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COMPARISON OF EFFICIENT DOMINATION NUMBER, INDEPENDENT DOMINATION NUMBER AND CONNECTED DOMINATION NUMBER WITH RADIUS OF A GRAPH G

Mathematics

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ABSTRACT

Domination in graphs has been an extensively researched branch of graph theory. Graph theory is one of the most flourishing branches of modern mathematics and computer applications. Different types of dominations like Efficient domination, independent domination and connected domination in graphs have applications to several fields. In this paper we compare Efficient domination number, Independent domination number and Connected domination number with radius of a graph G.

KEYWORDS

Eccentricity, Distance, Radius Of A Graph, Efficient Domination Number, Blast Domination Number, Independent Domination Number And Connected Domination Number.

INTRODUCTION:

Graph theory is rapidly moving into the mainstream of mathematics mainly due to its applications in diverse fields which include biochemistry (genomics), electrical engineering (communications networks and coding theory), computer science (algorithms and computations) and operations research (scheduling). Although graph theory is one of the younger branches of mathematics, it is fundamental to a number of applied fields.

Few subjects in mathematics have as specified an origin as graph theory. Graph theory originated with the Konigsberg Bridge Problem, which Leonhard Euler (1707-1783) solved in 1736. Over the past sixty years, there has been a great deal of exploration in the area of graph theory. Its popularity has increased due to its many modern day applications and it has become the source of interest to many researchers. High-speed digital computer is one of the main reasons for the recent growth of interest in graph theory and its applications.

Now a days graphs are really important in different fields. Probably, more important than we think. Graphs are among the most ubiquitous models of both natural and human-made structures. They can be used to model many types of relations and process dynamics in Physical, Biological and Social systems.

Many problems of practical interest can be represented by graphs and can be solved using graph theory. In Architecture, bipartite graphs play an important role in finding minimum number of cross beams required to make a grid of beams rigid, when the joints provide no rigidity to the structure. The structure is rigid if and only if the corresponding bipartite graph is connected. But the smallest connected graph is a tree and the largest possible tree in the bipartite graph with m, n vertices has m+n-1 edges. Hence an mxn graph is rigid iff the corresponding bipartite graph is connected. The rigid bracing will have minimum cross beams iff the bipartite graph is a tree with m+n-1 cross beams.

So, one can confidently put forward that a mere act of thinking about a problem in terms of a graph will certainly suggest insights and probable solution methods.

Graph theory is branch of mathematics, which has becomes quite rich and interesting for several regions. In last three decades thousand of research articles have been published in graph theory. There are several areas in graph theory which have reserved good attention from mathematicians. Graphs are very convenient tool for representing the relationship among objects, which are represented by vertices. In their term relationships among vertices are represented by connections. In general, any mathematical objects involving points and connection among them can be called a graph. For a great diversity of problems such pictorial representations may lead to a solution. For example data basics map coloring web graph, physical net work and organic molecular as well as less tangible interactions occurring in social net works in a flow of a computer program.

Domination is a rapidly developing area of research in graph theory, and its various applications to ad-hock net work, distributed computing, social net works and web graph, partly explain the increased interest. Domination in graphs has been an extensively researched branch of graph theory. Graph theory is one of the most flourishing branches of modern mathematics and computer applications. The last 30 years have witnessed spectacular growth of graph theory due to its wide application to discrete optimization problems, combinatorial problems and classical algebraic problems. It has a very wide range of application to many fields like engineering, physical, social and biological sciences, linguistics etc., the theory of domination has been the nucleus of research activity in graph theory in recent time.

Domination in graphs has applications to several fields. Domination arises in facility location problem, where the number of facilities like hospitals, fire stations is fixed and one attempts to minimize the distance that a person needs to travel to get to the closest facility. A similar problem occurs when the maximum distance to a facility is fixed and one attempts to minimize the number of facilities necessary so that ever one is serviced. Concepts from domination also appear in problems involving finding sets of representatives in monitoring communication or electrical networking, and in land surveying like minimizing the number of places a surveyor must stand in order to take high measure mints for an entire region.

A subset D of V is said to be a dominating set of G if every vertex in V\D is adjacent to a vertex in D. A dominating set with minimum cardinality is said to be a minimum dominating set. The domination number (G) of the graph G is the minimum cardinality of the dominating set in G.

