $X = x$, then $\tilde{P}(d)[\alpha] = P(x \leq c)$

$$= \sum_{d=0}^c \binom{n}{d} p^d (1-p)^{n-d} \quad (3)$$

Where $p \in \tilde{p}[1]$ and a $q \in \tilde{q}[1]$ with $p+q = 1$ and $\binom{n}{x} = \frac{n!}{r!(n-r)!}$

$$P^L[\alpha] = \min \left\{ \binom{n}{d} p^d (1-p)^{n-d} \right\} \text{ and}$$

$$P^U[\alpha] = \max \left\{ \binom{n}{d} p^d (1-p)^{n-d} \right\}$$

Definition 6: Let x be a random variable having the Poisson mass function. If $P(x)$ stands for the probability that $X = x$, then

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad (4)$$

For $x=0, 1, 2, \dots$ and $\lambda > 0$.

Now substitute fuzzy number $\tilde{\lambda} > 0$ for λ to produce the fuzzy Poisson probability mass function. Let $P(x)$ to be the fuzzy probability that $X = x$. Then α - cut of this fuzzy number as

$$\tilde{P}(x)[\alpha] = \left\{ \frac{e^{-\lambda} \lambda^x}{x!} \mid \lambda \in \lambda[\alpha] \right\} \quad (5)$$

For all $\alpha \in [0,1]$. Let X be a random variable having the fuzzy binomial distribution and \tilde{P} in the definition 4 are small. That is all are $p \in \tilde{p}$ sufficiently small. Then $\tilde{P}[a,b][\alpha]$ using the fuzzy poisson approximation.

$$\text{Then } \tilde{P}[a, b][\alpha] = \sum_{x=a}^b \frac{e^{-\lambda} \lambda^x}{x!}$$

ACCEPTANCE SAMPLING PLANS WITH FUZZY PARAMETER USING BINOMIAL DISTRIBUTION

In this section, the single sampling plan for classical attributes characteristics with fuzzy logic is introduced. Suppose to inspect a lot of size ‘ N ’, choose and inspect a random sample of size ‘ n ’, and count the number of defective items or damaged units (D). If the number of observed defective items (d) is less than or equal to the acceptance number ‘ c ’, the lot is accepted and otherwise it is rejected. If the size of the lot is very large, the random variable ‘ D ’ has a binomial distribution with parameters ‘ n ’ and ‘ p ’, where ‘ p ’ is the proportion of the defective items in the lot. So, the probability for the number of defective items to exactly equal ‘ d ’ is

$$P(D = d) = \binom{n}{d} p^d (1-p)^{n-d}$$

and hence the probability of acceptance of the lot is

$$P_a = P(D \leq c)$$

$$= \sum_{d=0}^c \binom{n}{d} p^d (1-p)^{n-d} \quad (8)$$

Suppose to inspect a lot of size of ‘ N ’, where the proportion of defective items is not known precisely then suppose that this parameter is the fuzzy number \tilde{p} as follows:

$$\tilde{p} = (a_1, a_2, a_3, a_4),$$

$$\tilde{p}[\alpha] = [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha]$$

A single sampling plan with a fuzzy parameter is defined by the sample size ‘ n ’, and acceptance number ‘ c ’, and if the number of observed defective items (d) is less than or equal to ‘ c ’, the lot will be accepted. If ‘ N ’ is a large number, then the number of defective items in this sample has a fuzzy binomial probability distribution. So for $0 \leq \alpha \leq 1$, the fuzzy probability that there will be exactly ‘ d ’ defective items in a sample of size ‘ n ’, is

$$\tilde{P}(D = d)[\alpha] = \left\{ \binom{n}{d} p^d q^{n-d} \mid S \right\}$$

$$= [P^L[\alpha], P^U[\alpha]]$$

$$P^L[\alpha] = \min \left\{ \binom{n}{d} p^d q^{n-d} \mid S \right\} \quad P^U[\alpha] = \max \left\{ \binom{n}{d} p^d q^{n-d} \mid S \right\}$$

where S stands for the statement “ $p \in \tilde{p}[\alpha], q \in \tilde{q}[\alpha], p + q = 1$ ”

SINGLE SAMPLING PLAN WITH FUZZY PARAMETER USING POISSON DISTRIBUTION

If the size of sample be large and ‘ p ’ is small then the random variable ‘ d ’ has a Poisson approximation distribution with $\lambda = np$. So, the probability for the number of defective items to be exactly equal to ‘ d ’ is:

$$P(d) = \frac{e^{-np} (np)^d}{d!}$$

and the probability for acceptance of the lot (P_a) is:

$$P_a = P(d \leq c)$$

$$= \sum_{d=0}^c \frac{e^{-np} (np)^d}{d!}$$

Suppose that we want to inspect a lot with the large size of ‘ N ’, such that the proportion of damaged items is not known precisely. So we represent this parameter with a fuzzy number \tilde{p} as follows:

