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SOME STATISTICAL PROPERTIES OF AREA BIASED GENERALIZED UNIFORM DISTRIBUTION

Statistics

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ABSTRACT

In this Paper, the area biased techniques of generalized uniform distribution has been obtained. The distribution has two parameters. The various structural properties of the newly model have been studied. The parameter estimation of area biased generalized uniform (ABGU) distribution by the method of maximum likelihood. And the Fisher information matrix has been discussed. Further, a likelihood ratio test of this distribution has been obtained.

KEYWORDS

Weighted distribution, generalized uniform distribution, Reliability analysis, Entropy, Order statistics, Maximum likelihood estimator, Likelihood ratio test

1.Introduction

The concept of weight distribution is discussed by Fisher (1934). After being modified by C R Rao (1965) in a different way, in which with a weighty distribution many situations can be resolved. Weighted distribution is used in a variety of research fields related to reliability, environment, engineering and biomedicine. If the weight function looks at the size of one, the weight distribution reduces the size of the average distribution. If the weight function looks at the size of two, the weight distribution reduces the area of the biased distribution. Patil and Ord (1976) examined the use of weight-bearing statistics related to population and the environment can be found in Patil and Rao (1978). Van Deusen's (1986) relevant data related to the extent to chest height from a sample of the above point in color distribution. Lappi and Bailey (1987), used a randomized distribution of research in the analysis of the ascending data rate of sample size. Recently, Elangovan *et al* (2020) obtained a new area biased Aradhana distribution with application in Cancer data. This is showing a more flexibility than the classical distribution. Characterization of generalized uniform distribution through expectation discussed by Bhatt (214). Characterization of generalized uniform distribution based on lower record values discuss by Khan and khan (2017).

Consider the probability density function (pdf) of generalized uniform distribution is given by $f(x;\alpha,\beta)=\frac{x^{\alpha}(\alpha+1)}{\beta^{\alpha+1}};\quad 0<x<\beta,\ -1<\alpha$ (1.1)

And its first and second raw moment

$$E(X)=\frac{\beta(\alpha+1)}{(\alpha+2))}$$

$$E(X^2)=\frac{\beta^2(\alpha+1)}{(\alpha+3)} \quad (1.2)$$

2. AREA BIASED GENERALIZED UNIFORM (ABGU) DISTRIBUTION

Suppose X is a non-negative random variable with probability density function $f(x)$. Let $w(x)$ be the non negative weight function, and then the probability density function of the weighted random variable X_w is given by:

$$f_w(x)=\frac{w(x)f(x)}{E(w(x))};\ x>0$$

Where $w(x)$ be a non-negative weight function and $E(w(x))=\int w(x)f(x)dx<\infty$.

For different weighted models, we have different choice of the weight function $w(x)$. When

$w(x)=x^c$, the resulting distribution is termed as weighted distribution. In this paper, we have to find the area biased version of weighted distribution, we will take c = 2 in weights x^2, in order to get the area biased distribution and its pdf is given by:

$$f_1(x)=\frac{x^2 f(x)}{E(x^2)};\ x>0 \quad (2.1)$$

Using the values of (1.1) and (1.2) in equation (2.1), we will get the pdf of area biased generalized uniform distribution

$$f_1(x)=\frac{x^{\alpha+2}(\alpha+3)}{\beta^{\alpha+3}} \quad (2.2)$$

And the cumulative distribution function of area biased generalized uniform (*ABGU)* distribution is obtained as

$$F_1(x)=\int_0^x f_1(x)dx$$

$$F_1(x)=\int_0^x \frac{x^{\alpha+2}(\alpha+3)}{\beta^{\alpha+3}}dx$$

After simplification, we will get the cumulative distribution function (cdf) of *ABGU* distribution is

$$F_1(x)=\left(\frac{x}{\beta}\right)^{\alpha+3} \quad (2.3)$$

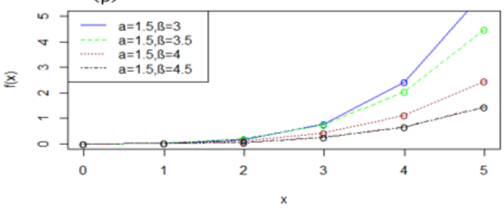

Fig 1. The Probability Density Function of *ABGUD* for the indicated values of α and β.

***Asymptotic Behavior*:**

We seek to investigate the behavior of the proposed model as in equation (2.) as $x\to 0$ and $x\to\infty$. This model involve considering $\lim_{x\to 0} f_1(x)$ and $\lim_{x\to\infty} f_1(x)$.