A trapezoid graph (T G) consists of two horizontal lines L_1 (top line) and L_2 (bottom line) and a set of trapezoids $T = \{T_1, T_2, \ldots, T_n\}$ with corner points lying on these two lines. An undirected graph G with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and edge set $E = \{e_1, e_2, \ldots, e_m\}$ is called a T G when a trapezoid representation exists with trapezoid set T, such that any vertex $v_k \in V$ corresponds to a trapezoid $T_k \in T$ and an edge $(v_k, v_l) \in E$ iff T_k and T_l intersect.

Any trapezoid T_k within these two lines is known by four corner points a_k, b_k, c_k and d_k which represent the upper left, upper right, lower left and lower right corner points respectively. Without loss of generality, we assume that no two trapezoids share a same endpoint. It is assumed that $T_p < T_q$ or $v_p < v_q$ iff $a_p < a_q$.

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The T G was established by Dagan. Corneil and Kamula established the same class but they refer to them as II (interval-interval) graphs. Felsner, Muller and Wernisch established an equivalent characterization for T G. After that, it has been broadly studied by many researchers. The class of T G is the generalization of the classes of I G and P G. It is very interesting that if $a_p = b_p$ and $c_p = d_p$ then the corresponding trapezoid T_p reduces to straight line. So, in that way, if each trapezoids cut down to a straight lines the corresponding T G reduces to a P G. Similarly, if $a_p = c_p$ and $b_p = d_p$, for every p, then the T G reduces to an I G. It is to be noted that any trapezoid T_u and the corresponding node u are one and same thing.

Preliminaries:

Let G be a graph. An edge which joins the same vertex is called a loop. An edge that joins two distinct vertices is called a link. If two or more edges of G have the same end vertices, then these edges are called parallel edges or multiple edges. A graph is said to be a simple graph if it has no loops and no parallel edges.

A graph G is said to be finite if both its vertex set and edge sets are finite, otherwise it is called an infinite graph. The number of edges incident with a vertex v of a graph is called the degree of v and is denoted by d(v). A graph is said to be connected if there is a path between every pair of vertices, otherwise it is said to be disconnected graph. Neighborhood of a vertex vV is a set consisting all vertices adjacent to v (including v), it is denoted by nbd [v]. i.e., nbd [v] = {the set of all vertices adjacent to v} {v}.

A subset S of V is called an independent set of G if no two vertices in S are adjacent. A maximum independent set of G is an independent set whose cardinality is largest among all independent sets of G. A subset D of V is said to be a dominating set of G if every vertex in V-D is adjacent to a vertex in D. A dominating set with minimum cardinality is said to be a minimum dominating set. The domination number (G) of the graph G is the minimum cardinality of the dominating set in G.

An independent set ***S*** of vertices in a graph is an efficient dominating set when each vertex not in ***S*** is adjacent to exactly one vertex in ***S***. A set $S \subset V$ is a dominating set if $N(S) = V$, that is, every vertex in *V-S* is adjacent to some vertex in *S*. The domination number (G) is the minimum cardinality of a dominating set in G. If the dominating set *S* is a stable set of G, then *S* is an independent dominating set. Also, when every vertex in *V-S* is adjacent to exactly one vertex in *S*, then *S* is a perfect dominating set. A dominating set *S* which is both independent and perfect is an efficient dominating set.

A subset S of V of a non-trivial connected graph G is called a Blast dominating set (or) BD-set, if S is a connected dominating set and the induced sub graph (V – S) is triple connected. The minimum cardinality taken over all such Blast Dominating sets is called the Blast Domination Number (BDN) of G and is denoted by γ_c^{tc} (G). Also, any Blast Dominating set with γ_c^{tc} vertices is called a γ_c^{tc} - set of G.

A graph is said to be triple connected if any three vertices lie on a path in G. A subset S of V of a nontrivial graph G is said to be triple connected dominating set, if S is a dominating set and <S> is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number of G and is denoted by γ_{tc}(G).

A connected dominating set D to be a dominating set D whose induced subgraph $\langle D \rangle$ is connected. The connected domination number (G) of a connected graph G is the minimum cardinality of a connected dominating set of G.

