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MHD CASSON FLUID FLOW PAST A STRETCHING CYLINDER SUBJECT TO NANOPARTICLES WITH CATTANEO-CHRISTOVE HEAT FLUX

Mathematics

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ABSTRACT

This article presents the steady, incompressible Casson nanofluid flow over a stretching cylinder subject to thermophoresis and Brownian motion in the presence of prescribed heat flux. Instead of Fourier's law Cattaneo-Christove heat flux theory is used to derive the energy equation. This theory can predict the characteristics of thermal relaxation time. The governing partial differential equations are transfigured into a system of ordinary differential equations by applying suitable similarity solutions and solved numerically by using Runge-Kutta fourth order method with shooting technique. The aim of the present study is to analyze the influence of various parameters, viz, Casson parameter, curvature parameter, thermal relaxation parameter, thermophoresis parameter, Prandtl number and Brownian motion parameter on the velocity profile, temperature and concentration profiles.

KEYWORDS

Casson nanofluid, Cattaneo-Christov Heat Flux model, MHD and prescribed heat

INTRODUCTON:

Heat transfer takes place when there is temperature difference between the two neighbouring objects. It has various usages like residential, industrial and engineering such as power production, air coolers, nuclear reactor cooling, heat and conduction in tissues etc. Fourier[1] propounded the well known law of heat conduction which is basis to know the behavior of heat transfer in different practical conditions. One of the major limitation of this model is it yields energy equation in parabolic form which shows the whole substance is instantly affected by initial disturbance. To overcome this limitation Cattaneo [2] upgraded Fourier's law from parabolic to hyperbolic partial differential equation by adding thermal relaxation time which allows the transmission of heat through propagation of thermal waves with finite speed. Later Christov [3] modified Cattaneo model by considering Oldroyd's upper convicted derivative to gain the material invariant formulation. Straughan [4] employed Cattaneo – Christov model in horizontal layer of isochoric flow with thermal convection. Tibullo and zampoli [5] perused the uniqueness of Cattaneo – Christov model for isochoric flow of fluids. Han et.al [6] investigated the heat transfer of viscoelastic fluid by using Cattaneo – Christov model over a stretching sheet. Mustafa [7] analyzed the rotating Maxwell fluid flow past a linearly stretching sheet with consideration of Cattaneo – Christov heat flux.

The study of non – Newtonian fluids is most important in the branches of engineering sciences. To characterize the flow and heat transfer, several rheological models have been proposed. Among these Casson fluid is one of the non – Newtonian fluids which fits rheological data better than other models for many materials as it acts like an elastic solid and which exhibits yield stress in the constitutive equation. The exemplars of this fluid are jelly, tomato sauce, human blood, honey etc. Many authors worked on this Casson fluid by considering over different geometries [8-10]. Fredrickson [11] analyzed the Casson fluid flow in a tube. Eldabe and salwa [12] investigated the flow of Casson fluid for between two rotating cylinders. The three dimensional MHD Casson fluid flow over a porous linearly stretching sheet was elucidated by Nadeem et.al [13]. The effect of viscous dissipation on radiative MHD free convection towards a laminar boundary layer flow of a Casson fluid due to a horizontal circular cylinder in non-darcy porous medium with partial slip conditions was elaborated by Makanda et al [14].

Now a days a continuous probe is going on in the flow analysis of nanofluids as it has many applications in heat transfer such as heat exchangers, radiators, hybrid – powered engines, solar collectors etc. In nanofluids the generally used nanoparticles are prepared by metals, carbides, oxides etc, and base fluids includes water, ethylene glycol and oil. Nanofluids show increase in thermal conductivity and convective heat transfer coefficient when estimated with base fluid. The investigations related to the rheology of nanofluids, International Nanofluid Property Benchmark Exercise (INPBE) revealed that nanofluid has both Newtonian and non – Newtonian behavior. Choi [15] was the first person who worked on this nanotechnology. Eastman observed amplification of thermal conductivity in nanofluids. Malik et.al [16] scrutinized the flow of Casson nanofluid over a vertical exponentially stretching cylinder. The study of heat and mass transfer over an exponentally stretching cylinder has many applications in piping and casting systems, fiber technology etc. Wang [17] studied the viscous flow and heat transfer past a stretching cylinder. Recently Majeed et.al [18] illustrated the effect of partial slip and heat flux moving over a stretching cylinder.