As $x\to 0$

$$\lim_{x\to 0} f_1(x)=\lim_{x\to 0}\left[\frac{x^{\alpha+2}(\alpha+3)}{\beta^{\alpha+3}}\right]=0$$

As $x\to\infty$

$$\lim_{x\to\infty} f_1(x)=\lim_{x\to\infty}\left[\frac{x^{\alpha+2}(\alpha+3)}{\beta^{\alpha+3}}\right]=0$$

These results confirm that the proposed distribution has a mode.

3. Reliability Analysis

In this session, we will introduce the reliability function or survival function, hazard rate function or failure rate and reverse hazard rate function, Mills ratio for the area biased generalized uniform distribution

3.1 Survival Function

The survival function of ABGUD is defined as

$$S(x) = 1 - F_1(x)$$

$$S(x) = 1 - \left(\frac{x}{\beta}\right)^{\alpha+3}$$

3.2 Hazard Rate Function of ABGUD

The hazard function is also known as the hazard rate is defined as

$$h(x) = \frac{f_1(x)}{S(x)}$$

$$h(x) = \frac{(\alpha+3)x^{\alpha+2}}{\beta^{\alpha+3} - x^{\alpha+3}}$$

3.3 Reverse Hazard Rate Function of ABGUD

The reverse hazard rate is given by

$$h_r(x) = \frac{f_1(x)}{F_1(x)}$$

$$h_r(x) = \frac{(\alpha+3)}{x}$$

3.4 Mills Ratio

The Mills ratio of the ABGU distribution is given by,

$$Mills\ ratio = \frac{1}{h_r(x)}$$

$$= \frac{x}{\alpha+3}$$

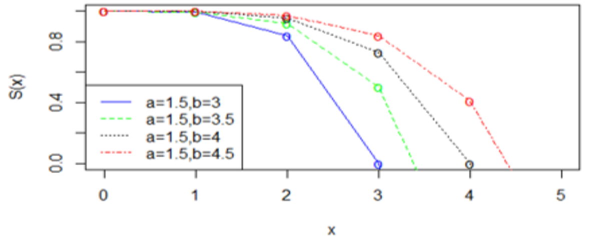

Fig.2 Survival function curve of ABGUD

4. MOMENTS AND ASSOCIATED MEASURES

Let X denotes the random variable of ABGU distribution with parameters α and β then the r^{th} order moment of ABGU distribution can be defined as

$$E(X^r) = \mu_r' = \int_0^\infty x^r f_1(x)dx$$

$$= \frac{(\alpha+3)}{\beta^{\alpha+3}} \int_0^\infty x^r \frac{x^{\alpha+2}(\alpha+3)}{\beta^{\alpha+3}}\ dx$$

$$= \int_0^\infty x^{r+\alpha+2}\, dx$$

$$= \frac{(\alpha+3)\beta^{\alpha+r+3}}{(\alpha+r+3)\beta^{\alpha+3}}$$

$$= \frac{(\alpha+3)\beta^r}{(\alpha+r+3)}$$

$$\mu_r' = \frac{(\alpha+3)\beta^r}{(\alpha+r+3)} \qquad (4.1)$$

Putting r = 1,2,3,4 in equation (4.1), we will get the mean of ABGU distribution which is given by

$$E(X) = \frac{\beta(\alpha+3)}{(\alpha+4)}$$

And putting r = 2 we obtain the second moment is

$$E(X^2) = \frac{(\alpha+3)\beta^2}{(\alpha+5)}$$

Therefore,

$$Variance = \sigma^2 = \frac{(\alpha+3)\beta^2}{(\alpha+4)^2(\alpha+5)} \quad Standard\ Deviation = \sigma = \frac{\beta}{(\alpha+4)}\sqrt{\frac{(\alpha+3)}{(\alpha+5)}}$$

$$Coefficient\ of\ Variation = \frac{\sigma}{\mu}$$

$$= \frac{1}{(\alpha+3)}\sqrt{\frac{(\alpha+3)}{(\alpha+5)}}$$

$$Coefficient\ of\ Dispersion(\gamma) = \frac{\sigma^2}{\mu}$$

$$= \frac{\beta}{(\alpha+4)(\alpha+5)}$$

4.1 Moment generation function and characteristic function

Let X be ABGU distribution, then the MGF of X is obtained as

$$M_X(t) = E(e^{tx}) = \int_0^\beta e^{tx} f_1(x)\, dx$$

$$= \int_0^\beta \left(1 + (tx) + \frac{(tx)^2}{2!} + \frac{(tx)^3}{3!} + \cdots + \cdots\right) f_1(x)dx$$

