



THE THEORY OF POPULATION GENETICS BEFORE AND AFTER MENDEL.

Mathematics

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ABSTRACT

The beginning of genetics and population genetics are same things. Both started with Mendel. Genetics emerged on the basis of classical experiments done on plants. Here we discuss pioneers contribution in the development of population genetics before and after Mendel.

KEYWORDS

Population genetics , genepool, heridity, dilution

1. INTRODUCTION

In ancient time man had some vague idea of some general aspect of population of heridity. Many considered heridity to be some sort of a blending process because of which the offspring showed different "dilution" of the parental characteristics. The concept of blending inheritance fitted with ancient thinking as it could explain why some children were more like their parents where as other were less. Heridity is one of the fundamental characteristics of all life on earth, which deals with the transmission of character from parent to offspring. Genetics and population genetics is very young branch barely one and quarter a century old. Both are deals with the study of changes within genepool of Mendelian population may undergo when it is exposed to systematic forces such as selection pressure, mutation, genetic drift, migration and relative frequencies. A genepool is the sum of genes in reproductive gametes of a population. The genepool is transferred from one generation to the other through the sample drawn from a gametic pool. This sample of gametes will form zygotes of next generation will depend upon the kind of random samples drawn to form zygotes. Genepool of a population will consist of a large number of genes which will vary in their frequencies. Gene frequencies are defined as proportion of different alleles of a gene in a population, and in a particular generation these frequencies will depend upon their frequencies in preceding generation. These frequencies also depend on proportion of various genotype in total population.

2. Status of population genetics before Mendel

The earliest indications about the matril basis of heridity came from plant breeding procedure practised by farmers who were aware of various techniques of hybridization and selection for developing new varieties. In the middle of the eighteenth century a Swedish taxonomist Carolus linnaeus and German plant breeders Josef Gottlieb Kolreuter and Karl Friedrich von Gaertner performed artificial cross pollinations in plants and obtained hybrid off spring. In crosses of maize plants Gaertner found that if one parent has red kernels the other yellow the hybrid were all of the yellow type. When these hybrid were self fertilized the second generation showed roughly three yellow to one red plant identical to the 3:1 ratio found later by Mendel. About at the same time Knight seton and Goss were experimenting with inheritance of seed color in garden peas. They observed that a cross between plants with yellow seeds and green seeds produced all yellow seeds plant in the first generation. On self fertilization they obtained second generation off spring with both yellow and green seeds. Plants with green seeds on self fertilization produced only green seeds offspring showing that green seed color always breed true.

The mathematical biology begin from Lamarck. He proposed "Theory of inheritance of acquired Characteristic". Also he introduced the word biology to emphasize the kinship between living beings. Who proposed in 1809 all that has been acquired by the organism during its life time due to direct or indirect environment effects is preserved by the generation and is transmitted to the off spring.

The charls Robert Darwin an English biologist proposed the theory of natural selection. He published a paper "on the origin of species by means of natural selection. Or The preservation of favoured races in the struggle for life" in 1859. He gave the concept of evolutionary changes and established that natural selection in the agent that brings about changes to result in the origin of species by evolution. Again he

proposed in 1868 every somatic cell and tissue of the body produces a tiny particle called gemmule or pangene which contains both the parental and the acquired character. According to his gemmule or pangenes of the body cell collect in the gametes and are passed the zygote where they guide the growth of different parts of the embryo to form an offspring.

3. Status of population genetics at the stage of Gregor John Mendel

Gregor Jhon Mendal, as his window emphasized was a father of genetics. He was interested in the grandest of all natural processes which a biologist can contemplate. Mendel chose the garden "pea" as his experimental organism. His work on pea was an important contribution to its study. He was the first to demonstrate the mechanism of transmission of characters from one generation to the other. He also gave generalization of principles, some of which principles or law of inheritance form the corner stone in genetics they are.

(A). Law of Dominance :- One factor in a pair may mask or prevent the expression of the other.

(B). Law of segregation or law of purity of gamete :- During gamete formation the genes of a particular character separate and enter different gametes.