This paper focuses on the study of heat and mass transfer behavior of MHD Casson nanofluid flow towards a stretching cylinder in the presence of prescribed heat flux using Cattaneo christov heat flux model. The model equations of the flow are solved numerically by utilizing Runge-Kutta fourth order method with shooting technique. Effects of the various parameters (such as Casson parameter, curvature parameter, Thermal relaxation parameter, Brownian motion parameter, thermophoresis parameter) on velocity temperature, concentration are discussed and illustrated through graphs.

Mathematical Formulation:

Consider a steady, laminar axisymmetric boundary layer flow of an isochoric Casson nanofluid along a horizontally stretching cylinder of radius 'a', where x-axis is along the axis of cylinder and the radial co-coordinate r is normal to the axis of cylinder using Buongiorno model. It is considered that the surface of the cylinder has the linear velocity $U_w(x)=\frac{U_0 x}{l}$ where U_0 s the reference velocity, l is the characteristic length, T_w is the constant temperature, C_w is the susceptibility of the concentration. Moreover, it is assumed that the uniform magnetic field is applied in the radial direction. Thermophoresis and Brownian motion are considered into account. The rheological equation of a Casson fluid can be defined as follows

$$\tau_{ij}=\begin{cases}2\left(\mu_B+\frac{p_y}{\sqrt{2\pi}}\right)e_{ij}, & \pi>\pi_c\\ 2\left(\mu_B+\frac{p_y}{\sqrt{2\pi_c}}\right)e_{ij}, & \pi<\pi_c\end{cases} \tag{2}$$

where $\tau_{ij}=e_{ij}e_{ij}$ is the component of stress tensor μ_B is the Casson viscosity coefficient, π is the product of the components of the deformation rate tensor with itself and π_c is the critical value of this product following the non-Newtonian model and P_y is the yield stress of the fluid. According Cattaneo-Christove model the heat flux (q) can be represented as

$$q+\lambda_2\left(\frac{\partial q}{\partial t}+V.\nabla q-q.\nabla V+(\nabla.V)q\right)=-k\nabla T \tag{2}$$

where λ_2 is thermal relaxation time, k is the thermal conductivity and V is the velocity vector. If λ_2=0 then Eq. (2) becomes classical Fourier's law. For steady incompressible fluid flow Eq. (2) reduces to

$$q+\lambda_2(V.\nabla q-q.\nabla V)=-k\nabla T \tag{3}$$

The governing equations of the flow can be denoted in the following mathematical model

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0 \quad (4)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \nu\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - \frac{\sigma B^2}{\rho}u \quad (5)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} + \lambda_2\left(u^2\frac{\partial^2 T}{\partial x^2} + v^2\frac{\partial^2 T}{\partial r^2} + 2uv\frac{\partial^2 T}{\partial r\partial x} + u\frac{\partial u}{\partial x}\frac{\partial T}{\partial x} + u\frac{\partial v}{\partial x}\frac{\partial T}{\partial r} + u\frac{\partial u}{\partial r}\frac{\partial T}{\partial x} + v\frac{\partial v}{\partial r}\frac{\partial T}{\partial r}\right) = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \tau\left(D_B\left(\frac{\partial T}{\partial r}\frac{\partial C}{\partial r}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial r}\right)^2\right) \quad (6)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial r} = D_B\frac{\partial^2 C}{\partial r^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial r^2} \quad (7)$$

The related boundary conditions are

$$u = U_w(x) + B_1\nu\left(1+\frac{1}{\beta}\right)\frac{\partial u}{\partial r},\ v = 0,\ k\frac{\partial T}{\partial r} = -q_w(x), C = C_w \quad \text{at } r = a \quad (8)$$

$$u \to 0, T \to T_\infty, C \to C_\infty \quad \text{as } r \to \infty \quad (9)$$

where u and v are the components of velocity in x and r directions respectively, $\beta = \mu_B\sqrt{2\pi_c}/P_y$ is the non-Newtonian Casson parameter, ν the coefficient of viscosity, σ is the electrical conductivity, D_B is the Brownian diffusion coefficient, D_T is thermophoresis diffusion coefficient, c_p is the specific heat at constant pressure, T is the temperature of the fluid, C is the local nano particle volume fraction, B is the uniform Magnetic field, ρ is the fluid density, B_1 is the velocity slip factor, $\tau = (\rho c)_p/(\rho c)_f$ is the ratio of the effective heat capacity of the ordinary fluid.

