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# STOCHASTIC MATRIX IN GENETICS

Mathematics

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## ABSTRACT

In this paper we express the mathematical representation of Genotypic structure in a randomly interbreeding population in equilibrium state through stochastic Matrix.

## KEYWORDS

Genotype, stochastic matrix, interbreeding.

## 1. INTRODUCTION :-

The beginning of genetics and population genetics are same things. Both started with Mendel. Population genetics deals with the study of changes within the gene pool of a mendalian population may undergo when it is exposed to systematic forces such as selection, mutation and migration. When the population is of limited size, the sample of genes transmitted to the next generation can deviate randomly from the true genetic composition of the parental generation and these random changes can accumulate over several generation. In other words, the change in the gene frequency over time due to systematic as well as random forces. Mendel considered the result of repeated self fertilization, he did not considered the result of a random mating system. The first pioneer mathematician G.H.Hardy and physian W.Weinberg [1] defining the genetic structure of population on the basis of random-mating system if no genetically determined fitness differences (no selection pressure), no mutation, no migration and no random fluctuation. This definition is the foundation of population genetics which states that "The relative frequencies of each allele (gene) tend to remain constant from generation to generation" and with random mating the arry of diploid in given by the binomial expansion of the square of the gematic arry. Consider a finite random-mating population of an individual with two [allels dominant a1 and recessive a2. The 2-ploid gametes produced by the population are $a_1a_1$, $a_1$ $a_2$ = $a_2$ $a_1$, $a_2$ $a_2$ which will be abbreviated as $a_{11}$, $a_{12}$ = $a_{21}$ , $a_{22}$ called dominant hybrid and recessive respectively.

## 2. Population in $F_0$ generation (Fusion Stage)

**2.1** If the population $a_{11}$ interbreeding with population $a_{11}$ , $a_{12}$ and $a_{22}$ respectively. Then population are $(a_{11})(a_{11}) = (a_{11}, a_{11}, a_{11}, a_{11})$. Here probability of population are $p(a_{11}) = 1, P(a_{12}) = 0, P(a_{22}) = 0$

Similarly

$(a_{11})(a_{12}) = (a_{11}, a_{12}, a_{11}, a_{12})$ ; $P(a_{11}) = ½, P(a_{12}) = ½, P(a_{22}) = 0$

$(a_{11})(a_{22}) = (a_{12}, a_{12}, a_{12}, a_{12})$ ; $P(a_{11}) = 0, P(a_{12}) = 1, P(a_{22}) = 0$

Transition Probability matrix 'A' obtained for the above population

| | $a_{11}$ | $a_{12}$ | $a_{22}$ |
|---|---|---|---|
| $a_{11}$ | 1 | 0 | 0 |
| $a_{12}$ | ½ | ½ | 0 |
| $a_{22}$ | 0 | 1 | 0 |

**2.2** If the population $a_{12}$ interbreeding with population $a_{11}$, $a_{12}$ & $a_{22}$ respectively. Then population are

$(a_{12})(a_{11}) = (a_{11}\,a_{11}, a_{21}\,a_{21})$ ; $P(a_{11}) = ½, P(a_{21}) = ½, P(a_{22}) = 0$

$(a_{12})(a_{12}) = (a_{11}\,a_{12}, a_{21}\,a_{22})$ ; $P(a_{11}) = ¼, P(a_{12}) = ½, P(a_{22}) = ¼$

$(a_{12})(a_{22}) = (a_{12}\,a_{12}, a_{22}\,a_{22})$ ; $P(a_{11}) = 0, P(a_{12}) = ½, P(a_{22}) = ½$

Transition probability matrix 'B' obtained for the above population.

| | $a_{11}$ | $a_{12}$ | $a_{22}$ |
|---|---|---|---|
| $a_{11}$ | ½ | ½ | 0 |
| $a_{12}$ | ¼ | ½ | ¼ |
| $a_{22}$ | 0 | ½ | ½ |

**2.3** If the population $a_{22}$ interbreeding with population $a_{11}$, $a_{12}$ & $a_{22}$ respectively. Then population are

$(a_{22})(a_{11}) = (a_{21}\,a_{21}\,a_{21}a_{21})$ ; $P(a_{11}) = 0, P(a_{21}) = 1, P(a_{22}) = 0$

$(a_{22})(a_{12}) = (a_{21}\,a_{22}\,a_{21}a_{22})$ ; $P(a_{11}) = 0, P(a_{21}) = ½, P(a_{22}) = ½$

$(a_{22})(a_{22}) = (a_{22}\,a_{22}\,a_{22}a_{22})$ ; $P(a_{11}) = 0, P(a_{12}) = 0, P(a_{22}) = 1$

Transition probability matrix 'C' obtained for the above population.

| | $a_{11}$ | $a_{12}$ | $a_{22}$ |
|---|---|---|---|
| $a_{11}$ | 0 | 1 | 0 |
| $a_{12}$ | 0 | ½ | ½ |
| $a_{22}$ | 0 | 0 | 1 |

Evidently each of these transition probability matrix for the population is stochastic matrix, since all of its elements are non-negative and the row sums are unity i.e. sum of population of a11, a12, & a22 be unity. Let the population of a11, a12 and a22 be denoted by x, y & z respectively then. x+y+z = 1

let X=(x,y,z) be a probability vector of F0 generation.

