ORIGINAL RESEARCH PAPER
Volume-9 | Issue-4 | April-2020 | PRINT ISSN No. 2277 - 8179 | DOI : 10.36106/ijsr
INTERNATIONAL JOURNAL OF SCIENTIFIC RESEARCH

ON FUZZY SOFT NOWHERE DENSE SETS

Mathematics

Dr. E. Poongothai* Department of Mathematics, Shanmuga Industries Arts & science college, Tiruvannamalai *Corresponding Author

S. Divyapriya Research scholar, Department of Mathematics, Shanmuga Industries Arts & science college, Tiruvannamalai.

ABSTRACT

Some new concept of Baireness in fuzzy soft topological spaces are introduced, and their characterizations and properties are investigated in this work. Several examples are given to illustrate the concepts introduced in this paper.

KEYWORDS

Fuzzy soft nowhere dense set, fuzzy soft first category, and fuzzy soft second category.

INTRODUCTION

The concept of Baire spaces have been studied extensively in classical topology in [14]. In 2013 the concept of baire space in fuzzy setting was introduced and studied by G.Thangaraj and S.Anjalmose [17] Molodtsov [13] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainities which traditional mathematical tools cannot handle. He has shown several applications of this theory in solving any practical problems in economics, engineering, social sciences, medical science etc. Later other authors like Maji et al. [9-13] have further studied the theory of soft sets and used this theory to solve some decision making problems.

Next, the concept of fuzzy soft set is introduced and studied [5-8,14] a more generalized concept, which is a combination of fuzzy set and soft set and studied its properties. Since then much attention has been paid to generalize the basic concepts of fuzzy topology in soft setting and this modern theory of fuzzy soft topology has been developed. In recent years, fuzzy soft topology has been found to be very useful in solving any practical problems. The aim of this paper is to introduce the concepts of fuzzy soft Baireness in fuzzy soft topological spaces. we introduce fuzzy soft nowhere dense set, fuzzy soft, first category set, fuzzy soft second category set, fuzzy soft first category space, fuzzy soft second category space, and study several characterizations of fuzzy soft setting.

2. Preliminaries

Definition 2.1[5]

The fuzzy soft set $F_\phi \in FS(U, E)$ is said to be null fuzzy soft set and it is denoted by ϕ, if for all $e \in E$, $F(e)$ is the null fuzzy soft set $\bar{0}$ of U, where $\bar{0}(\chi) = 0$ for all $\chi \in U$.

Definition 2.2 [5]

Let $F_E \in FS(U,E)$ and $F_E(e) = \bar{1}$ all $e \in E$, where $\bar{1}(x) = 1$ for all $x \in U$. Then F_E is called absolute fuzzy soft set. It is denoted by $\bar{E}$.

Definition 2.3[5]

A fuzzy soft set FA is said to be a fuzzy soft subset of a fuzzy soft set G_B over a common universe U if $A \subseteq B$ and $F_A(e) \subseteq G_B(e)$ for all $e \in A$, i.e., if $\mu^e_{FA}(x) \leq \mu^e_{GB}(x)$ for all $x \in U$ and for all $e \in E$ and denoted by $F_A G_B$.

Definition 2.4[5]

Two fuzzy soft sets F_A and G_B over a common universe U are said to be fuzzy soft equal if FA is a fuzzy soft subset of G_B and G_B is a fuzzy soft subset of F_A.

Definition 2.5[5]

The union of two fuzzy soft sets $FA$and GB over the common universe U is the fuzzy soft set HC, defined by $H_C(e) = \mu^e_{HC} = \mu^e_{FA} \cup \mu^e_{GB}$ for all $e \in E$, where $C = A \cup B$. Here we write $H_C = F_A \tilde{\vee} G_B$.

Definition 2.6[5]

Let F_A and G_B be two fuzzy soft set, then the intersection of F_A and G_B is a fuzzy soft set H_C, defined by $(e) = \mu^e_{FC} = \mu^e_{FA} \cap \mu^e_{GB}$for all $e \in E$, where $C = A \cap B$. Here we write $H_C = F_A \tilde{\wedge} G_B$.

Definition 2.7[15]

Let $F_A \in FS(U,E)$ be a fuzzy soft set. Then the complement of F_A, denoted by F_A^C, defined by

$$F_A^C(e) = \begin{cases} \bar{1} - \mu^e_{FA}, & if \ e \in A \\ \bar{1}, & if e \notin A \end{cases}$$

Definition 2.8[16]

Let ψ be the collection of fuzzy soft sets over U. Then ψ is called a fuzzy soft topology on U if ψ satisfies the following axioms:

(i)ϕ, $\bar{E}$ belong to ψ.

