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LEFT JORDAN AND LEFT DERIVATION ON RINGS

Mathematics

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ABSTRACT

I.N.Herstein [1] showed that every Jordan derivation on a prime ring not of characteristic 2 is a derivation, and it is proved in [2] that every continuous Jordan derivation on a semi simple banach algebra is a derivation. We shall extend these results to the case of ring R which implies and which has a commutator which is not a zero devisor then every left Jordan derivation on R is a left derivation.

KEYWORDS

Derivations, Jordan derivations, Left Jordan derivations, Commutator, Characteristic of a ring.

INTRODUCTION:

An additive mapping D of an (associative) ring into itself is a derivation if $D(ab) = aDb + (Da)b$ for all elements a and b of the ring. An additive mapping $D: R \to R$ will be called a derivation if $D(ab) = D(a)b + aD(b)$holds for all pairs$x, y \in R$. For a ring R In which $2x = 0$ implies $x = 0$, an additive mapping D of R into itself is said to be a Jordan derivation if $D(aob) = ao\,D(b) + (Da)ob$ for all a and b in R , where $xoy = xy + yx$ is the Jordan product of x and y in R. An additive mapping $D: R \to R$ will be called a Left Jordan derivation if $D(aob) = aoD(b) + boD(a)$ holds For all a and b in R .Thus every Left Jordan derivation on R is a Joran Derivation, and the aim of this paper is extend the class of rings for which it is known that the converse of this is true. This class contains all commutative rings in which $2x = 0$ implies $x = 0$ and all rings R with identity such that every Jordan homomorphism of R is the sum of a homomorphism and an anti homomorphism. I.N.Herstein [1] showed that every Jordan derivation on a prime ring not of characteristic 2 is a derivation, and it is proved in [2] that every continuous Jordan derivation on a semi simple Banach algebra is a derivation. We shall extend these results to the case of ring R which $2x = 0$ implies $x = 0$ and which has a commutator which is not a zero devisor then every left Jordan derivation on R is a left derivation.

Throughout this section D will denote a left Jordan derivation on a ring in which $2x = 0$ implies $x = 0$ and d will be the mapping from $R^2 = \{(a,b): a, b \in R\}$ to R defined by $d(a,b) = D(ab) - aD(b) - bD(a)$.

the mapping d is additive with respect to point wise addition on R^2 and is zero if D is a derivation. We shall use the notation $[a,b] = ab - ba, [a,b,c] = abc + cba$. The main results follow from the fact that $[a,b]od(a,b)$ and $[[a,b], r, d(a,b)]$ are zero for all a , b and r in R. This can be proved directly, but is more easily obtained from the following analogous results for Jordan homomorphisms proved in [3]. An additive mapping J of R into a ring S is a Jordan homomorphism if $J(aob) = (Ja)o(Jb)$ for all a, b in R.

MAIN RESULTS:

LEMMA 1: Let J be a Jordan homomorphism of R into a ring S in which $2x = 0$ implies $x = 0$.then for all a, b and r in,

$$(J(ab) - (Ja)(Jb))(J(ba) - (Ja)(Jb) = 0 \tag{1}$$

$$[J(ab) - (Jb)(Ja), Jr, J(ab) - (Ja)(Jb)] = 0 \tag{2}$$

PROOF: Let S be the ring obtained from R^2 by defining the product of (a,b) and (s,t) to be $(as, at + sb)$. Then the mapping J from R into S, defined by $Ja = (a, Da)$, is a Jordan homomorphism.

Then

$Ja = (a, Da)$, $Jb = (b, Db)$

$J(ab) = (ab, D(ab))$

Then

$(Ja)(Jb) = (a, Da)(b, Db)$

$= (ab, aDb + bDa)$

There fore

$(J(ab) - (Ja)(Jb)) =$

$(ab, D(ab)) - (ab, aDb + bDa) = (0, d(a,b))$

And $(J(ba) - (Ja)(Jb)) =$

$(ba, D(ba)) - (ab, aDb + bDa) = ([b,a], d(a,b))$

Now $(J(ab) - (Ja)(Jb))\,(J(ba) - (Ja)(Jb)) =$

$(0, d(a,b))\,([b,a], d(a,b)) = 0$

Hence proposition (1) is proved.

