



TOTAL DOMINATOR CHROMATIC NUMBER OF PATHS AND CYCLES THROUGH COMPUTER PROGRAMMING

Mathematics

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ABSTRACT

A total dominator coloring of a graph $G=(V,E)$ without isolated vertices is a proper coloring together with each vertex in G properly dominates a color class. The total dominator chromatic number of G is a minimum number of color classes with additional condition that each vertex in G properly dominates a color class and is denoted by $\chi_{td}(G)$. In this paper we introduce C++ programmes that are able to efficiently determine on approximation to the total dominator chromatic number of Paths and Cycles.

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KEYWORDS

Total dominator coloring, Total dominator chromatic number.

INTRODUCTION

In this paper we only consider Paths and Cycles. Further details in graph theory can be found in F.Harry[4]

Let $G=(V,E)$ be a graph with minimum degree atleast one. The Path and Cycle of order n are denoted by P_n and C_n respectively.

A proper coloring of G is an assignment of colors to the vertices of G , such that adjacent vertices have different colors. The smallest number of colors for which there exists a proper coloring of G is called a chromatic number of G , and is denoted by $\chi(G)$. A total dominator coloring (td-coloring) of G is a proper coloring of G with extra property that every vertex in G properly dominates color class. The total dominator chromatic number is denoted by $\chi_{td}(G)$ and is defined by the minimum number of colors needed in a total dominator coloring of G . This concept was introduced by A.Vijayalekshmi in [1]. This notion is also referred as smarandachely k -dominator coloring of G , ($k \geq 1$) and was introduced by A.Vijayalekshmi in [2]. For an integer $k \geq 1$, a smarandachely k -dominator coloring of G is a proper coloring of G , such that every vertex in a graph G properly dominates a k color class. The smallest number of colors for which there exists a smarandachely k -dominator coloring of G is called the smarandachely k -dominator chromatic number of G and is denoted by $\chi_{td}^k(G)$.

In a proper coloring C of a graph G , a color class of C is a set consisting of all those vertices assigned the same color. Let C be a minimum td-coloring of G . We say that a color class is called a non-dominated color class (n -d color class) if it is not dominated by any vertex of G and these color classes are also called repeated color classes.

The total dominator chromatic number of Paths and Cycles were found in [3]. We have the following observation from [3]

Theorem A [3]

Let G be P_n or C_n . Then

$$\chi_{td}(P_n) = \chi_{td}(C_n) = \begin{cases} 2 \left\lfloor \frac{n}{4} \right\rfloor + 2 & \text{if } n \equiv 0 \pmod{4} \\ 2 \left\lfloor \frac{n}{4} \right\rfloor + 3 & \text{if } n \equiv 1 \pmod{4} \\ 2 \left\lfloor \frac{n+2}{4} \right\rfloor + 2 & \text{Otherwise} \end{cases}$$

Main Results

Section 1.1

In this section we have to find the total dominator chromatic number of paths by using C++ programme.

A path of 'n' vertices, denoted by P_n , is a connected graph where all but two vertices have degree 1. We label the vertices of P_n as v_i for $(1 \leq i \leq n)$. Furthermore let (v_i, v_{i+1}) be an edge of P_n for $i < n$.

Here is the source code of C++ programme to find the total dominator chromatic number of Paths. The C++ programme is successfully compiled and run on C++ platform.