We introduce the following similarity transformations

$$\eta = \frac{r^2-a^2}{2a}\frac{U_0}{\nu l}, \psi = \sqrt{\nu x U_w}\,a f(\eta), T = T_\infty + \frac{q_w}{k}\sqrt{\frac{\nu x}{U_w}}\theta(\eta), \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty} \quad (10)$$

Using above non dimensional variables (10), equations (5) – (9) are transformed into the following system of ordinary differential equations

$$\left(1+\frac{1}{\beta}\right)\left((1+2\gamma\eta)f''' + 2\gamma\eta f''\right) + (ff'' - f'^2) - Mf' = 0 \quad (11)$$

$$(1+2\gamma\eta - Pr\lambda f^2)\theta'' + 2\gamma\theta' - \Pr(f'\theta - f\theta' + \lambda(f'^2\theta - ff'\theta' - ff''\theta) + (1+2\gamma\eta)\left(Nb\theta'\phi' + Nt\theta'^2\right) = 0 \quad (12)$$

$$(1+2\gamma\eta)\phi'' + 2\gamma\phi' + \frac{Nt}{Nb}\left((1+2\gamma\eta)\theta'' + 2\gamma\theta'\right) + Lef\phi' = 0 \quad (13)$$

Subject to the boundary conditions

$$f(0) = 0, f'(0) = \left(1 + B\left(1+\frac{1}{\beta}\right)f''(0)\right),\ \theta'(0) = -1, \phi(0) = 1 \ at\ \eta = 0 \quad (14)$$

$$f' = 0, \theta = 0, \phi = 0 \ at\ \eta = \infty \quad (15)$$

where $\gamma = \sqrt{\frac{l\nu}{a^2U_0}}$ is a curvature parameter, $M = \frac{\sigma B^2 l}{\rho U_0}$ is the Magnetic parameter, $\lambda = \lambda_2\frac{U_0}{l}$ is the thermal relaxation parameter, $Pr = \frac{\nu}{\sigma}$ is the Prandtl number, $b = \frac{\tau D_B C_\infty}{\nu}$ is the Brownian motion parameter, $Nt = \frac{\tau D_T \Delta T}{T_\infty \nu}$ is the thermophoresis parameter $Le = \frac{\nu}{D_B}$ is the Lewis number, $B = B_1\sqrt{\frac{U_0\nu}{l}}$ is the slip parameter.

The expression for local Nusselt number and Sherwood number in dimensionless form are defined as $\frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0)$ and $\frac{Sh_x}{\sqrt{Re_x}} = -\phi'(0)$ (16)

RESULTS AND DISCUSSIONS: In the present paper the characteristics of Cattaneo-Christov heat flux model for Casson nanofluid due to a stretching cylinder is analyzed graphically for different parameters on velocity, temperature and concentration distributions shown in figs. (1-16). The present results are compared with Majeed et al and remarkable agreement can be seen in Table.1.

Table 1: Comparison table for $\theta(0)$ for disparate values of Pr with $\beta\to\infty$, $Nt=Nb=\lambda=$ Le $=0$,

γ	Pr	θ(0)	
		Majeed et al [22]	Present results
0	0.72	1.2367	1.231421
	1	1.0000	0.999516
	6.7	0.3333	0.333322
	10	0.2688	0.268780
1	0.72	0.8701	0.807961
	1	0.7439	0.717881
	6.7	0.2966	0.298178
	10	0.2422	0.245124