$$= \int_0^\beta \sum_{r=0}^\infty \frac{(tx)^r}{r!} f_1(x)\, dx$$

$$= \sum_{r=0}^\infty \frac{(t)^r}{r!} \int_0^\beta x^r f_1(x)\, dx$$

$$= \sum_{r=0}^\infty \frac{(t)^r}{r!} E(x^r) \qquad (4.1.1)$$

Using equation (4.1) in equation (4.1.1), we have

$$M_X(t) = \sum_{r=0}^\infty \frac{(t)^r}{r!} \frac{\beta^r}{(\alpha+r+3)}$$

Similarly, the characteristic function of ABGU distribution can be obtained as

$$\varphi_X(t) = M_X(it)$$

$$\Rightarrow M_X(it) = \sum_{r=0}^\infty \frac{(it)^r}{r!} \frac{\beta^r}{(\alpha+r+3)}$$

4.2 Harmonic Mean

Let X be a ABGUD, then the harmonic mean is obtained as

$$\frac{1}{H} = \int_0^\infty \frac{1}{x} f_1(x)\, dx$$

$$= \int_0^\beta \frac{1}{x} \frac{x^{\alpha+2}(\alpha+3)}{\beta^{\alpha+3}} dx$$

$$= \frac{(\alpha+3)}{\beta^{\alpha+3}} \int_0^\beta x^{\alpha+1} dx$$

$$= \frac{(\alpha+3)}{\beta^{\alpha+3}} \frac{\beta^{\alpha+2}}{(\alpha+2)}$$

$$= \frac{(\alpha+3)}{\beta(\alpha+2)}$$

Thus,

$$H = \frac{\beta(\alpha+2)}{(\alpha+3)}$$

5. ENTROPIES

The idea of entropy is significant in various territories, for example, likelihood and insights, material science, correspondence hypothesis and financial matters. Entropies evaluate the decent variety, vulnerability, or arbitrariness of a framework. Entropy of an arbitrary variable X is a proportion of variety of the vulnerability.

5.1 Renyi Entropy

Renyi entropy is important in nature and mathematics as an indicator of diversity. Renyi entropy is also important in quantum data, where it can be used as a catch measure. With a distributed distribution, Renyi entropy is provided by

$$Re(\delta) = \frac{1}{1-\delta}\log\left(\int_0^\infty f_1{}^\delta(x)dx\right)$$

Where $\delta > 0$ and $\delta \neq 1$

$$Re(\delta) = \frac{1}{1-\delta}\log\left(\int_0^\beta \left(\frac{x^{\alpha+2}(\alpha+3)}{\beta^{\alpha+3}}\right)^\delta dx\right)$$

$$Re(\delta) = \frac{1}{1-\delta}\log\left(\left(\frac{(\alpha+3)}{\beta^{\alpha+3}}\right)^\delta \int_0^\beta x^{\delta(\alpha+2)}dx\right)$$

$$Re(\delta) = \frac{1}{1-\delta}\log\left(\left(\frac{(\alpha+3)}{\beta^{\alpha+3}}\right)^\delta \frac{\beta^{\delta(\alpha+3)+1}}{\delta(\alpha+2)+1}\right)$$

$$Re(\delta)=\frac{1}{1-\delta}log\left(\frac{(\alpha+3)^{\delta}\beta^{1-\delta}}{\delta(\alpha+2)+1}\right)$$

5.2 Tsallis Entropy

A speculation of Boltzmann-Gibbs (B-G) measurable mechanics started by Tsallis has centered a lot to consideration. The mathematical expression of Tsallis entropy (Tsallis, 1988) for a continuous random variable x is given by

$$S_{\lambda}=\frac{1}{1-\lambda}\left(1-\int_0^{\infty}f_1{}^{\lambda}(x)dx\right)$$

$$S_{\lambda}=\frac{1}{1-\lambda}\left(1-\int_0^{\beta}\left(\frac{x^{\alpha+2}(\alpha+3)}{\beta^{\alpha+3}}\right)^{\lambda}dx\right)$$

$$S_{\lambda}=\frac{1}{1-\lambda}\left(1-\left(\frac{(\alpha+3)}{\beta^{\alpha+3}}\right)^{\lambda}\int_0^{\beta}x^{\lambda(\alpha+2)}dx\right)$$

$$S_{\lambda}=\frac{1}{1-\lambda}\left(1-\left(\frac{(\alpha+3)}{\beta^{\alpha+3}}\right)^{\lambda}\frac{\beta^{\lambda(\alpha+3)+1}}{\lambda(\alpha+2)+1}\right)$$

$$S_{\lambda}=\frac{1}{1-\lambda}\left(1-\frac{(\alpha+3)^{\lambda}\beta^{1-\lambda}}{\lambda(\alpha+2)+1}\right)$$

