INTERNATIONAL JOURNAL OF SCIENTIFIC RESEARCH

N-FUZZY BH-SUBALGEBRAS OF BH-ALGEBRAS

Mathematics

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ABSTRACT

In this paper, we introduce the notion of fuzzy BH-subalgebra of a BH-algebra with respect to a triangular norm. Then we show that the Cartesian product of two fuzzy BH-subalgebras of BH-algebra X with respect to a triangular norm is a fuzzy BH-subalgebra of the product of X with respect to the norm.

KEYWORDS

fuzzy BH-subalgebra, BH-ideal, t-norm

1. INTRODUCTION

Y. Imai and K. Iseki introduced BCK-algebras and BCI-algebras, classes of abstract algebras [4, 5]. In [2, 3] Q.P. Hu and x. Li introduced another class of algebra which is called BCH-algebra and also showed that a BCI-algebra is a proper subclass of BCH algebra. Later J. Neggers and H. S. Kim [6] introduced the notion of d-algebras, which is another useful generalization of BCK –algebras. L. A. Zadeh [7] introduced the notion of fuzzy sets. M. Akram and K. H. Dar [1] introduced the notion of fuzzy d-ideals of d-algebras. Y. B. Jun, E. H. Roh and H. S. Kim, [8] introduced On BH-algebras, Q. Zhang, E. H. Roh and Y. B. Jun, [9] introduced on fuzzy BH-algebras. Recently, several researchers have apply the notion of fuzzy set to algebraic structures such as BCC/BCI/BCK/MV/TM/d/BH-algebras. In this paper, we introduce the notion of fuzzy sub algebra of BH-algebras with respect to a triangular norm and investigate some related properties.

2 PRELIMINARIES

In this section we cite the fundamental definitions that will be used in the sequel:

Definition 2.1 [2, 3]
A nonempty set x with a constant 0 and a binary operation * is called a BCH - algebra if it satisfies the following axioms.

1. $x*x=0$
2. $(x*y)*z=(x*z)*y=0$
3. $x*y=0, y*x=0 \Rightarrow x=y$ for all $x,y,z \in x$

Definition 2.2 [4, 5] A nonempty set x with a constant 0 and a binary operation * is called a BCI - algebra if it satisfies the following axioms.

1. $((x*y)*(x*z))*(z*y)=0$
2. $(x*(x*y))*y=0$
3. $x*x=0$
4. If $x*y=0$ and $y*x=0 \Rightarrow x=y$ for all $x,y,z \in x$

Definition 2.3[4, 5] A nonempty set X with a constant 0 and a binary operation * is called a BCK- algebra if it satisfies the following axioms.

1. $((x*y)*(x*z))*(z*y)=0$
2. $(x*(x*y))*y=0$
3. $x*x=0$
4. If $x*y=0$ and $y*x=0 \Rightarrow x=y$
5. $0*x=0$ for all $x,y,z \in X$

Example 2.4 Let X= {0, 1, 2, 3} be a set with the following Cayley table

*	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	3	1	0

Then (X, *,0) is a *BCK-algebra*

Definition 2.5 [6]: A d-algebra is a non-empty set *X* with a constant 0 and a binary operation * satisfying the following axioms:

1. $x * x = 0$
2. $0 * x = 0$
3. $x * y = 0$ and $y * x = 0$ Imply that $x = y$ for all x,y in X .

Example 2.6
Let X= {0, 1, 2, 3} be a set with the following Cayley table

*	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	3	1	0

Then (X, *,0) is a *d-algebra*

Definition 2.7 [6] Let S be a nonempty subset of a d-algebra X, then S is called subalgebra of X if $x*y \in S$ for all $x,y \in S$.

Definition 2.8 [8]: A BH-algebra is a non-empty set X with a constant 0 and a binary

Operation * Satisfying the following axioms:

1. $x * x = 0$
2. $x * 0 = x$
3. $x * y = 0$ and $y * x = 0$ Imply that $x = y$ for all x, y in X.