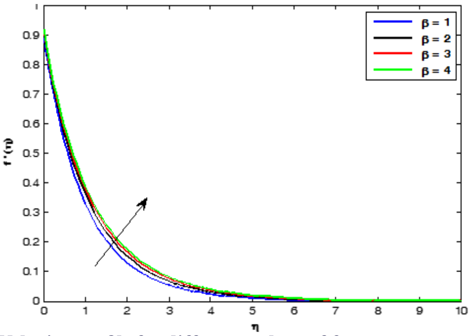

Fig. 1. Velocity profile for different values of β

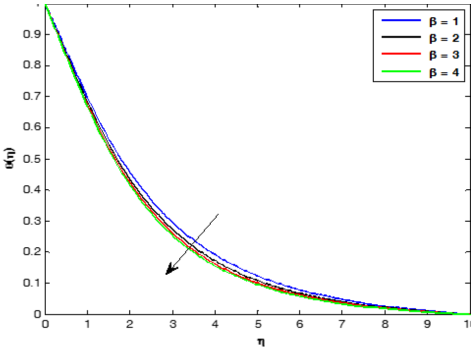

Fig. 2. Temperature profile for different values of β

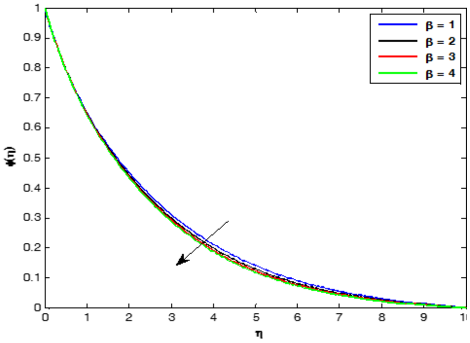

Fig. 3. Concentration profile for different values of β

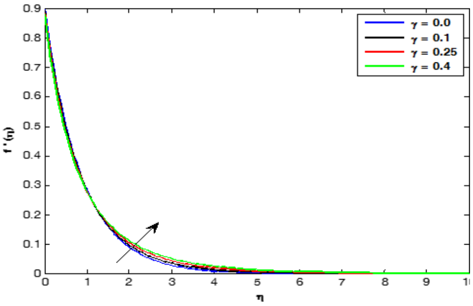

Fig. 4. Velocity profile for different values of γ

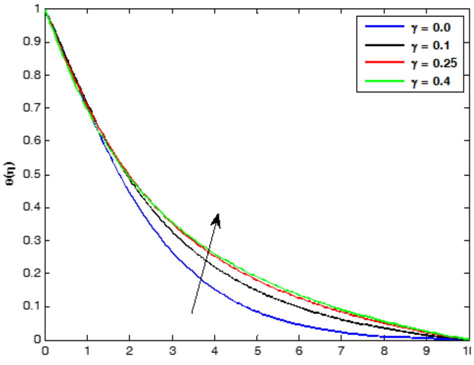

Fig. 5. Temperature profile for different values of γ

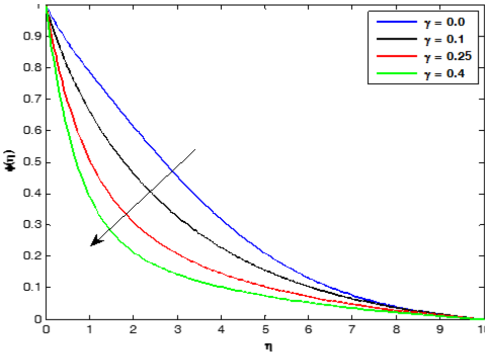

Fig. 6. Concentration profile for different values of γ

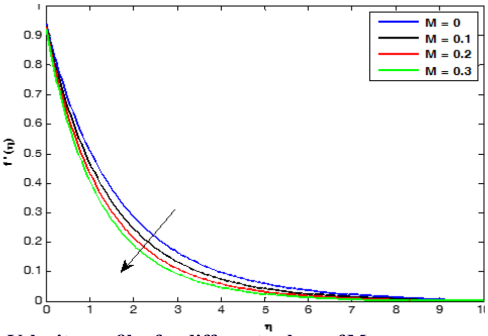

Fig. 7. Velocity profiles for different values of M

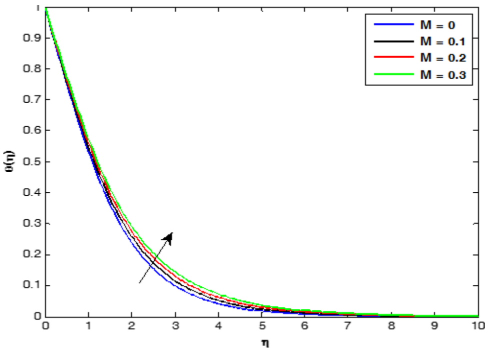

Fig. 8. Temperature profiles for different values of M

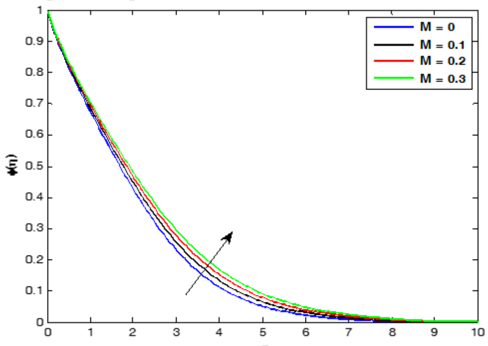

Fig. 9. Concentration profile for different values of M

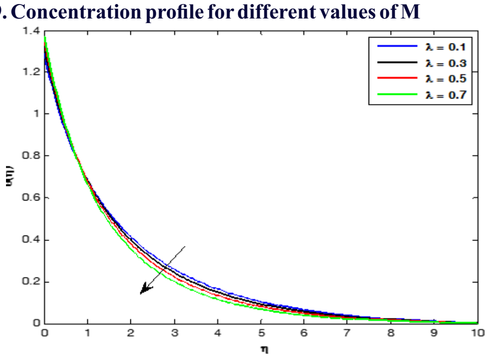

