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AN OPTIMAL REPLACEMENT POLICY FOR A DETERIORATING SYSTEM WITH INCREASING REPAIR TIMES USING ALPHA SERIES PROCESSES

Statistics

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ABSTRACT

This paper studies an optimal maintenance policy for a reparable deteriorating system subject to random shocks. It assumes that the successive repair time form an increasing alpha series process and the failure mechanism by a generalized δ-shock process. Based on these assumptions, an explicit expression for the long-term average cost per unit time under a threshold type replacement policy is determined and minimized the long run average cost per unit time such that an optimal replacement policy N* is obtained analytically.Finally, numerical results are provided to strengthen the obtained theoretical results.

KEYWORDS

Working Reward cost, Renewal theorem, Replacement, Poisson process, Exponential process, Alpha series repair process, δ-Shock model

1. Introduction

Maintenance is a routine activity of keeping an equipment or a machine or a facility in its normal working condition so that it can perform its intended function satisfactorily without causing any loss to the service time on account of its breakdown. Maintenance is any activity designed to keep the resources in good working condition or restore them to operating status. Therefore, the machinery should be maintained in good working state. If not, there may be too much down time and break for production, if the machinery is a part of an assembly line. Improper functioning of system may lead to issues related to quality and cost and maintenance of the system. So, it would be good if the equipment or machinery is always maintained in working condition with minimum possible cost. Therefore, minimization of overall maintenance cost needs a holistic approach. Decades ago, the equipment or machinery is used till its failure and then it would be repaired if possible, or else discarded.

In order to maximize the availability and improve reliability, the sophisticated equipment must be maintained properly throughout the life. Maintenance activities pertaining to facilities and equipment in good working condition are indispensable to achieve specified level of quality, reliability and efficient working. Thus, Maintenance activity contributes to revenue by reducing the operating costs and increasing the effectiveness of production.

Further, the failure will have negative impacts on a firm's performance such as the revenue and customer service. To minimize the negative effects of machine failures, researchers are interested in investigating the appropriate machine maintenance and replacement policies. Obviously, a system, experience two stages namely 1. Productive stage 2. Non-Productive stage. Both stages are modeled and consider a threshold replacement policy for the system subject to random shocks.

Much research work has been carried out on the maintenance problems for the system with operating and repair stages. This is mainly because that some classical assumptions are not realistic in modeling the real systems. These assumptions include repaired system becoming "as good as new" and a failed system being replaced by a new one immediately. Barlow and Proschan [6] introduced an imperfect repair model, where the repair is prefect with probability 'p' and minimal with probability '1- p'. Other studies along this line include Block et al. [3], Kijima [4], Makis and Jardine [13], Dekker [8], Stadje and Zukerman [14],Lam and Zhang [16] Sheu et al. [10], Lam[17], Zhang and Wang [15].

In most practical situations, to reflect the aging process of the system, the consecutive repair times are assumed to become longer and longer till the system is replaced with a new one according to some replacement rule. Lam [18,19] first introduced the geometric processes (GP) to study the maintenance for such a deteriorating system. Finkelstein [5] generalized Lam's work based on a scale transformation after each repair. Zhang and Wang [15] applied the GP repair model to analyze a system with multi failure modes consisting of two components (working and standby) and one repairman. They further found the optimal replacement policy N*for component 1.

Many systems are subject to random shocks from external environment in their working. Due shocking a certain amount of damage may happen to a system and eventually deteriorates the working time, or break the system down immediately. Thus, the probability of a system to survive the shocks in time interval [0, t] is a main problem in the field of shock model theory. Generally, shock models can be classified into two major types namely 1. Cumulative shock model and 2.Extreme shock model. In the cumulative shock model, system fails because of the cumulative effect of shocks, while in the extreme shock model, the system breaks down due to a single shock with a large amount of magnitude, see Hameed and Proschan [1], Shanthikumar and Sumita [12] for references. On the basis of the two major types of shock models, some extensions have been developed by many researchers.