Program as follows

```
#include "stdafx.h"
#include <Windows.h>
#include <conio.h>

#include <iostream> using namespace std; int main() {
int inpt;

cout << "Enter the Value" << endl;
cin >> inpt;
while (inpt >= 11)

{
int n = inpt; // matrix row
int m = inpt; // matrix column
// dynamic allocation

int** ary = new int*[n]; //logic matrix int** mat = new int*[n];
//adjacency matrix for (int i = 0; i < n; ++i)

{
ary[i] = new int[m];
mat[i] = new int[m];
}

// fill ary

for (int i = 0; i < n; ++i) for (int j = 0; j < m; ++j) ary[i][j] = i;
system("pause"); system("cls");
cout << "\n";

cout << "The Adjacency Matrix for P" << n << "\n" << "\n";
for (int i = 0; i < n; i++)
{
for (int j = 0; j < n; j++)
{
if (ary[j][i] == i + 1 | ary[j][i] == i - 1)
{
mat[i][j] = 1;
cout << mat[i][j] << " ";
}
}
else
{
mat[i][j] = 0;
cout << mat[i][j] << " ";
}
}
cout << "\n";
}
```

```

system("pause");
//-----END LOGIC TO FORM MATRIX----- int d;
cout<<"\n";

if(n%4==3)// CONDITION FOR GETTING SUB MATRIX
{
cout<<"Order of sub matrices for the above Matrix is : "<<(2*(n/4))
<<"x"<<(2*(n/4))<<"\n";

d=2*(n/4);
cout<<"\n";
}

else// CONDITION FOR GETTING SUB MATRIX

{
cout<<"Order of sub matrices for the above Matrix is : "<<(2*(n/4)
-1)
<<"x"<<(2*(n/4)-1)<<"\n";
d=2*(n/4)-1;
cout<<"\n";
}

cout<<"for sub matrices"<<" ";
system("pause");
system("cls");
cout<<"Sub Matrices"<<" "<<d<<"x"<<d;
cout<<"\n"<<"\n";
int r=0;// matrix row
int c=0;// matrix column int k;
int l;
if(n%4==0)

{
l=n/2+2;
}

else

{
l=n/2+3;
}

HANDLE hConsole=GetStdHandle(STD_OUTPUT_HANDLE);
for(k=0;k<l;k++)

{
cout<<"Sub Matrix"<<" "<<k+1;
cout<<"\n"<<"\n";
for(int i=0;i<n;i++)
{
for(int j=0;j<n;j++)
{
if(j>=k&&j<d&&i>=k&&i<d)
{
SetConsoleTextAttribute(hConsole, FOREGROUND_RED |
FOREGROUND_INTENSITY);
cout<<mat[i][j]<<" ";
}

else

{
SetConsoleTextAttribute(hConsole, FOREGROUND_GREEN |
FOREGROUND_INTENSITY);
//cout<<" "<<" ";
cout<<mat[i][j]<<" ";
}
}
}
cout<<"\n";
}

if(k==l-1)

{
SetConsoleTextAttribute(hConsole, FOREGROUND_GREEN |
FOREGROUND_INTENSITY);
cout<<"\n";
}
    
```

```

cout<<"No. of Sub Matrices are"<<" "<<l;
cout<<"\n";
cout<<"\n";
cout<<" Total Dominator Chromatic Number of " <<"P" <<n<<" "
<<"is"
<<" "<<l;
cout<<"\n";
cout<<"\n";
} system("pause"); system("cls"); d++;
}
return 0;
for(int i=0;i<n; ++i)
delete[] ary[i]; delete[] ary; return 0;
}
return main();
}
    
```

Runtime Test



```

Sub Matrix 4
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1
Press any key to continue . . .
    
```

```

Sub Matrix 5
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1
Press any key to continue . . .
    
```

```

Sub Matrix 6
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0
Press any key to continue . . .
    
```

```

Sub Matrix 7
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0
Press any key to continue . . .
    
```

```

Sub Matrix 8
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0
Press any key to continue . . .
    
```

```

Sub Matrix 9
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1
No. of Sub Matrices are 9
Total Dominator Chromatic Number of P13 is 9
Press any key to continue . . .
    
```

Section 1.2

Program source code

In this section, we give the source code of the C++ program to find the td-chromatic number of a cycle. A cycle on n vertices denoted by C_n is a connected graph where each vertex has degree two. We label the vertices of C_n as v_i for $1 \leq i \leq n$ and let (v_i, v_{i+1}) be an edge of C_n for $1 \leq i \leq n - 1$. We let (v_n, v_1) be the remaining edge of C_n . The program is successfully compiled and tested under C++ platform. The program output is also shown below

Program as follows

```

#include "stdafx.h"
#include <Windows.h>
#include <conio.h>F

#include <iostream> using namespace std; int main() {
int inpt;

cout << "Enter the Value" << endl;
cin >> inpt;
while (inpt >= 11)

{
// dimensions
int n = inpt; // matrix row
int m = inpt; // matrix column
// dynamic allocation
int** ary = new int*[n]; //logic matrix

int** mat = new int*[n]; //adjacency matrix for (int i = 0; i < n; ++i)
{
ary[i] = new int[m];
mat[i] = new int[m];
}

// fill ary

for (int i = 0; i < n; ++i) for (int j = 0; j < m; ++j) ary[i][j] = i;
system("pause"); system("cls");
cout << "\n";
cout << "The Adjacency Matrix for C" << n << "\n" << "\n";
// ----LOGIC TO FORM MATRIX----- for (int i = 0; i < n; I++)
{
for (int j = 0; j < n; j++)

{
if (ary[j][i] == i + 1 | ary[j][i] == i - 1 | ary[j][i] == i + (n - 1)
| ary[j][i] == i - (n - 1))
{
mat[i][j] = 1;
cout << mat[i][j] << " ";
}
else

{
mat[i][j] = 0;
cout << mat[i][j] << " ";
}
}
}
}
    
```


CONCLUSION

In this paper, we investigated the hard counting problems, the total dominator chromatic number of paths and cycles. By introducing and equivalent rare-event estimation problem. We were able to apply the evolutionary C++ approach. We showed the C++ program provides a provable probabilistic performance lower bound guarantee which is easy to calculate on line. Our numerical output indicate that the proposed method is successful.

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