6. ORDER STATISTICS OF ABGU DISTRIBUTION

The probability density function of the j^{th} order statistics $X_{(j)}$ for $1\le j\le n$ is

$$f_{X_{(j)}}(x)=\frac{n!}{(j-1)!(n-j)!}[F(x)]^{j-1}\,[1-F(x)]^{n-j}f_1(x) \tag{6.1}$$

Substitute the value of pdf and cdf of area biased generalized uniform distribution in equation (6.1), we get

$$f_{X_{(j)}}(x)=\frac{n!}{(j-1)!(n-j)!}\left[\left(\frac{x}{\beta}\right)^{\alpha+3}\right]^{j-1}\times\left[1-\left(\frac{x}{\beta}\right)^{\alpha+3}\right]^{n-j}\frac{x^{\alpha+2}(\alpha+3)}{\beta^{\alpha+3}} \tag{6.2}$$

Put $j=1$ in equation (6.2), we will get the probability density function of first order statistics of ABGUD.

$$f_{X_{(1)}}(x)=n\left[1-\left(\frac{x}{\beta}\right)^{\alpha+3}\right]^{n-1}\frac{x^{\alpha+2}\,(\alpha+3)}{\beta^{\alpha+3}}$$

Put $j=n$ in equation (6.2), we will get the probability density function of n^{th} order statistics of ABGUD.

$$f_{X_{(n)}}(x)=n\left[\left(\frac{x}{\beta}\right)^{\alpha+3}\right]^{n-1}\frac{x^{\alpha+2}\,(\alpha+3)}{\beta^{\alpha+3}}$$

7. BONFERRONI AND LORENZ CURVES

The Bonferroni and Lorenz curves is obtained by

$$B(p)=\frac{1}{p\mu}\int_0^q x\,f_1(x)dx$$

$$\&\ L(p)=pB(p)=\frac{1}{\mu}\int_0^q x\,f_1(x)dx$$

$$Where\ \mu=E(x)=\frac{\beta(\alpha+3)}{(\alpha+4)}\ and\ q=F^{-1}(p)$$

$$\therefore\ B(p)=\frac{(\alpha+4)}{p\beta(\alpha+3)}\int_0^q\frac{x^{\alpha+3}\,(\alpha+3)}{\beta^{\alpha+3}}dx$$

$$B(p)=\frac{(\alpha+4)}{p\beta(\alpha+3)}\frac{(\alpha+3)}{\beta^{\alpha+3}}\int_0^q x^{\alpha+3}\,dx$$

After simplification

$$B(p)=\frac{1}{p}\left(\frac{q}{\beta}\right)^{\alpha+4}$$

And

$$L(p)=pB(p)=\left(\frac{q}{\beta}\right)^{\alpha+4}$$

8. Maximum likelihood Estimation (MLE)

In this section, we will discuss the maximum likelihood estimators (MLE) of the parameters of area biased generalized uniform (ABGU) distribution. Consider be the random sample of size n from the ABGU distribution, then the likelihood function is given by

$$L(x;\alpha,\beta)=\frac{(\alpha+3)^n}{\beta^{n(\alpha+3)}}\prod_{i=1}^{n}x_i{}^{\alpha+2}$$

The log likelihood function is given by

$$logl=nlog(\alpha+3)-n(\alpha+3)log(\beta)+(\alpha+2)\sum_{i=1}^{n}logx_i \tag{8.1}$$

The MLE of α and β can be obtained by differentiating equation (8.1) with respect to α and β, we get

$$\frac{\partial log\,l}{\partial\alpha}=\frac{n}{(\alpha+3)}-nlog(\beta)+\sum_{i=1}^{n}logx_i \tag{8.2}$$

$$\frac{\partial log\,l}{\partial\beta}=\frac{-n(\alpha+3)}{\beta} \tag{8.3}$$

$$\frac{\partial log\,l}{\partial\alpha}=0\ \text{ and }\ \frac{\partial log\,l}{\partial\beta}=0$$

From 8.2 we get

$$\frac{n}{(\alpha+3)}-nlog(\beta)+\sum_{i=1}^{n}logx_i=0$$

after solving

$$\hat{\alpha}=\frac{n-3nlog(\beta)+3\sum_{i=1}^{n}logx_i}{nlog(\beta)-\sum_{i=1}^{n}logx_i}$$

From 8.3 we get

$$\frac{-n(\alpha+3)}{\beta}=0$$

$\hat{\beta}=\infty$, which is an absurd result.

