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ANALYSIS OF FLOW OF BLOOD AND FLOW IN LUNG AIRWAYS

Mathematics

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ABSTRACT

The cardiovascular system consists, the heart, which acts as a pump whose elastic muscular walls making possible the pulsatile flow of blood through the vascular system; the distributary system comprising arteries and arterioles for sending blood to the various organs of the body; the diffusing system made of fine capillaries which are in contact with the cells of the body; the collecting system of veins which collects blood depleted of oxygen and full of products of metabolic processes of the system. The organs which supplement the function of the cardiovascular system are the lungs which provide a region of inter phase transfer of oxygen to the blood and removal of carbon dioxide from it and the kidney, liver, spleen which help in maintaining the chemical quality of blood under normal conditions and under conditions of extreme stress.
The main goal of this paper is comparison between flows of blood and flows in lung airways.

KEYWORDS

viscosity, blood, strain rate, stress tensor, Reynolds number

Blood consists of a suspension of cells in an aqueous solution called plasma which is composed of about 90 percent water and 7 percent protein. There are about 5×10^{9} cells in a millilitre (1cc) of healthy human blood, of which about 95 percent are red cells or erythrocytes whose main function is to transport oxygen from the lungs to all the cells of the body and the removal of carbon dioxide formed by metabolic processes in the body to the lungs. About 45 percent of the blood volume in an average man is occupied by red cells. This fraction is known as the hematocrit. Of the remaining, white cells or leucocytes consists about one six or 1 percent of the total, and these play a role in the resistance of the body to infection; platelets form 5 percent of the total, and they perform a function related to blood clotting. In a 5 litres of blood in the human body, there are about 25×10^{12} red cells. The mean life of a red cell is about 120 days and the total number of red cells which die per second is $(25\times10^{12})/(120\times24\times60\times60)=2.4\times10^{6}$ Surface area of a red blood cell is about $140\mu m^2$ Hence the total surface area of all red cells is about $25\times10^{12}\times 40\mu m^2 = 3500m^2$ This surface area is important because diffusion of oxygen takes place all along this surface.

Oxygen is carried by blood in two forms-

1. Dissolved in plasma
2. Bound with hemoglobin in red cells

For the simple motions we shall consider, there is only one non zero component τ of the stress tensor and only one non-zero component e of the rate of strain tensor. In general, each of these tensors has six distinct components. The functional relations between the components of the two tensors depend on the fluid under consideration and determine the constitutive equations for the fluid.

For Newtonian viscous fluids,
$\tau=\mu e$ where μ is the constant coefficient of viscosity. We have fluids for which μ itself may be a function of strain rate. Such fluids are called non- Newtonian fluids.

Blood is neither homogeneous nor Newtonian. Plasma in isolation may be considered Newtonian with a viscosity of about 1.2 times that of water. For whole blood, we can measure effective viscosity, and this is found to depend on shear rate. The constitutive equations proposed for whole blood are as follows:
(I) $\tau=\mu e^{n}$ (i)power law equation). This is found to hold good for strain rates between 5 and 200 sec^{-1}, with n having a value between 0.68 and 0.80.

(ii) $\tau=\mu e^{n}+\tau_0$ $(\tau>\tau_0)$(Herschel -Bulkley equation)

(ii)$\tau^{1/2}=\mu^{1/2} e^{1/2}+\tau_0^{1/2}$(Casson equation). This hold for strain rates between 0 and 100,000 sec^{-1}.

Weibel's Model for flows in Lung Airways-In mammalian lungs, airways branch by irregular dichotomy, i.e. there can be considerable difference in lengths and diameters of the two daughter tubes arising from bifurcation of a mother tube. E Weibel gave a theoretical model with regularized dichotomy and systematic branching pattern by simply taking the mean value of the lengths and diameters for the airways of each generation. . Most theoretical workers use this model on account of its simplicity in spite of some criticism that it is at best an approximation to reality.