(ii) The union of any number of fuzzy soft sets in ψ belongs to ψ.

(iii) The intersection of any two fuzzy soft sets ψ belongs to ψ. The triplet (U, E, ψ) is called a fuzzy soft topological space over U. The members of ψ are called fuzzy soft open sets in U and complements of them are called fuzzy soft closed sets in U.

Definition 2.9[16]

The union of all fuzzy soft open subsets of F_A over (U, E) is called the interior of F_A and is denoted by $int^{fs}(F_A)$.

Proposition 2.1[16]

$int^{fs}(F_A \tilde{\wedge} G_B) = in(F_A) \tilde{\wedge} int^{fs}(G_B)$.

Definition 2.10 [16]

Let $F_A \in FS(U,E)$ be a fuzzy soft set. Then the intersection of all closed sets, each containing F_A, is called the closure of F_A and is denoted by $cl^{fs}(F_A)$.

Remarks 2.1 [16]:

(1) For any fuzzy soft set F_A in a fuzzy soft topological space (U, E, ψ), it is easy to see that $((F_A))^c = int^{fs}(F_A^C)$ and $(int^{fs}(F_A))^c = cl^{fs}(F_A^C)$.

(2) For any fuzzy soft F_A subset of a fuzzy soft topological space (U, E, ψ) ,we define the fuzzy soft subspace topology. ψ on F_A by $K_D \epsilon \psi_{F_A}$if $K_D = F_A \tilde{\wedge} G_B$ for some $G_B \epsilon \psi$.

(3)For any fuzzy soft H_C in F_A fuzzy soft subspace of a fuzzy soft topological space, we denote to the interior and closure of H_C in F_A by $int^{fs}_{F_A}(H_C)$ and $cl^{fs}_{F_A}(H_C)$, respectively

CHAPTER-3

3.1 ON FUZZY SOFT NOWHERE DENSE SETS

Definition 3.1

A fuzzy soft set F_A in a fuzzy soft topological space (U,E,ψ) is called a fuzzy soft nowhere dense set if there exists no non-zero fuzzy soft open set G_B in (U,E,ψ) such that $G_B < cl^{fs}(F_A)$. That is, $int^{fs} cl^{fs}(F_A) = 0$.

EXAMPLE 3.1

Suppose the fuzzy soft sets $F_E, G_E, H_E, I_E, K_E, L_E, T_E, J_E$ describe colours of the bikes with respect to the given parameters.

$U = \{x_1, x_2, x_3\}$ which is the set of all bikes under consideration.

Let I^U be the collection of all fuzzy subsets of U. Also

Let $E = \{e_1, e_2, e_3\}$ where e_1, e_2, e_3 stand for the attributes "nice", "good", "excellent".

Let $F_E, G_E, H_E, I_E, K_E, L_E, T_E, J_E$ defined as follows,

$$F_E = \begin{bmatrix} .2 & .3 & .6 \\ .25 & .15 & .10 \\ .3 & .6 & .12 \end{bmatrix}, G_E = \begin{bmatrix} .7 & .3 & .4 \\ .8 & .75 & .7 \\ .8 & .10 & .9 \end{bmatrix}, H_E = \begin{bmatrix} .1 & .5 & .3 \\ .5 & .75 & .4 \\ .8 & .1 & .4 \end{bmatrix}$$

$$I_E = \begin{bmatrix} .2 & .5 & .6 \\ .5 & .75 & .4 \\ .8 & .4 & .6 \end{bmatrix}, K_E = \begin{bmatrix} .2 & .3 & .3 \\ .15 & .25 & .10 \\ .3 & .10 & .6 \end{bmatrix}, L_E = \begin{bmatrix} .1 & .3 & .3 \\ .15 & .25 & .10 \\ .3 & .12 & .1 \end{bmatrix}$$

$T_E=\begin{bmatrix} .4 & .7 & .6 \\ .8 & .75 & .7 \\ .8 & .12 & .9 \end{bmatrix}$, $J_E=\begin{bmatrix} .2 & .5 & .6 \\ .5 & .75 & .4 \\ .8 & .12 & .6 \end{bmatrix}$

Then ψ ={0, F_E,G_E, H_E, I_E, K_E, L_E, T_E, J_E,1} is clearly a fuzzy soft topology on (U, E,ψ).The non-zero fuzzy soft open sets is (U, E, ψ) are F_E, G_E, H_E, I_E, K_E, L_E, T_E, J_E, ($F_E \vee G_E$), ($G_E \vee H_E$), ($H_E \vee I_E$), ($I_E \vee K_E$), ($K_E \vee L_E$), ($L_E \vee T_E$), ($T_E \vee J_E$), ($J_E \vee F_E$).