Recently, shock models were utilized to model the operating time. Conceptually, a system fails due to the shock effect on the system. While most of existing shock models were based on the accumulated or extreme damage causing a system failure, Li [20] first introduced the -shock model to avoid measuring amount of damage which may not be easy in many situations. Therefore, the -shock model focuses mainly on the frequency of shocks rather than the magnitude of shocks. In the -shock model, if the time interval between two consecutive shocks is smaller than a threshold value , the system fails. Thus the operating time is equal to the time to failure caused by a shock that follows the previous shock too closely. The threshold is usually set to be a constant.

Shengliang Zong, Guorong Chai, Zhe George Zhang Lei Zhao, [9] considered an optimal maintenance policy for a reparable deteriorating system subject to random shocks. Modeling the repair time by a geometric process and the failure mechanism by a generalized -shock process, an explicit expression of the long-term average cost per time unit for the system under a threshold type replacement policy is determined. Based on this average cost function, they proposed a finite search algorithm to locate the optimal replacement policy N*to minimize the average cost rate. Further they proved that the optimal policy N*is unique and present some numerical examples.

However the geometric process is more useful model for deteriorating system, Braun et.al [2] introduced an alternative model, the -series process ,which contributes these characteristics. Furthermore Braun et al [2] explained the increasing geometric process grows at most logarithmically in time, while the decreasing geometric process is almost certain to have a time of explosion. The -series process grows either as a polynomial in time or exponential in time. It also noted that the geometric process doesn't satisfy a central limit theorem, while the -series process does. Braun et.al [2] also presented that both the increasing geometric process and the -series process have a finite first moment under certain general conditions.

Based on this understanding, we study an optimal maintenance policy for a reparable deteriorating system subject to random shocks. It assumes that the successive repair time form an increasing alpha series process and the failure mechanism by a generalized -shock process. Based on these assumptions, an explicit expression of the long-term average cost per time unit for the system under a threshold type replacement policy is determined and minimized the long run average cost per unit time such that an optimal replacement policy N* is obtained analytically.Finally, numerical results are provided to strengthen the obtained theoretical results.

This paper is organized as follows. Section 1 describes the some review needed to understand the problem under considered; Section 2 presents the assumptions and definitions, Section 3 develops the long-run average cost per unit time while in Section 4, Numerical examples are provided.

2. Model definitions and assumptions

In this section, to formulate the d-shock model, we need the alpha series process first introduced in Lam [18,19].

Definition:

A stochastic process {X_n, n=1, 2, 3,..... } is called a alpha series increasing or decreasing process if there exists a real number 'α' (α>0 , α<0.), thus, {$n^\alpha X_n$, n= 1, 2, 3,..... }forms a new renewal process. The real 'α' is exponent of the alpha series process (AP).

The present model is developed under the following assumptions.

Assumptions:

1. Assume that a system is installed at t = 0. If a system fails, it is either repaired or replaced with a new and identical one.
2. Assume that the shocks arrive according to a Poisson process with rate $EX_i = \frac{1}{\lambda_1}$ where X_i is the i^{th} inter-arrival time of two consecutive shocks. Let δ_i be another exponentially distributed random variable associated with Xi. We assume that the $\{\delta_i, i=1,2,3,....\}$forms an increasing alpha series process with α < 0. Then δ_i has cumulative distribution function $Q(n^\alpha X)$,where Q(x) is the cumulative distribution function of δ_i. {Xi, δ_i} follows a δ -shock model if the system fails at i^{th} shock which satisfies $X_i <= \delta_i$, and then the life time or equivalently the operating time is the sum of all X_i until the one satisfying the above condition. Further, we assume that X_i is independent of δ_i.
3. Let T_n be the operating time after the (n -1) th repair. $\{T_n, n=1,2,3,.....\}$ is a stochastically decreasing random variable sequence induced by the δ -shock model.
4. Let Yn be the repair time after the (n-1)th failure and forms an increasing geometric process with $\beta < 0$ .Then Y_n has cumulative distribution function $G(n^\beta y)$ $G(b^{n-1}y)$ where G(y) is the cumulative distribution function of Y_1 with $EY_1 = \mu$, *where* $\mu > 0$.
5. Assume that T_n and Y_n, n=1,2,3,…are two independent sequences.
6. Assume that the repair cost rate is c, the operating reward rate is r, and the replacement cost consists of fixed cost R and variable cost$\upsilon = r_p Z$, where Z is the replacement time and r_p is the rate of cost per time unit during replacement.
7. Let E(Z) = t.
8. A threshold replacement policy N is adopted. Under such a policy, the system will be replaced with a new one after it fails for N times.