(C). Law of independent assortment :- The genes for each pair of characters separate independently from those of other characters, during gamete formation.

Mendel had used probability theory to determine model for the mechanism of segregation. The law of probability apply to the genetic mechanism as well as to other processes in which uncertainty exist. Evidently genetic transmission of character from parent to offspring is much like tossing a coin.

Let us consider population of an individual with two alleles dominant 'a₁' and recessive 'a₂'.

Then evidently, Probability of a₁, P(a₁) = 1/2 and Probability of a₂, P(a₂) = 1/2

The 2-ploid gametes produced by the population are a₁a₁, a₁a₂, a₂a₁, a₂a₂ called dominant, hybrid and recessive respectively.

P(a₁a₁) = P(a₁) P(a₁) = 1/2 . 1/2 = 1/4 offspring is dominant

P(a₁a₂) = P(a₁) P(a₂) = 1/2 . 1/2 = 1/4 offspring is hybrid

P(a₂a₁) = P(a₂) P(a₁) = 1/2 . 1/2 = 1/4 offspring is hybrid

P(a₂a₂) = P(a₂) P(a₂) = 1/2 . 1/2 = 1/4 offspring is recessive.

When two individuals heterozygotes for the same gene a₁a₂ are crossed (a₁a₂ x a₁a₂), they segregate in the ratio 1a₁a₁ : 2a₁a₂ : 1a₂a₂ ie 1a₁ : 2a₁ : 1a₂. This ratio can also be obtained by simple expansion of Binomial ie

$$\begin{aligned}(a_1 + a_2)^2 &= 2c_0 a_1^2 + 2c_1 a_1 a_2 + 2c_2 a_2^2 \\ &= 2c_0 a_1 a_1 + 2c_1 a_1 a_2 + 2c_2 a_2 a_2 \\ &= 2c_0 a_{11} + 2c_1 a_{12} + 2c_2 a_{22}\end{aligned}$$

Where the population $a_1, a_2 = (a_1, a_2)$ are abbreviated as a_1, a_2 (a_1), a_2 respectively and ' a_1 ' is the probability that an event will occur and ' a_2 ' is the probability that it will not occur. August weisman established in 1892, that the germ (cell) cells are apart from the body (somatics) cells early in the embryonic development and it is only the changes in the germ plasm that can effect the characteristic of future generations.

4. Status of Population genetics after mendel

Mathematics has helped in the under standing of the genetic of quantative character in the evolutionary context a field of biology which attracted statisticians and physician such as G.H. Hardy, W. weingberg, J.B.S. Haldan, R.A Fisher, S. Wright etc. Genetics, in the real sense, was born on 1900 when Hugo devaries, corrence and von. Tschermak independently rediscovered mendel's law of particular inheritance. The considerable genetic variability exists within a population as well as between different population of the same species. It is these genetic differences caused by mutation and recombination and acted upon by selection and other factors like migration that contribute to evolution. Hugo devaries was the first hybridist who used the term mutation to describe mutation is the ultimate sources of all genetic variation it provides the raw matrial for evolution ie genetic variability present in individual chromosomes. He published a book in 1901 entitled "The mutation theory" and express the phenotypic changes which were heritable.

Mendel's concept of the segregation and recombination of unit characters was advanced by Boveri and sutton in 1902. They pointed out the importance of the chromosome mechanism in the hereditary system. Boveri and Sutton also studied the behavior of chromosomes during cell division. They found a parallelism in the behavior of Mendelian factors (genes) and chromosome. They found that chromosomes occur in pair and each member of a pair seperates or segregates from each other at the time of reduction division.