Proposition3.3

The complement of a fuzzy soft nowhere dense set in a fuzzy soft topological space (U, E, ψ) need not be a fuzzy soft nowhere dense set.

Proof

For, in example 3.2, (1–F_E) is a fuzzy soft nowhere dense set in (U, E, ψ) whereas F_E= 1–(1–F_E) is not a fuzzy soft nowhere dense set in (U, E, ψ).

Proposition 3.4

If F_A and G_B are fuzzy soft nowhere dense sets in a fuzzy soft topological space (U, E, ψ), then $F_A \vee G_B$ need not be fuzzy soft nowhere dense set in ,E,ψ).

Proof

For, in example 3.2, 1– F_E, 1–G_E, are fuzzy soft nowhere dense sets in ,E,ψ). , But (1– F_E) ∨ (1–G_E)= α implies that int^{fs} cl^{fs} (α)≠0. Therefore union of fuzzy soft nowhere dense sets need not be fuzzy soft nowhere dense set in ,E,ψ).

Proposition 3.5

If the fuzzy sets F_A and G_B are fuzzy soft nowhere dense sets in a fuzzy soft topological space (U,E,ψ) then $F_A \wedge G_B$ is a fuzzy soft nowhere dense set in (U,E, ψ).

proof

Let the fuzzy soft sets F_A and G_B are fuzzy soft nowhere dense sets in (U,E, ψ). Now int^{fs} cl^{fs} ($F_A \wedge G_B$) ≤ int^{fs} cl^{fs} (F_A) ∧ int^{fs} cl^{fs} (G_B) ≤ 0 ∧ 0 (since int^{fs} cl^{fs} (F_A)=0 and int^{fs} cl^{fs} (G_B)=0). That is, int^{fs} cl^{fs} ($F_A \wedge G_B$)=0. Hence($F_A \wedge G_B$) is a fuzzy soft nowhere dense set in (U,E, ψ).

Proposition 3.6

If F_A is a fuzzy soft nowhere dense set is a fuzzy soft topological space (U,E,ψ), then int^{fs} (F_A)=0.

Proof

Let F_A be a fuzzy soft nowhere dense set in (U,E,ψ).Then we have int^{fs} cl^{fs} (F_A)=0. Now $F_A \leq cl^{fs}$ (F_A). We have $int^{fs} \leq$ (F_A) ≤ int^{fs} cl^{fs} (F_A) =0. Hence int^{fs} (F_A)=0.

Proof

Let F_A be a fuzzy soft nowhere dense set in (U, E, ψ). Then we have int^{fs} cl^{fs} (F_A) = 0 and int^{fs} (F_A) =0. Now cl^{fs} (F_A) =F_A, since F_A is fuzzy soft closed set in (U, E, ψ) implies that int^{fs} (F_A) = int^{fs} (F_A)=0. Hence int^{fs} cl^{fs} (F_A) =0.

Proposition 3.8

If F_A is a fuzzy soft nowhere dense set and G_B is any fuzzy soft set is a fuzzy soft topological space (U, E, ψ) then ($F_A \wedge G_B$) is a fuzzy soft nowhere dense set in (U,E,ψ).

Proof

Let F_A be a fuzzy soft nowhere dense set in (U, E,ψ).Then, int^{fs} cl^{fs} (F_A)=0.Now int^{fs} cl^{fs} ($F_A \wedge G_B$) ≤ int^{fs} cl^{fs} (F_A) ∧ int^{fs} cl^{fs} (G_B) ≤ 0 ⋀ int^{fs} cl^{fs} (G_B)=0 that is, int^{fs} cl^{fs} ($F_A \wedge G_B$)=0. Hence ($F_A \wedge G_B$) is a fuzzy soft nowhere dense set in (U,E, ψ).