Due to aging process and accumulated wearing, it is reasonable to assume that the successive operating times after repairs will become shorter and shorter, and the corresponding repair times of the system will become longer and longer. Such a behavior is also be studied by alpha series process model (see Braun et.al [2]). In this paper, we assume that the successive working times forma a decreasing alpha series process, while the consecutive shocks forma an increasing alpha series process and also the consecutive repair time form an increasing alpha series process. Under the assumptions we developed an expression for the long run average cost per unit time in the following section.

3. Long-run average cost per unit time

According to renewal reward theorem (see Ross), the long run average cost per unit time i.e., C (N) under the replacement policy N is developed as follows.

Let C(N) be the long-run average cost per unit time of the system.

$$C(N) = \frac{Expected\ cost\ incurred\ in\ a\ cycle}{Expected\ length\ in\ a\ cycle} \tag{3.1}$$

Now, let W be the length of renewal cycle under the replacement N. Thus, we have

$$W = \sum_{j=1}^{N} T_j + \sum_{j=1}^{N-1} Y_j + Z \tag{3.2}$$

To evaluate the expected cost in a cycle, we first evaluate E(Tn), the expected time of the system after the (n-1) the failure. Let l_{ni} be the inter-arrival time between (i-1)th shock and the i^{th} shock following (n-1) the repair. Where i=1,2,3,…. .

Define:

$$Mn = \min\{m / l_{n_1} > n^\alpha\delta_1, l_{n2} > n^\alpha\delta_1, l_{n3} > n^\alpha\delta_1, ... l_{nm} < n^\alpha\delta_1 \tag{3.3}$$

And

$$T_n = \sum_{i=1}^{M_n} l_{ni} \tag{3.4}$$

Let M_n denotes the number of shocks till the first deadly shock occurs. Obviously, M_n has a Geometric distribution with probability function

$$p(M_n = k) = q_n^{k-1} p_n \ ; \ k = 1,2,3,..... \tag{3.5}$$

Where P_n is the probability a shock following n-1th repair and q_n=1-p_n There fore we have:

$$EM_n = \frac{1}{p_n}. \tag{3.6}$$

As M_n is stopping time with respect to the random sequence {δ_i,i=1, 2, 3,.... } which are independent and identically distributed random variables, using the Wald equation, we have

$$E(T_n) = E(\sum_{i=1}^{M_n} l_{ni}) = El_{ni} EM_n = \frac{El_{ni}}{p_n} \tag{3.7}$$

According to the assumption (2) As F(x) and Q(x) are all identically distributed, we have

$$F(x) = 1 - \exp(\lambda_1 x), x > 0 \quad Q(n^\alpha x) = 1 - \exp(n^\alpha \lambda_2 x),\ x > 0 \tag{3.8}$$

And

$$El_{n1} = \int_0^\infty x\, dF(x) = \int_0^\infty x\, d(1 - e^{-\lambda_1 x}) = \frac{1}{\lambda_1} \tag{3.9}$$

Further more, as l_{ni} *and* $\delta_n(n^\alpha\delta_1)$ are independent and have the marginal distributions with means $\frac{1}{\lambda_1}$ *and* $\frac{1}{n^\alpha\lambda_2}$ respectively, we obtain

$$p_n = P(l_{ni} < \delta_n) = \int_0^\infty e^{-n^\alpha\lambda_2 x}\lambda_1 e^{-\lambda_1 x} dx = \lambda_1 \int_0^\infty e^{-(n^\alpha\lambda_2 + \lambda_1)x} dx = \frac{\lambda_1}{(n^\alpha\lambda_2 + \lambda_1)} \tag{3.10}$$

