



ESTIMATION OF SHEARING FORCE AND VERIFICATION OF STOKES' LAW OF VISCOSITY

Physics

Arup Shit

Undergraduate, Bankura Christian College, Burdwan University

KEYWORDS

INTRODUCTION -

We know that Scientist George Gabriel Stokes gave his law in the year 1851 and after that there was a problem, how to derive it theoretically! so here specially I thought that why it cannot be proved in general way and also with the help of pressure term and the shearing force term as derived theoretically in this theorem it can be shown easily that Stokes' law is even valid theoretically also. But since it is a general way to prove the shearing term so it is really a new one and also this theorem gives a proof of Stokes' theorem.

ESTIMATION OF SHEARING FORCE -

When a spherical solid ball or sphere having radius a is allowed to drop through a liquid, having viscosity coefficient n then, for totally immersed condition when it is moving at downward under the influence of gravity then, tangential force will act on the surface of that sphere and it is given by the liquid for its viscous nature,

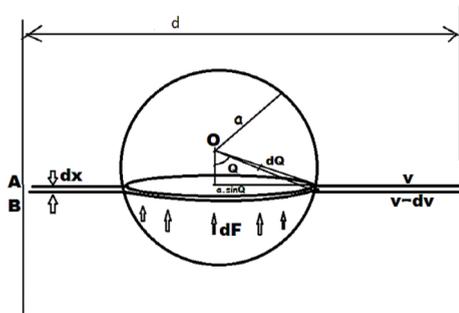
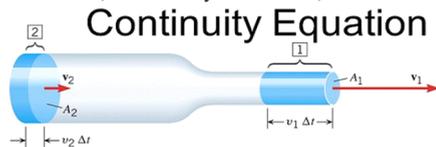


fig-1- the sphere is falling through the viscous medium]

Now for the lower portion of the sphere, we consider a circular elementary ring having radius $a \sin Q$ and width dx of it. Let, dx be an elementary segment [as shown in figure] of the liquid layer, such that the velocity of it at the layer A is v and at the layer B is $(v-dv)$, [since, from the continuity equation we know for the greater cross-sectional area the velocity of liquid flow must be lower than that for the smaller cross sectional area, more clearly if $A_1/A_2 > 1$, then $V_1/V_2 < 1$,



$$\rho_2 A_2 v_2 = \rho_2 A_1 v_1$$

Same, incompressible, fluid so rho drops out!

$$A_1 v_1 = A_2 v_2$$

fig:2- proof of equation of continuity]

we have the velocity gradient over this layer must be $= dv/dx$. Now, also if one can stay just on the sphere, then with respect to the frame of sphere, the liquid is flowing just opposite to the motion of sphere and for the corresponding elementary width dx of this liquid layer, it can be supposed that the liquid with a certain viscosity coefficient will pass through a region having width $(d-2a \sin Q)$ with velocity v , SO, We can apply here the Reynolds' law related with viscosity.

Here it should be,

$$v = \frac{k\eta}{\rho(d-2a \sin Q)} \text{ and } dx \approx a.dQ$$

And hence the change in velocity between two layers be given by

$$dv = \frac{2k\eta \cos Q dQ}{\rho(d-2a \sin Q)^2}$$

Where, k is Reynold's number and ρ is the density of the fluid medium,

Here the tangential force will act on that layer (At downward hemisphere of the sphere) along upward on the elementary area segment of it, and here this area be,

Now, the tangential stress on that elementary segment from Newton's law of viscosity which is tangential stress is proportional to the velocity gradient (dv/dx) ,

$$\frac{dv}{dx} = \frac{2k\eta \cos Q}{\rho(d-2a \sin Q)^2}$$

$$f \propto \frac{dv}{dx}$$

so,

$$F_1 = \int dF$$

$$\Rightarrow F_1 = \int_0^{\pi/2} \frac{4\pi k\eta^2 a^2 \sin Q \cdot \cos Q \cdot dQ}{\rho(d-2a \sin Q)^2}$$