**3. AIM:-** Probability vector for the first generation and all the succeeding generation are same.

## 4. Population in F1 generation

**4.1** If each individual in this population interbreeding with population A, then probability vector for the F1 generation is

$$XA = [x,y,z]\begin{bmatrix} 1 & 0 & 0 \\ ½ & ½ & 0 \\ 0 & 1 & 0 \end{bmatrix} = [x+(y/2), (y/2)+z, 0]$$

Similarly

XB = [(x/2)+(y/4) ,(x+y+z)/2 , (y/4)+(z/2)]

XC = [0, x+(y/2), (y/2)+z]

Let XA , XB and XC be the probability vector for interbreeding with $a_{11}$, $a_{12}$ & $a_{22}$ respectively and the relative probability of $a_{11}$, $a_{12}$ & $a_{22}$ in the population be [x,y,z]. then probability vector for the first generation $F_1$ be

x.XA + y.XB+ z.XC =

$[x\{x+(y/2)+(y/2)+z\}+y\{(x/2)+(y/4)+(x+y+z)/2+(y/4)+(z/2)\}+z\{x+(y/2)+(y/2)+z\}]$

$=[\{x+(y/2)\}^2+2\{x+(y/2)\}\{(y/2)+z\}+\{(y/2)+z\}2]$

$=[x^1y^1z^1]=X^1$

## 5 Population of $F_2$ generation

**5.1** Probability vector for the second generation $F_2$ be

$[\{x^1+(y^1/2)\}^2, 2[\{x^1+(y^1/2)\}\{z^1+(y^1/2)\}, \{z^1+(y^1/2)\}^2]$

Now

$\{x^1+(y^1/2)\}^2=[\{x+(y/2)\}^2+(1/2).2\{x+(y/2)\}\{(y/2)+z\}]^2$

$= \{x+(y/2)\}^2[\{x+(y/2)\}\{(y/2)+z\}]^2$

$= \{x+(y/2)\}^2[x+y+z]^2=\{x+(y/2)\}^1; x+y+z=1$

$= x^1$

$2\{x^1+(y^1/2)\}\{(y^1/2)+z^1\} = 2[x+(y/2)^2+(1/2).2\{x+(y/2)\}\{(y/2)+z\}]$

$[\{(y/2)+z\}^2+(1/2)2.\{x+(y/2)\}\{(y/2)+z\}]$

$=2\{x+(y/2)\}\{(y/2)+z\}[x+(y/2)+(y/2)+z][(y/2)+z+x+(y/2)]$

$=2\{x+(y/2)\}\{(y/2)+z\}[x+y+z][x+y+z]$

$=2\{x+(y/2)\}\{(y/2)+z\}=y^1$

$\{(y^1/2)+z^1\}^2= \{(y/2)+z\}^2+1/2.2\ \{x+(y/2)\}\{(y/2)+z\}$

$= \{(y/2)+z\}[(y/2)+z+x+(y/2)]$

$=\{(y/2)+z\}(x+y+z)= \{(y/2)+z\} = z^1$

**6 Population in $F_n$ generation**

**6.1** If probability vector X is interbreeding with population $a_{11}$,$a_{12}$ and $a_{22}$ the

$P(a_{11})$ at nth generation be $1.x+y\{1-(1/2^n)\}+z\{1-(1/2^{n-1})\}$

$= \{x+y+z-(y/2^n)-(z/2^{n-1})\}$

$=\{1-(y/2^n)-(z/2^{n-1})\}$

$P(a_{12})$ at $n^{th}$ generation $x.0+y.1/2^n+z.1/2^{n-1}$

$=(y/2^n)+(z/2^{n-1})$

$P(a_{22})$ at $n^{th}$ generation be $x.0+y.0+z.0=0$

Where

$$A^n = \begin{bmatrix} 1 & 0 & 0 \\ 1-(1/2^n) & 1/2^n & 0 \\ 1-(1/2^{n-1}) & 1/2^{n-1} & 0 \end{bmatrix}$$

$Xa^n = [\{1-(y/2^n)-(z/2^{n-1}), (y/2^n)+(z/2^{n-1}), 0\}]$

Similarly

$$B^n = \begin{bmatrix} (1/4)+(1/2^{n+2}) & 1/2 & (1/4)-(1/2^{n+2}) \\ 1/4 & 1/2 & 1/4 \\ (1/4)-(1/2^{n+2}) & 1/2 & (1/4)+(1/2^{n+2}) \end{bmatrix}$$

$$C^n = \begin{bmatrix} 0 & 1/2^{n-1} & 1-(1/2^{n-1}) \\ 0 & 1/2^n & 1-(1/2^n) \\ 0 & 0 & 1 \end{bmatrix}$$

Hence $XB^n=[(1/4)+(y/2^{n+2})-(z/2^{n+2}), 1/2, (1/4)-(x/2^{n+2})+(z/2^{n+2})]$

$Xc^n = [0, (x/2^{n-1})+(y/2^n), 1-(z/2^{n-1})-(y/2^n)]$

**7. CONCLUSION**

If XA=X then x=1, y=0, z=0

XB=X then x=1/4, y=1/2, z=1/4

XC=X then x=0, y=0, z=1

Also at $n^{th}$ generation when $n \to \infty$

$XA^n=[1,0,0]$, $XB^n=[1/4,1/2,1/4]$, $XC^n=[0,0,1]$

Which satisfy hardy weing being law that the probability vector for the first generation and all succeeding generation are same.

**REFERENCES**

1. Hardy, G.H. (1908). Mendelian Proportion in a mixed population. Science 28:49