From equation (3.6) to equation (3.9) we have:

$$E(T_n) = \frac{n^\alpha\lambda_2 + \lambda_1}{\lambda_1^2} \tag{3.11}$$

As a result of this

$$E(\sum_{n=1}^{N} T_n) = \sum_{n=1}^{N} \frac{n^\alpha\lambda_2 + \lambda_1}{\lambda_1^2} \tag{3.12}$$

Again , Y_n, n=1,2,3,…. is an increasing alpha series process with exponent $\beta < 0$. We have

$$E(Y_n) = \frac{\mu}{n^\beta}, \quad \mu > 0. \tag{3.13}$$

From equation (3.1), the long run average cost C(N) of the system under policy N is given by

$$C(N) = \frac{E(C\sum_{n=1}^{N-1} Y_n - r\sum_{n=1}^{N} T_n + R + r_p Z)}{E(\sum_{n=1}^{N-1} Y_n + \sum_{n=1}^{N} T_n + Z)} \tag{3.14}$$

$$C(N)=\frac{\left[C\sum_{n=1}^{N-1}EY_n-r\sum_{n=1}^{N}ET_n+R+E(r_pZ)\right]}{\sum_{n=1}^{N-1}EY_n+\sum_{n=1}^{N}ET_n+EZ)} \qquad (3.15)$$

From the assumptions (6,7) and equations (3.12) and (3.13) we have:

$$C(N)=\frac{\left(C\sum_{n=1}^{N-1}\frac{\mu}{n^{\beta}}-r\sum_{n=1}^{N}\frac{n^{\alpha}\lambda_2+\lambda_1}{\lambda_1^2}+R+r_pt\right)}{\left(\sum_{n=1}^{N-1}\frac{\mu}{n^{\beta}}+\sum_{n=1}^{N}\frac{n^{\alpha}\lambda_2+\lambda_1}{\lambda_1^2}+t\right)} \qquad (3.16)$$

This is an expression for the long run average cost per unit time of a deteriorating system.

In the next section we determine the long run average cost per unit time with hypothetical parameters of the model.

4. Numerical Results

Table: 4.1 The long run average cost per unit time is computed for the hypothetical values.

In this section, we present a numerical example to demonstrate the determination of the optimal replacement policy. Consider a system with the following parameters: $\alpha = 0.35$, $\beta = -0.75$, $c = 10$, $r = 500$, $R = 8000$, $t = 10$, $r_p = 5$, $\lambda_1=10$, $\lambda_1=20$ *and* $\mu = 4$ Using Eq.(3.16) , we plot C(N) in 4.1

N	C(N)	N	C(N)
1	0.995715	13	0.940013
2	0.985511	14	0.942466
3	0.976073	15	0.945986
4	0.967572	16	0.950557
5	0.960097	17	0.956162
6	0.953696	18	0.96278
7	0.948396	19	0.970393
8	0.94421	20	0.978981
9	0.941144	21	0.988522
10	0.939199	22	0.998995
11	0.938367	23	1.010377
12	0.938643	24	1.022646

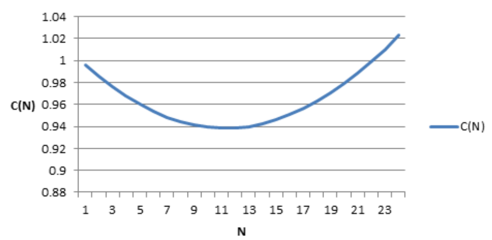

Fig:4.1 The long run average cost per unit time

Conclusions:

From Table 4.1 and fig 4.1 it can be observed that the long run average cost per unit time is minimum i.e C(N)= 0.938367 when the number of failure N reaches 11. Similar results can be obtained with different parameter values of the model. Further if a repair man is more experiences with repair the successive repair times form a decreasing alpha series process. Therefore, we can develop an optimal replacement policy N for an improved system.

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