Let us now consider $(d-2a \sin Q) = t$,

$$\text{hence; } -2a \cos Q \cdot dQ = dt$$

$$\text{so, } \cos Q \cdot dQ = \frac{(-dt)}{2a} \text{ and } \sin Q = \frac{d-t}{2a}$$

$$\Rightarrow F_1 = \int_d^{d-2a} \frac{4\pi k\eta^2 a^2 \cdot (d-t) \cdot (-dt)}{\rho t^2 \cdot \frac{2a}{2a}}$$

$$= \int_d^{d-2a} \frac{4\pi k\eta^2 a^2 \left(\frac{1}{t} - \frac{d}{t^2} \right) dt}{\rho \cdot Aa^2}$$

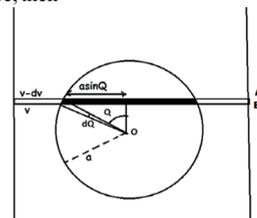
$$= \frac{4\pi k\eta^2 a^2}{Aa^2 \rho} \left[\log_e \left(\frac{d-2a}{d} \right) + \frac{d}{d-2a} - \frac{d}{d} \right]$$

$$\text{so, } F_1 = \frac{\pi k\eta^2}{\rho} \left[\frac{2a}{d-2a} - \log_e \left(\frac{d}{d-2a} \right) \right]$$

This is the force acting on the sphere along upward direction (as shown in figure) at the lower hemisphere,

On the other hand, for the upper portion of it the attractive force will act on the upper hemisphere along upward and the effect of this force must change layer to layer when the sphere is falling down through the viscous medium,

Similarly, for this case if we choose an elementary area segment on the upper hemisphere, then



So, here effectively we see that the velocity gradient is here actually negative, and it is $(-dv/dx)$ and the net force acting (along the upward due to adhesive attraction to the sphere by the liquid layers) on the upper hemisphere be given by

$$F_2 = \int_{\frac{\pi}{2}}^0 dF$$

$$= \int_{\frac{\pi}{2}}^0 \eta \cdot dA \cdot \left(-\frac{dv}{dx}\right)$$

$$\Rightarrow F_2 = \frac{\pi k \eta^2}{\rho} \left[\frac{2a}{d-2a} - \log_e \left(\frac{d}{d-2a} \right) \right]$$

Now, net force acting on the sphere along upward sence be

$$F = F_1 + F_2$$

$$\Rightarrow F = \frac{2\pi k \eta^2}{\rho} \left[\frac{2a}{d-2a} - \log_e \left(\frac{d}{d-2a} \right) \right]$$

This is the actual effective force acting on the spherical ball;

Now for $d \gg 2a, [(d-2a) \sim d]$

We have,

$$F = \frac{2\pi k \eta^2}{\rho} \cdot \frac{2a}{d}$$

$$= 4\pi \eta a \cdot \frac{k\eta}{\rho d}$$

This is the effective force on the whole spherical ball along upward, when the diameter of the vessel of liquid be much greater than that of the sphere or spherical ball, Now in case, if the spherical ball will achieve the constant velocity (So called terminal velocity) then for velocity of the ball v , which may be considered as the constant velocity of flow of liquid with respect to the frame of spherical ball or sphere,

So, we can apply

$$v = \frac{k\eta}{\rho d}$$

hence,

$$\text{the force will be } F = 4\pi \eta a v$$

Hence it will be the actual shearing force acting on the sphere or spherical ball along upward when, it will fall along downward through a viscous medium under gravity and the force acting due to pressure must be

$$F_p = 2\pi \eta a v$$

[which was proved before]

Total force along upward sence be

$$F_{tot} = F + F_p$$

$$= 4\pi \eta a v + 2\pi \eta a v$$

$$= 6\pi \eta a v$$

Hence

Which is accurate and actual Stokes' law in viscosity, and this estimation of shearing force.