Example 2.9
Let X= {0, 1, 2, 3} be a set with the following Cayley table

*	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	3	1	0

Then (X, *,0) is a *BH-algebra*

Definition 2.10 [8] Let S be a nonempty subset of a BH-algebra X, then S is called subalgebra of X if $x*y \in S$ for all $x,y \in S$. *

We define $x*y=0$ if and only if $x \le y$, the $(X, \le)$ is an ordered set. Let $\{(X_i,*,0_i/i \in I\}$ be a non empty family of d-algebras. Then $(\Pi X_i,*,0)$ is a d-algebra, so called the direct product of d-algebras.

Definition 2.11 [8] Let X be a BH-algebra and I be a subset of X, then I is called an ideal of X if

(d1) $0 \in I$
(d2) $x*y \in I$ and $y \in I \Rightarrow x \in I$ for all $x,y \in I$
(d3) $x \in I$ and $y \in X \Rightarrow x*y \in I$

Definition 2.12 [8] A mapping f: X → Y of BH-algebras is called a homomorphism if $f(x*y) = f(x)*f(y)$ for all x,y X ∈X.

Note that if $f : X \to Y$ is homomorphism of BH-algebras, then $f(0)= 0$

Definition 2.13[7] Let X be a nonempty set. A fuzzy (sub) set μ of the set X is a mapping $\mu:X\to[0,1]$

Definition 2.14[7] Let μ be the fuzzy set of a set X. For a fixed $s\in[0,1]$, the set $\mu\_s=\{x\in X:\mu(x)\geq s\}$ is called an upper level of μ or level subset of μ

Definition 2.15[1] A fuzzy set μ in d-algebra X is called a fuzzy subalgebra of X if it satisfies $\mu(x*y)\geq\min\{\mu(x),\mu(y)\}$ for allx,y∈X.

Definition 2.16[9] A fuzzy set μ in BH-algebra X is called a fuzzy subalgebra of X if it satisfies $\mu(x*y)\geq\min\{\mu(x),\mu(y)\}$ for allx, y∈X.

Example 2.17 Let X = {0, 1, 2…} be a set given by the following cayley table

*	0	1	2
0	0	0	0
1	1	0	0
2	2	1	0

$$x*y=\begin{cases}0 & if \quad x\leq y\\ x-y & if \quad y<x\end{cases}$$

Then (X, *0) is an infinite BH-algebra. If we define a fuzzy set $\mu:X\to[0,1]$ by $\mu(0)=t_1$ $\mu(x)=t_2$ for all $x\neq0$,where $t_1>t_2$ then μ is a fuzzy BH-subalgebra of X.

Definition 2.18 [6] A triangular norm is a binary operation N on the unit interval [0, 1] which is commutative, associative, and monotone and has 1 as neutral element. It is a function $N:[0,1]^2\to[0,1]$ satisfying the following properties:

1.$N(x,1)=x$

2.$N(x,y)=N(y,x)$

3. $N(x,N(y,z))=N(N(x,y),z)$,

4.$N(x,y)\leq N(x,z)$ Whenever $y\leq z$, for all $x,y,z\in[0,1]$

3. N-FUZZY BH-SUBALGEBRAS

Definition 3.1. Let μ be a fuzzy set of a BH-algebra X. Then μ is called a fuzzy BH-subalgebra of X with respect to a triangular norm N (N-fuzzy BH – subalgebra of X) if for all

$$x,y\in X,$$
$$\mu(x*y)\geq N(\mu(x),\mu(y))$$

Examples 3.2: Let X= {0, 1, 2, 3} be a set with the following cayley table

*	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	2	0	0
3	3	3	3	0

Then (X, *,0) is a fuzzy BCK-algebra. We define fuzzy set μ in X by $\mu(0)=0.5,\mu(1)=0.2$, for $allx\neq0$. Then we define a triangular norm $N:[0,1]^2\to[0,1]$ by $N(X,Y)=\max\{x+y-1,0\}$ for all x,y∈ X. Then we can check that N is a triangular norm, and also an N-fuzzy BH-subalgebra of X.