Now $J(b)J(a) = (b, Db)(a, Da)$

$= (ba, bDa + aDb)$

And $J(ab) - (Jb)(Ja) = ([a,b].d(a,b))$

Now

$$[J(ab)-(Jb)(Ja),Jr,J(ab)-(Ja)(Jb)]$$
$$=[([a,b],d(a,b)),(r,Dr),(0,d(a,b))]$$
$$=\big([a,b],d(a,b)\big)(r,Dr)\big(0,d(a,b)\big)+$$
$$(0,d(a,b))\,(r,Dr)([a,b],d(a,b))$$
$$=0$$

Hence lemma is proved. ♦

Lemma 2: For all a,b in R,

$[a,b]d(a,b)=0=d(a,b)[a,b]$ (3)

$[[a,b],r,d(a,b)]=0$ (4)

Proof: By (1) of lemma 1 we have,

$$(0,d(a,b))([b,a],d(a,b)]=0$$

And, therefore $[a,b]d(a,b)=0$

For all a and b in R .Since D is also a Jordan derivation on the ring obtained from R by reversing the product, it follows that $d(a,b)[a,b]=0$.

Hence (3) is obtained.

Similarly, by (2) of lemma 1 gives,

$$[([a,b],d(a,b)),(r,Dr),(0,d(a,b)]=0$$

And, therefore $[[a,b],r,d(a,b)]=0$ for all a,b and r in R. ♦

LEMMA 3: Let R be a 2-torsion semi prime ring and let $a,b\in R$ if for all $x\in R$ the relation holds

$axb+bxa=0$ (5)

holds then $axb=bxa=0$ is fulfilled for all $x\in R$.

PROOF: Let x , y be arbitrary elements from R.

Using (5) three times we obtain

$$[a(x)b]yaxb=-[bxa]yaxb$$
$$=-[b(xay)a]xb$$
$$=[a(xay)b]xb$$
$$=[ax(ayb)]xb$$
$$=-[ax(bya)]xb$$
$$=-(axb)y(axb)$$

Thus $2(axb)y(axb)=0$ for all $x,y\in R$.

Since R is a 2-torsion free semi prime ring, it follows that $axb=0$ for all $x\in R$. ♦

Lemma 4: Let P be a prime ideal in R then $[a,r,b]=0$ for all r in R implies$a\in P$ *or* $b\in P$.

Proof: Let k be arbitrary elements of R.

Then

$ahakb+bhaka=0$

$-ahbka-ahbka=0$

$-2ahbka=0$ (Since above lemma)

But then $ahbRa\in P$ for all h in R, and if a does not belongs to P, then $aRb\subset P$ so $b\in P$. ♦

Theorem 1: Let R be ring where $2x=0$ implies $x=0$ and which has a commutator which is not a zero devisor. Then every left Jordan derivation on R is a left derivation.

Proof: By (4) we have for all a,r,s and t in R,

$$[[s+a,t],r,d(s+a,t)]=0.$$

It follows that

$$[[s,t],r,d(a,t)]+[[a,t],r,d(s,t)]=0$$

And, on replacing t by $t+b$, that

$[[s,t]r,d(a,b)]+[[s,b],r,d(a,t)]+[[a,t],r,d(s,b)]+[[a,b],r,d(s,t)]=0$

For all a,b,r,s and t in R.

Let s,t be elements of R such that $[s,t]z=0$ or $z[s,t]=0$ implies $z=0$.

Then by (3) we have $d(s,t)=0$. So the substitution $a=s$ and $b=t$ give,

$$\big[[s,t]r,d(s,b)\big]=0$$

and $[[s,t],r,d(a,t)]=0$.

But then, as in the proof of lemma (4) with $h=k=[s,t]$ we have

$$[s,t]^2\,d(s,b)[s,t]^2=0$$

And $[s,t]^2\,d(a,t)[s,t]^2=0$

And so $d(s,b)=0$ and $d(a,t)=0$ for all a and b in R.

Finally,$[[s,t],r,d(a,b)]=0$ and therefore, by same argument $d(a,b)=0$ for all a and b in R, and D is a derivation. ♦

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