If D_n, L_n, v_n *and* $(Re)_n$ denote respectively the average diameter, length, velocity, and Reynolds flow number of a tube of the *n-th* generation, then flow rate $Q = 2^n \frac{\pi D_n^2 v_n}{4\times1000}$ and $(Re)_n = \frac{v_n D_n}{\nu} = \frac{1}{\nu}\frac{4000\,Q}{\pi D_n\times2^n} = \frac{1}{.152}\frac{4000\,Q}{\pi D_n\times2^n}$ where the kinematic viscosity ν of air is taken as $\nu = 1.52\times10^{-5}m^2sec^{-1} = .152\,cm^2sec^{-1} = 15.2mm^2sec^{-1}$
Now if β_n denotes the ratio of the total cross sectional area of all tubes of the *n-th* generation to the total cross- sectional area of all tubes of the (*n*-1) th generation, A_n denotes the total cross sectional area of all tubes of the *n-th* generation.

Then $A_n = 2^n \frac{\pi D_n^2}{4}$

$$\beta_n = \frac{2^n\times\pi D_n^2/4}{2^{n-1}\times\pi D_{n-1}^2/4} = \frac{2D_n^2}{D_{n-1}^2} = \frac{A_n}{A_{n-1}};$$

$$V_n = 2^n \frac{\pi D_n^2}{4} L_n$$

$$, \frac{V_n}{V_{n-1}} = \beta_n \frac{L_n}{L_{n-1}}$$

We now draw the following conclusions:

(i) For a given Q,

$$v_n \propto \frac{1}{2^n D_n^2}, \qquad (Re)_n \propto \frac{1}{2^n D_n}$$

Both velocity and Reynolds number decreases as n increases, but Reynolds number decrease faster.

During normal breathing, a man breathes about 15 times a minute, and during each inspiration and expiration, the flow direction is reversed. The angular frequency of this periodic flow is given by $\frac{2\pi}{\omega} = \frac{60}{15}$ or $\omega = \frac{\pi}{2}\ sec^{-1}$ If the breathing is $\frac{1}{2}\pi k\ sec^{-1}$. The thickness of the oscillating boundary layer is proportional to $(\nu/\omega)^{1/2}$, where ν is the kinematic viscosity of air.

The extent of time dependence of flow depends on Womersley's number $\alpha = \frac{1}{2}d\sqrt{\omega/\nu}$. This is the ratio of the tube to the thickness of the oscillating boundary layer. If a is large, the oscillating boundary layer is thin relative to the radius of the tube, and most of the flow is dominated by inertial terms. Viscosity effects are small and cannot adjust the flow to periodic effects so that the flow is not quasi –steady. However, if a is unity or less, the viscous effects are dominant and the flow can be regarded as quasi steady.

In addition to periodicity of breathing, the periodicity of heart beats (72 times a minute of 1.2 Hz) also has its effect on the time dependence of the flow. For normal breathing, the flow may be regarded as quasi – steady from fourth or fifth generation onwards, but for really heavy breathing, the flow may not be quasi- steady up to tenth generation.

We now discuss similarities and dissimilarities between blood flow

and airways flow.

Blood is a suspension of particles in plasma and is a non homogeneous, non Newtonian fluid. Air is essentially Newtonian and viscous. It contains dust particles.

Air is compressible, whereas blood is incompressible. However, since the velocities in the lung airways are always small as compared with the speed of sound, compressibility effects can be neglected.

Air flow reverses its direction with expiration and inspiration. For very short time velocity can be zero when the dust particles can be deposited in the lungs but blood flow in a tube is essentially unidirectional, though some reversed flow, localized near an arterial wall may occur.

Blood flow is essentially laminar through there may be turbulence in the arota. Air flow in the trachea is usually turbulent and turbulence may persist four four and five generations.

Blood flow is pulsatile and under normal conditions, its oscillatory boundary

layer thickness is $\sqrt{\frac{\nu}{\omega}} = \sqrt{\frac{4}{8}} = 0.7\ mm$. The corresponding thickness for airflow

is $\sqrt{\frac{15.2}{1.57}} = 3.1\ mm.$

Thus the pulsatility effect is felt in blood in blood vessels whose diameter is five times smaller than that of the corresponding airways. When a mother tube divides symmetrically into two daughter tubes, the area increases in the ratio β:1,On the other hand , the mean value of velocity is also divided by β, and hence a large value of β may lead to large retardation and flow separation. In the lung airways, β exceeds unity when the tube diameter is about 50 mm.

In cardiovascular system pressure drop occurs in microcirculation in tubes of diameter which is less than 0.1mm, but in lung airways, it takes place in the first 10 generations with diameters greater than 1mm.

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