Theorem 3.3 Let X be a BH-algebra. Given a∈[0,1] and let μ be an N –fuzzy BH-subalgebra of X.

Then we have
1. If a=1, then the upper level of μ in X, U(μ,a) is either empty or a BH-subalgebra of X.
2. If N=min, then U(μ,a) is either empty or a BH-subalgebra of X.
3. If N=min, then $\mu(0)\geq\mu(x)$ for all x∈X

Proof

Suppose that $U(\mu,1)$ is not empty. Then we let $x,y\in U(\mu,1)$.
Thus we have $\mu(x)\geq1$ and $\mu(y)\geq1$. Since μ is an N –fuzzy BH-subalgebra of X, we obtain $\mu(x*y)\geq N(\mu(x),\mu(y))\geq N(1,1)=1$
Hence $x*y\in U(\mu,1)$.
Therefore $U(\mu,1)$ is a BH-subalgebra
(2)Suppose that $U(\mu,a)$ is not empty, and let $x,y\in U(\mu,a)$. Then $\mu(x)\geq a$ And $\mu(y)\geq a$. It follows that $\mu(x*y)\geq N(\mu(x),\mu(y))=\min\{\mu(x),\mu(y)\}\geq\min\{a,a\}=a$

(3) Since $x*x=0$, we have $\mu(0)=\mu(x*x)\geq N(\mu(x),\mu(x))=\min\{\mu(x),\mu(x)\}=\mu(x)$ Hence $\mu(0)\geq\mu(x)$ for all $x\in X$

Definition 3.4. Let μ_1 and μ_2 be N-fuzzy BH–subalgebras of a BH-algebra X. The direct product of N-fuzzy BH-subalgebras μ_1 and μ_2 is defined by $\mu_1\times\mu_2(x,y)=N(\mu_1(x),\mu_2(y))$ for all x,y∈ X

Theorem 3.5

Let X be a BH-algebra. Let μ_1 and μ_2 be N-fuzzy BH –subalgebras of X. Then $\mu_1\times\mu_2$ is an N-fuzzy BH –subalgebras of X.

Proof

Let μ_1 and μ_2 be N-fuzzy BH –subalgebras of X.
Put $\mu=\mu_1\times\mu_2$ and let x,y∈ X
Then, we have $\mu((x_1,x_2)*(y_1,\ y_2))$

$$=\mu(x_1*y_1,x_2*y_2)$$
$$=(\mu_1\times\mu_2)(x_1*y_1,x_2*y_2)$$
$$=N(\mu_1(x_1*y_1),\mu_2(x_2*y_2))$$
$$\geq N(N(\mu_1(x_1),\mu_1(y_1)),N(\mu_2(x_2),\mu_2(y_2)))$$
$$=N(N(\mu_1(x_1),\mu_2(x_2)),N(\mu_1(y_1),),\mu_2(y_2)))$$
$$=N((\mu_1\times\mu_2)(x_1,x_2),(\mu_1\times\mu_2)(y_1,y_2))$$
$$=N(\mu(x_1,x_2),\mu(y_1,y_2))$$

Hence $\mu_1\times\mu_2$ is an N-fuzzy BH –subalgebras of X.

Theorem 3.6 Let f:X→Y be an epimorphism of BH-algebras, and μ be an N-fuzzy BH –subalgebras of Y. If μ of is an N-fuzzy BH –subalgebras of X.

Proof

Since f:X→Y is an epimorphism, μ of is obviously a fuzzy set of X.
Let x,y∈ X

Then $\mu\ of(x*y)=\mu(f(x*y))$
$=\mu(f(x)*f(y))$
$\geq N(\mu(f(x)),\mu(f(y)))$
$=N((\mu\ of)(x),(\mu\ of)(y)$

Hence μ of is an N-fuzzy BH –subalgebras of X.

Definition 3.7 Let f be a mapping on a set X, and μ be a fuzzy set of f (X) . Then the fuzzy set μ of is called Preimage of μ under f.

Corollary 3.8 An epimorphism Preimage of an N-fuzzy BH-subalgebras of a BH-algebra X is an N-fuzzy BH-subalgebra.

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