Definition 3.9

A Fuzzy soft set F_A is a fuzzy soft topological space (U,E,ψ) is called fuzzy soft dense if there exists no fuzzy soft closed set G_B in (U,E,ψ),such that F_A< G_B <1,that is cl^{fs} (F_A)=1

Proposition 3.10

If F_A is a fuzzy soft dense and fuzzy soft open set in a fuzzy soft topological space (U,E,ψ) and if $F_A \leq 1-G_B$, then G_B is a fuzzy soft nowhere dense set in (U,E,ψ).

Proof

Let F_A be a fuzzy soft dense set in (U, E, ψ) then we have Cl^{fs} (F_A) =1 and int^{fs} (F_A) = F_A .Now $G_B \leq 1-F_A$, implies that cl^{fS} (G_B) ≤ 1 cl^{fs} (1−F_A) .Then cl^{fS} (G_B) ≤ 1−int^{fs} (F_A) = 1−F_A which implies that int cl^{fs} (G_B) ≤ int^{fs} (1−F_A)=1−cl^{Fs} (F_A)=1 – 1=0.That is int cl^{fS} (G_B) =0. Hence G_B is a fuzzy semi nowhere dense set in (U, E, ψ).

Proposition3.11

If F_A is a fuzzy soft nowhere dense set in a fuzzy soft topological space (U, E, ψ), then 1−F_A is a fuzzy soft dense set in (U, E,).

Proof

Let F_A be a fuzzy soft nowhere dense set in (U, E, ψ).Then, int^{fs} cl^{fs} (F_A)=0. Now F_A ≤ cl^{fs} (F_A) implies that int^{fs} (F_A) ≤ int^{fs} cl^{fs} (F_A) = 0. Then int^{fs} (F_A) = 0 and cl^{fs} (1−F_A)=1− int^{fs} (F_A)=1 – 0=1 and hence 1− F_A is a fuzzy soft dense set in (U,E, ψ).

Proposition 3.12

If F_A is a fuzzy soft nowhere dense set in a fuzzy soft topological space (U, E, ψ), then cl^{fs} (F_A) is also a fuzzy soft nowhere dense set in (U,E,ψ).

Proof

Let F_A be a fuzzy soft nowhere dense set in (U, E, ψ).Then, int^{fs} cl^{fs} (F_A) = 0. Now cl^{fs} cl^{fs} (F_A) = cl^{fs} (F_A). Hence int^{fs} cl^{fs} (cl^{fs} (F_A))= int^{fs} cl^{fs} (F_A)=0. Therfore cl^{fs}(F_A) is also a fuzzy soft nowhere dense set in (U,E, ψ).

Proposition 3.13

If F_A is a fuzzy soft nowhere dense set in a fuzzy soft topological space (U, E, ψ). Then 1−cl^{fs} (F_A) is a fuzzy soft dense set in (U, E, ψ).

Proof

Let F_A be a fuzzy soft nowhere dense set in(U, E, ψ). Then by proposition 3.12, cl^{fs} (F_A) is a fuzzy soft nowhere dense set in (,E,ψ,). Also by proposition 3.11, 1−cl^{fs} (F_A) is a fuzzy soft dense set in (U, E, ψ).

Definition 3.14

Let (U, E, ψ) be a fuzzy soft topology. A fuzzy soft set F_A

In (U,E,ψ) is called fuzzy soft first category. If $F_A=\vee^{\infty}_{i=1}(F_{Ai})$, where F_{Ai}'s are fuzzy soft nowhere dense sets in (U,E, ψ). Any other fuzzy soft set in (,E,ψ,) is said to be of fuzzy soft second category.

Proposition 3.15

If F_A is a fuzzy soft first category set in a fuzzy soft topological space (U, E,), then $1-F_A=\wedge^{\infty}_{i=1}(F_{Ai})$, where cl^{fs} (G_{Bi}) =1.

Proof

Let F_A be a fuzzy soft first category set in (U,E, ψ).Then we have $F_A=\vee^{\infty}_{i=1}(F_{Ai})$,where F_{Ai}'s are fuzzy soft nowhere dense sets in (U,E,ψ). Now $1-F_A=\vee^{\infty}_{i=1}(F_{Ai})$ $=\wedge^{\infty}_{i=1}(F_{Ai})$. Let G_{Bi} = 1−F_{Ai}. Then $1-F_A=\wedge^{\infty}_{i=1}(G_{Bi})$. Since F_{Ai}'s are fuzzy soft nowhere dense sets in (U,E, ψ),by proposition 3.11,

We have 1−F_{Ai}'s are fuzzy soft dense sets in (U, E, ψ). Hence cl^{fs} (G_{Bi}) = cl^{fs} (1−F_{Ai}) =1, Therefore we have $1-F_A=\wedge^{\infty}_{i=1}(G_{Bi})$, where cl^{fs} (G_{Bi}) = 1.