A subset S of V is called an independent set of G if no two vertices in S are adjacent. A maximum independent set of G is an independent set whose cardinality is largest among all independent sets of G. A subset D of V is said to be a dominating set of G if every vertex in V\D is adjacent to a vertex in D. A dominating set with minimum cardinality is said to be a minimum dominating set. The domination number (G) of the graph G is the minimum cardinality of the dominating set in G. An *independent dominating set* of G is a set that is both dominating and independent in G. The *independent domination number* of G, denoted by i(G), is the minimum size of an independent dominating set.

Distance between Two Vertices in a graph G is number of edges in a shortest path between Vertex u and Vertex v. If there are multiple paths connecting two vertices, then the shortest path is considered as the distance between the two vertices. Denoted as d(u,v).

Eccentricity of a Vertex is the maximum distance between a vertex to all other vertices is considered as the eccentricity of vertex. Denoted as e(V). The distance from a particular vertex to all other vertices in the graph is taken and among those distances, the eccentricity is the highest of distances.

Radius of a Connected Graph G is the minimum eccentricity from all the vertices is considered as the radius of the Graph G. The minimum among all the maximum distances between a vertex to all other vertices is considered as the radius of the Graph G. denoted as r(G). From all the eccentricities of the vertices in a graph, the radius of the connected graph is the minimum of all those eccentricities.

Let G be a graph and v be a vertex of G. The eccentricity of the vertex v is the maximum distance from v to any vertex. That is, e(v)=max{d(v,w):w in V(G)}. The radius of G is the minimum eccentricity among the vertices of G. Therefore, radius(G)=min{e(v):v in V(G)}.

Distance between *u* and *v* in the graph G is the length of the shortest *u* – *v* path in G if such a path exists. If no *u* – *v* path exist, define $d(u, v) = \infty$.

For each vertex *v* in a graph G, the eccentricity, *e*(*v*), is the distance from *v* to the vertex furthest from *v*, that is, $e(v) = \max\{d(v, u) \mid u$ in V(G)}.

Radius rad G of a connected graph G is defined as rad G = min $\{e(v) \mid v$ in V(G)}.

Main theorems:

Theorem 1: If T = {$T_1, T_2, \ldots, T_n$} be a trapezoid family and TG is a trapezoid graph corresponding to a trapezoid family 'T'. Then the efficient domination number is less than the radius of a trapezoid graph TG

Proof: An independent set ***S*** of vertices in a graph is an efficient dominating set when each vertex not in ***S*** is adjacent to exactly one vertex in ***S***. A set $S \subset V$ is a dominating set if $N(S) = V$, that is, every vertex in *V-S* is adjacent to some vertex in *S*. The domination number (G) is the minimum cardinality of a dominating set in G. If the dominating set *S* is a stable set of G, then *S* is an independent dominating set. Also, when every vertex in *V-S* is adjacent to exactly one vertex in *S*, then *S* is a perfect dominating set. A dominating set *S* which is both independent and perfect is an efficient dominating set.

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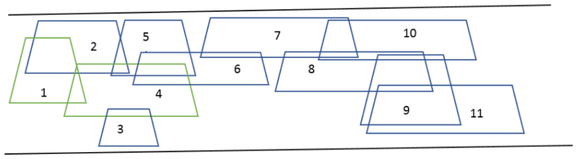
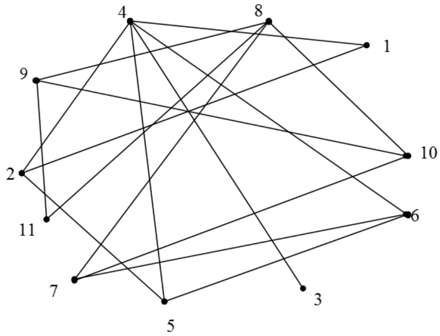
Experimental Problem





Find the efficient domination number of G

An independent set ***S*** of vertices in a graph is an efficient dominating set when each vertex not in ***S*** is adjacent to exactly one vertex in ***S***.

Vertex v_1 is adjacent to exactly one vertex v_4, Vertex v_2 is adjacent to exactly one vertex v_4,

Vertex v_3 is adjacent to exactly one vertex v_4, Vertex v_5 is adjacent to exactly one vertex v_4,

Vertex v_6 is adjacent to exactly one vertex v_4, Vertex v_7 is adjacent to exactly one vertex v_8,

Vertex v_9 is adjacent to exactly one vertex v_8, Vertex v_{10} is adjacent to exactly one vertex v_8,

Vertex v_{11} is adjacent to exactly one vertex v_8.