$$\tilde{p} = (a_1, a_2, a_3), p \in \tilde{p}[1], q \in \tilde{q}[1],$$

$$p + q = 1.$$

A single sampling plan with a fuzzy parameter if defined by the sample size ‘ n ’, and acceptance number ‘ c ’, and if the number of observation defective product is less than or equal to ‘ c ’, the lot will be acceptance. If ‘ N ’ is a large number, then the number of defective items in this sample (d) has a fuzzy binomial distribution, and if \tilde{p} is a small, then random variable ‘ d ’ has a fuzzy Poisson distribution with parameter $\tilde{\lambda} = n\tilde{p}$. So the fuzzy probability for the number of defective items in a sample size that is exactly equal to ‘ d ’ is $\tilde{P}(d - defective)[\alpha] = [P^L[\alpha], P^U[\alpha]]$

$$P^L[\alpha] = \min \left\{ \frac{e^{-\lambda} \lambda^d}{d!} \mid \lambda \in n\tilde{p}[\alpha] \right\},$$

$$P^U[\alpha] = \max \left\{ \frac{e^{-\lambda} \lambda^d}{d!} \mid \lambda \in n\tilde{p}[\alpha] \right\}$$

and fuzzy acceptance probability is as follows:

$$\tilde{p}_\alpha = \left\{ \sum_{d=0}^c \frac{e^{-\lambda} \lambda^d}{d!} \mid \lambda \in \tilde{\lambda}[\alpha] \right\} = [P^L[\alpha], P^U[\alpha]]$$

$$P^L[\alpha] = \min \left\{ \sum_{d=0}^c \frac{e^{-\lambda} \lambda^d}{d!} \mid \lambda \in \tilde{\lambda}[\alpha] \right\},$$

$$P^U[\alpha] = \max \left\{ \sum_{d=0}^c \frac{e^{-\lambda} \lambda^d}{d!} \mid \lambda \in \tilde{\lambda}[\alpha] \right\}$$

OC Band with Fuzzy Parameter

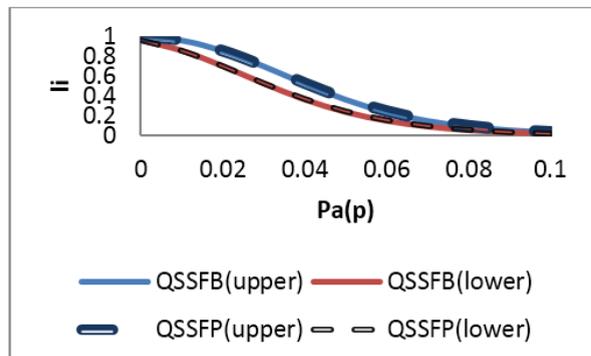
Operating characteristic curve is one of the important criteria in the sampling plan. By this curve, one could be determined the probability of acceptance or rejection of a lot having some specific defective items. The OC curve represents the performance of the acceptance sampling plans by plotting the probability of acceptance a lot versus its production quality, which is expressed by the proportion of nonconforming items in the lot. OC curve aids in selection of plans that are effective in reducing risk and indicates discriminating power of the plan.

The fuzzy probability of acceptance a lot in terms of fuzzy fraction of defective items would be as a band with upper and lower bounds. The uncertainty degree of a proportion parameter is one of the factors that bandwidth depends on that. The less uncertainty value results in less bandwidth, and if proportion parameter gets a crisp value, lower and upper bounds will become equal, which that OC curve is in classic state. Knowing the uncertainty degree of proportion parameter and variation of its position on horizontal axis, we have different fuzzy number (p) and hence we will have different proportion (p) which the OC bands are plotted in terms of it.

Table 1: Comparison of QSS_{FB} & QSSP_{FP}
(n= 30; k= 1.75 c₀= 1)

Li	\tilde{p}	QSS _{FB}	QSS _{FP}
0	[0,0.01]	[1,0.9615]	[1,0.9607]
0.01	[0.01,0.02]	[0.9615,0.8561]	[0.9607,0.8548]
0.02	[0.02,0.03]	[0.8561,0.7002]	[0.8548,0.7009]
0.03	[0.03,0.04]	[0.7002,0.5246]	[0.7009,0.5295]
0.04	[0.04,0.05]	[0.5246,0.3633]	[0.5295,0.3726]
0.05	[0.05,0.06]	[0.3633,0.2368]	[0.3726,0.2487]
0.06	[0.06,0.07]	[0.2368,0.1481]	[0.2487,0.1604]
0.07	[0.07,0.08]	[0.1481,0.0903]	[0.1604,0.1013]
0.08	[0.08,0.09]	[0.0903,0.0541]	[0.1013,0.0633]
0.09	[0.09,0.10]	[0.0541,0.0321]	[0.0633,0.0393]
0.1	[0.10,0.11]	[0.0321,0.0189]	[0.0393,0.0244]

Figure 1: OC Band for QSS using Fuzzy Binomial distribution and QSS using Fuzzy Poisson Distribution



CONCLUSION

In this article, construction and designing of Quick Switching System - QSS_{FP} (n; k; c₀) with reference to single sampling plan using Fuzzy Poisson Distribution for various Fuzzy Quality Characteristics are studied. These systems are well defined since if the fraction of defective items is crisp they reduce to classical plans. The uncertainty degree of a proportion parameter is one of the factors that bandwidth depends on that. The less uncertainty value results in less bandwidth, and greater uncertainty values results in wider bandwidth. From this it is suggested that, can adopt this system to predict the uncertainty level in a easy way and from the composite OC curve we can understand that the Quick switching system using fuzzy Binomial gives the better outcome than Fuzzy Binomial Distribution. Based on this system, the better outcome can be achieved in the shop floor situations.

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