Definition 3.16

A fuzzy soft topological space (U, E, ψ) is called a fuzzy soft first category space if the fuzzy soft set 1_x is a fuzzy soft first category set in (U, E, ψ). That is, $1_x=\vee^{\infty}_{i=1}(F_{Ai})$, where F_{Ai}'s are fuzzy soft nowhere dense sets in (U, E, ψ). Otherwise (U, E, ψ) will be called a fuzzy soft second category space.

Proposition 3.17

If F_A be a fuzzy soft closed set in a fuzzy soft topological space (U, E, ψ) and if int^{fs} (F_A)=0, then F_A is a fuzzy soft nowhere dense set in (U,E,ψ).

Proof

Let F_A be a fuzzy soft closed set in (U, E, ψ). Then we have cl^{fs} (F_A)= F_A.

Now int^{fs} cl^{fs} (F_A)= int^{fs} (F_A) and int^{fs} (F_A)=0 implies that int^{fs} cl^{fs} (F_A)=0. Hence F_A is a fuzzy soft nowhere set in (U, E,).

REFERENCES

[1] Cagan,N., Enginoglu s. & citak, F. 2011, Fuzzy set theory and its applications. Iranian journal of fuzzy systems, 8(3),137-147.

[2] Frolik,Z.1961. Baire spaces and some generalizations of complete metric spaces, Czech.Math.Journal,11, 237-247.

[3] Gruenhage.G and Lutzer.D, Baire and Volterra Spaces, Proc.Amer.soc. 128 (2000), 3115-3124.

[4] Haworth, R.C. & McCoy, R.A. 1977. Baire space. Dissertations. Math., 141 (1977), 1-77.

[5] Mahmood, S, 2014. Soft Regular Generalized b-closed sets in soft Topological spaces. Journal of Linear and Topological Algebra, 3[4], 195-200.

[6] Mahmood, S, 2015. On intuitionistic fuzzy soft b- closed sets in intuitionistic fuzzy soft topological spaces. Annals of fuzzy Mathematics and informatics, 10[2],221-233.

[7] Mahmood, S. & Al-Batat, Z, 2016. Intuitionistic Fuzzy Soft LA-Semigroups and intuitionistic Fuzzy Soft Ideals. International Journal of Applications of Fuzzy Sets and Artificial Intelligence,6, 119-132.

[8] Mahmood, S, 2016. Dissimilarity Fuzzy Soft Points and their Applications. Fuzzy Information and Engineering, 8, 281-294.

[9] Maji, P.K., Roy, A.R., Biswas, R, 2001. Fuzzy soft sets. Journal of Fuzzy Mathematics 9 (3), 589-602.

[10] Maji, P.K.,Roy, A.R., Biswas, R, 2002.An application of soft sets in a decision making problem. Comput. Math. Appl. 44,1077-1083.

[11] Maji, P.K., Roy, A.R., Biswas, R., 2003.Soft set theory. Comput. Math. Appl. 45, 555-562.

[12] Maji, P.K., Roy, A.R., Biswas, R. 2001.Intuitionistic fuzzy soft sets. Journal of fuzzy Mathematics, 9(3), 677-692.

[13] Molodtsov, D, 1999. Soft set theory First results. Comput. Math. Appl. 37, 19-31.

[14] T.Neubrunn, A Note on Mappings of Baire spaces, Math slovaca. Vol.27(1977), N0.2, 173-176.

[15] Roy, S., Saanta, T.K, 2013. An introduction to open and closed sets on fuzzy soft topological spaces. Ann. Fuzzy Math. Inform., 6 (2), 425-431.

[16] Tanay, B., Kander, M.B. 2011.Topological structure of fuzzy sets comput. Math.Appl., 61(10), 2952-2957.

[17] Thangaraj.G, Anjalmose.S , 2013. On Fuzzy Baire Spaces, J. Fuzzy Math., 21(3), 667-676.

[18] Zadeh. L.A.,Fuzzy sets, Information and Control, Vol. 8 (1965), 338-353.