Mendel considered the consequence of repeated self fertilization but he did not consider the consequences of random mating system. In this connection the first pioneer Hardy and Weinberg 1908 discovered law of equilibrium population. This law is the foundation of population genetics. The law states that gene frequencies in a population remains constant from generation to generation if no evolutionary processes like migration, mutation, selection and drift are operating. Thus if mating are random and no other factors disturbe the reproductive abilities of any genotype, the equilibrium genotypic frequencies are given by the square of the allelic frequencies. This law provides a basic algebraic formulæ for describing the expected frequencies of various genotypes in a population. In a Mendel population the frequencies of two alleles belonging to a particular gene be describe 1:2:1 but in a random mating population the ratio of 1:2:1 is not always available. If there are only two alleles a_1 and a_2 the proportion of gametes carrying a_1 will be p and the proportion of gametes carrying a_2 will be q . The probabilities of individuals obtained from random mating form such a population be

	Sperm		
		$a_1 (p)$	$a_2 (q)$
EGGS	$a_1 (p)$	$a_1 a_1 (p^2)$	$a_1 a_2 (pq)$
	$a_2 (q)$	$a_2 a_1 (pq)$	$a_2 a_2 (q^2)$

$$a_1 = a_{21}; \\ (p+q)=1$$

This leads to the formulation of hardy-weing berg equation $a_1^2 (p^2) + a_2 (2pq) + a_2^2 (q^2)$.

The similarty of their work however remain unnoticed until stern 1943 drew attention to both papers and recommended that names of both discoverers' be attached to the population formulae.

Pearl, Jennings and wrights work on inbreeding in (1912-1920) and established process of marting of individuals which are more closely related than the average of the population. In a population at equilibrium inbreeding changes the frequency of genotypes ($a_1 a_1 = P^2$, $a_1 a_2 = 2pq$, and $a_2 a_2 = q^2$) to ($a_1 a_1 = p^2 + pqf$, $a_1 a_2 = 2pq + 2pqf$ and $a_2 a_2 = q^2 + pqf$) where f is inbreeding coefficient (or Wright coefficient of inbreeding) or F is the probability that two homozygous alleles united in a zygote have descended from an ancestor common to both parents.

J.B.S. Haldane R.A. fisher and sewal wright the credit for originating the field of population genetics. J.B.S. Haldane's early papers in population genetics called "Beanbag genetics". In his first paper on the mathematical theory of natural selection 1924, he derived a formulæ for measuring the intensity of selection operating in a population under various biological situations. Haldane (1931) investigated situation where mutant genes are disadvantageous singly but become advantageous in combination. Haldane (1937) most important papers were on the effect of variation in fitness ie genetic load and the cumulative cost of an allelic substitution. In this paper Haldane demonstrated that the effect of recurrent mutation on population fitness does not depend on the harmfulness of the individual mutation but on their rate of occurrence. For independently active recessive mutations the load is simply the gametic mutation rate and for completely dominant mutation it is twice this value.

R.A. Fisher a great mathematical and statistical skill. In 1930 his book the genetical theory of natural selection, he reconcited Mendelian inheritance with biometry. He showed correlation between physical measurements of closed relatives with Mendelism and realized deviation from perfect correlation with proportion of shared genes caused either by dominance and epistasis or by environmental effects and showed that dominance contributes to the correlation, but not to that of parent and offspring. He showed for the first time that selection in favour of heterozygotes leads to include the accidents of gene sampling that occure during reproduction on finite population.

Sewal wright gave shifting balance theory grew out of his studies on the guinea pig's coat colour his observation on the effects of inbreeding and his analysis of the history.

Sewal wright and J.B.S. Haldane around 1930 began to calculate the evolutionary effects of mutation, they showed that in a species with several hundred thousand members mutation could not be a pacemaker. Almost all mutations occurred several times in a generation in one member or another of a species.

In 1955 kimura proposed diffusion equation solution to the distribution of alleles frequincies. He also calculated that the average number of generation necessary for the fixation of a new neutral mutation is approximately four times the number of parents in each generation.

Molecular genetic came into existence in 1953 with the discovery of genetics matrial as a macromolecule DNA by Watson and crick. DNA in any organism serves two fundamental purposes duplication of a gene and the protein synthesis. In each process it preserves the biological information. Kimura developed the mathematical theory of molecular evolution by mutation and random drift.

5. Purpose of the work

The knowledge of the early history of population ganeties is essential for student's of Bio-mathematics and researcher's

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