Fig. 10. Temperature profile for different values of λ

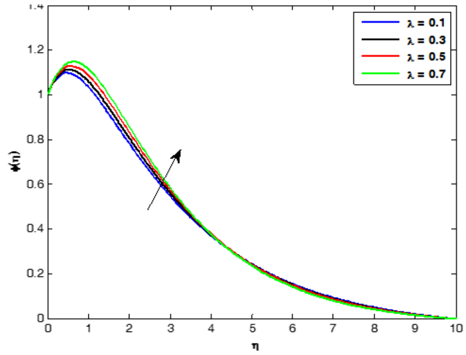

Fig. 11. Concentration profile for different values of λ

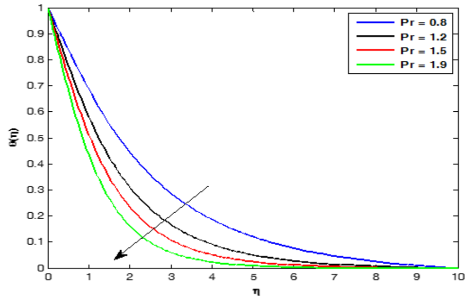

Fig. 12. Temperature profile for different values of Pr

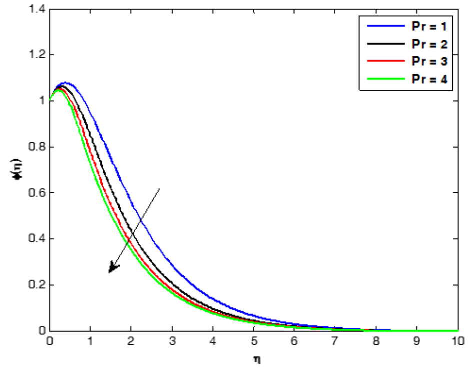

Fig. 13. Concentration profile for different values of Pr

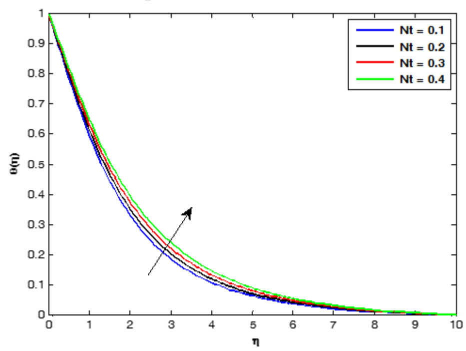

Fig. 14. Temperature profile for different values of Nt

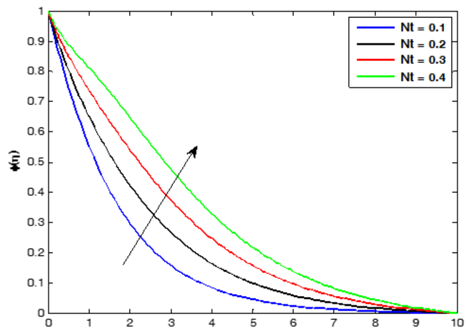

Fig. 15. Concentration profile for different values Nt

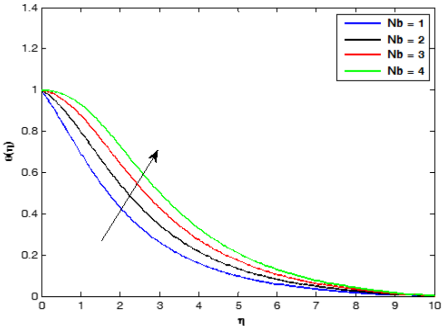

Fig. 16. Temperature profile for different values of Nb

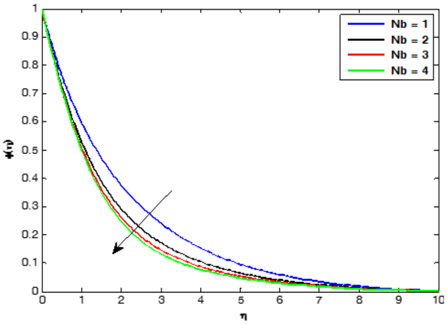

Fig. 17. Concentration profile for different values of Nb

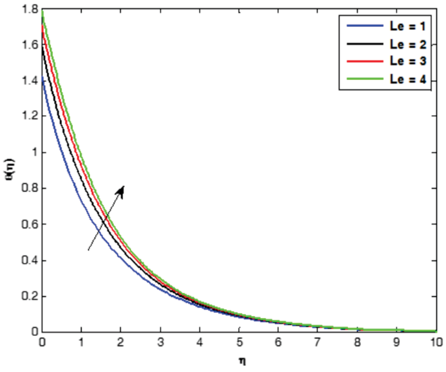

Fig. 18. Temperature profile for different values of Le

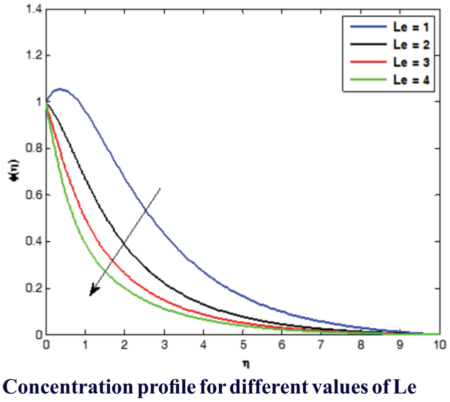