Here we can apply inspection method. Let us consider the n[th] ordered samples,

$$0\le X_{(1)}\le X_{(2)}\le X_{(3)}\ \le\cdots\le X_{(n)}\le\beta$$

$$\Rightarrow\beta\ge X_{(n)}$$

Therefore MLE of $\hat{\beta}=X_{(n)}=$ there highest sample observation.

In order o obtain confidence interval we use the asymptotic normality results. Suppose that if $\hat{\lambda}(\hat{\alpha},\hat{\beta})$ denotes the MLE of $\lambda=(\alpha,\beta)$ then Since the MLE of $\hat{\lambda}$ follows asymptotically normal distribution which is given as

$$\sqrt{n}(\hat{\lambda}-\lambda)\rightarrow N(0,I^{-1}(\lambda))$$

Where $I^{-1}(\lambda)$ is Fisher's Information Matrix, i.e.

$$I(\lambda)=-\frac{1}{n}\begin{bmatrix}E\left(\frac{\partial^2log\,l}{\partial^2\alpha}\right) & E\left(\frac{\partial^2log\,l}{\partial\alpha\partial\beta}\right)\\ E\left(\frac{\partial^2log\,l}{\partial\beta\partial\alpha}\right) & E\left(\frac{\partial^2log\,l}{\partial^2\beta}\right)\end{bmatrix}$$

Where

$$E\left(\frac{\partial^2log\,l}{\partial^2\alpha}\right)=\frac{-n}{(\alpha+3)^2},$$

$$E\left(\frac{\partial^2log\,l}{\partial^2\beta}\right)=\frac{n(\alpha+3)}{\beta^2}$$

$$E\left(\frac{\partial^2log\,l}{\partial\alpha\partial\beta}\right)=E\left(\frac{\partial^2log\,l}{\partial\beta\partial\alpha}\right)=\frac{-n}{\beta}$$

Since λ being unknown, we estimate $I^{-1}(\lambda)$ by $I^{-1}(\hat{\lambda})$ and this can be used to obtain asymptotic confidence intervals for α and

β .

9. Likelihood ratio test:

Let $x_1, x_2, x_3, \ldots\ldots, x_n$ be a random sample from the ABGU distribution. We use the hypothesis

$H_0: f(x) = f(x,\alpha,\beta)$ Against $H_1: f(x) = f_1(x,\alpha,\beta)$

In order test the random sample of size n comes from generalized uniform distribution and area biased generalized uniform(ABGU) distribution then following test statistics is used

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{f_1(x,\alpha,\beta)}{f(x,\alpha,\beta)}$$

$$\Delta = \left(\frac{(\alpha+3)}{\beta^2(\alpha+1)}\right)^n \prod_{i=1}^{n} x_i^2$$

We reject the null hypothesis, if

$$\Delta = \left(\frac{(\alpha+3)}{\beta^2(\alpha+1)}\right)^n \prod_{i=1}^{n} x_i^2 > k$$

$$\Delta = \prod_{i=1}^{n} x_i^2 > k\left(\frac{\beta^2(\alpha+1)}{(\alpha+3)}\right)^n$$

or

$\Delta^* = \prod_{i=1}^{n} x_i^2 > k^*$ Where $k^* = k\left(\frac{\beta^2(\alpha+1)}{(\alpha+3)}\right)^n > 0$

For large sample size n, $2log\Delta$ distributed as chi-square distribution with 1 degree of freedom (df) and also p-value is obtained from the chi-square distribution. Thus we reject the null hypothesis, when the probability value is given by

$$p(\Delta^* > \theta^*)$$

Where $\theta^* = \prod_{i=1}^{n} x_i$ is less than specified level of significance and $\prod_{i=1}^{n} x_i^2$ is observed value of the statistics Δ^*.

CONCLUSION:

In this paper, we have studied a new distribution called as the area biased generalized Uniform distribution. By using certain special functions, its statistical properties, moments, failure rate, survival function, entropy has been obtained. The parameters have been estimated by using maximum likelihood method and also order statistics and fisher's information matrix have been obtained.

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