Fig. 19. Concentration profile for different values of Le

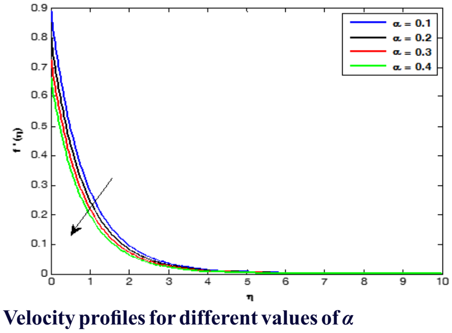

Fig. 20. Velocity profiles for different values of *α*

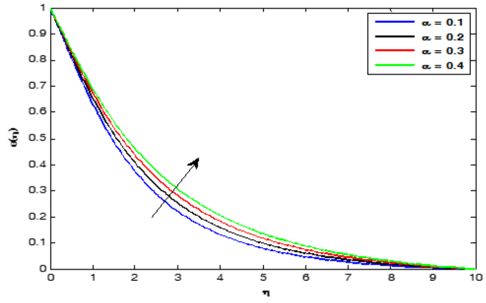

Fig. 21. Temperature profile for different values of *α*

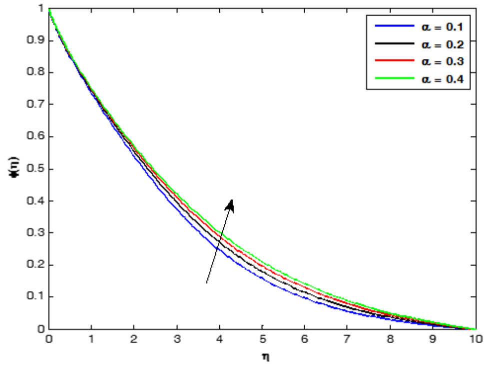

Fig. 22. Concentration profile for different values of *α*

Figs. (1) – (3) illustrate the change of velocity, temperature and concentration for higher values of Casson parameter β. A raise in β tends to decrease in yield stress of the Casson fluid. This serves to make the flow of the fluid easily so that the boundary layer thickness shows augmentation behavior near the cylindrical surface. However for higher values of β the fluid acts as Newtonian fluid and further withdraws from plastic flow. The temperature and concentration shows decreasing effect for increasing β.

Fig. 4 demonstrates the variation of velocity with curvature parameter γ. It is seen that there is growth in boundary layer thickness and velocity hikes with higher values of curvature parameter. Fig.5 illustrates the influence of temperature with γ and it is noticed that with the rise of curvature parameter the cylinderical surface area will squeeze, hence lesser surface area gives low heat transfer rate i.e, the temperature profile diminishes with enhancement of curvature parameter γ. In Fig.6 the impact of curvature parameter γ on the concentration profile is sketched. It is noted that the concentration decreases with increase of γ.

Fig (7) – (9) exhibits the velocity, temperature and concentration for distinct values of magnetic parameter. It is clear that the presence of magnetic field decreases the velocity. This is due to the higher value of the Lorentz force diminishes the velocity and consequently the boundary layer thickness diminishes. However the impact of magnetic parameter on temperature and concentration shows opposite trend to the velocity.

Effect of thermal relaxation parameter λ on temperature distribution is shown in fig.10. It is noticed that the temperature profile decreases with higher values of thermal relaxation parameter λ. For larger thermal relaxation parameter, particles of the material will takes long time to transfer heat to its surrounding particles and hence reduces the temperature. In Fig. 11 the effect of thermal relaxation parameter λ on concentration is shown and it is noted that the concentration increases with the enhancing values of thermal relaxation parameter λ.

Effect of Prandtl number Pr on temperature and concentration distributions are displayed in figs. 12 and 13. Higher values of Prandtl number Pr reduce both temperature and thermal boundary layer thickness. Since Prandtl number is inversely proportional to thermal diffusivity higher prandtl number corresponds to lower thermal diffusivity which reduces the temperature profile. It is also clear that the concentration profile decreases with increasing values of Prandtl number Pr.

Fig. 14 and Fig.15 exhibts the temperature and concentration distributions for various values of thermophoresis parameter Nt. Increasing values of thermophoresis parameter Nt tends to an increase in temperature and concentration profiles. In this case solutal boundary layer thickness decreases with increase in thermophoresis parameter Nt. Fig.16 is drawn the influence of Brownian motion parameter Nb on temperature. It is seen that the increase of thermal conductivity of a nanofluid is owing to Nb which facilitates micromixing, so we can say that the temperature is an increasing function of Brownian parameter Nb therefore the temperature increases with the increase of Nb. Fig.17 depicts that the concentration profile diminishes with the higher values of Brownian parameter Nb.

From Fig.18 it is observed that the temperature raises with the increasing values of Lewis number Le whereas Fig.19 shows that for larger values of Lewis number Le the concentration decreases and there will be reduction in the concentration boundary layer thickness.

Figs. (20) – (22) show the influence of velocity slip parameter on velocity, temperature and concentration profiles. The velocity distribution is decreasing function of the velocity slip parameter. This tends that in slip condition the fluid velocity near the wall of the sheet is no longer equal to the stretching cylinder velocity. Increasing The temperature and concentration hike for increasing values of α. Table 1 shows that the present results are in good agreement with Majeed et al in the absence of cattaneo heat flux for Casson nanofluids.

CONCLUSIONS:

Cattaneo-Christov heat flux model with thermal relaxation time is employed to analyze casson nanofluid past a stretching cylinder. The problem is modeled and then solved using shooting technique which was compared with previous results. The main results are summarized as follows:

- With the increase of Casson parameter β the velocity decreases whereas inverse relationship is found for temperature and concentration
- The velocity and boundary layer thickness increases with the increase of curvature (γ) of cylinder whereas temperature and concentration profiles decrease.
- Higher values of Prandtl number Pr reduce both temperature and concentration profiles.
- Temperature decreases with the increasing values of thermal relaxation parameter λ and concentration increases.
- Temperature increases with the increase of Brownian parameter Nb.
- For larger values of Lewis number Le, the temperature increases and Concentration decreases.

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