S={v_4, v_8} is an independent set and each vertex not in S is adjacent to exactly one vertex in S. so S is an efficient dominating set of G.

Efficient dominating set of G {v_4, v_8}

Efficient domination number of G is 2

Find the radius of G

Find the distance between u ,v such that u,v are in V

d(1,1)=0	d(2,1)= 1	d(3,1)=2
d(1,2)=1	d(2,2)= 0	d(3,2)=2
d(1,3)=2	d(2,3)=2	d(3,3)=0
d(1,4)=1	d(2,4)=1	d(3,4)=1
d(1,5)=2	d(2,5)=1	d(3,5)=2
d(1,6)=2	d(2,6)=2	d(3,6)=2
d(1,7)=3	d(2,7)=3	d(3,7)=3
d(1,8)=4	d(2,8)=4	d(3,8)=4
d(1,9)=5	d(2,9)=5	d(3,9)=5
d(1,10)=5	d(2,10)=5	d(3,10)=5
d(1,11)=5	d(2,11)=5	d(3,11)=5

d(4,1)=1	d(5,1)=2	d(6,1)=2
d(4,2)=1	d(5,2)=1	d(6,2)=2
d(4,3)=1	d(5,3)=2	d(6,3)=2
d(4,4)=0	d(5,4)=1	d(6,4)=1
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=1	d(5,6)=1	d(6,6)=0
d(4,7)=2	d(5,7)=2	d(6,7)=1
d(4,8)=3	d(5,8)=3	d(6,8)=2
d(4,9)=4	d(5,9)=4	d(6,9)=3
d(4,10)=4	d(5,10)=4	d(6,10)=3
d(4,11)=4	d(5,11)=4	d(6,11)=3

d(7,1)=3	d(8,1)=4	d(9,1)=5
d(7,2)=3	d(8,2)=4	d(9,2)=5
d(7,3)=3	d(8,3)=4	d(9,3)=5
d(7,4)=2	d(8,4)=3	d(9,4)=4
d(7,5)=2	d(8,5)=3	d(9,5)=4
d(7,6)=1	d(8,6)=2	d(9,6)=3
d(7,7)=0	d(8,7)=1	d(9,7)=2
d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=2	d(8,9)=1	d(9,9)=0
d(7,10)=2	d(8,10)=1	d(9,10)=1
d(7,11)=2	d(8,11)=1	d(9,11)=1

d(10,1)=5	d(11,1)=5
d(10,2)=5	d(11,2)=5
d(10,3)=5	d(11,3)=5
d(10,4)=4	d(11,4)=4
d(10,5)=4	d(11,5)=4
d(10,6)=3	d(11,6)=3
d(10,7)=2	d(11,7)=2
d(10,8)=1	d(11,8)=1
d(10,9)=1	d(11,9)=1
d(10,10)=0	d(11,10)=2
d(10,11)=2	d(11,11)=0

Find the eccentricity, *e*(*v*) of G

$e(v) = \max\{d(v, u) \mid u \text{ in } V(G)\}$.

$e(1) = \max\{d(1, u) \mid u \text{ in } V(G)\} = \max\{0, 1, 2, 1, 2, 2, 3, 4, 5, 5, 5\} = 5$

$e(2) = \max\{d(2, u) \mid u \text{ in } V(G)\} = \max\{1, 0, 2, 1, 1, 2, 3, 4, 5, 5, 5\} = 5$

$e(3) = \max\{d(3, u) \mid u \text{ in } V(G)\} = \max\{2, 2, 0, 1, 2, 2, 3, 4, 5, 5, 5\} = 5$

$e(4) = \max\{d(4, u) \mid u \text{ in } V(G)\} = \max\{1, 1, 1, 0, 1, 1, 2, 3, 4, 4, 4\} = 4$

$e(5) = \max\{d(5, u) \mid u \text{ in } V(G)\} = \max\{2, 1, 2, 1, 0, 1, 2, 3, 4, 4, 4\} = 4$

$e(6) = \max\{d(6, u) \mid u \text{ in } V(G)\} = \max\{2, 2, 2, 1, 1, 0, 1, 2, 3, 3, 3\} = 3$

$e(7) = \max\{d(7, u) \mid u \text{ in } V(G)\} = \max\{3, 3, 3, 2, 2, 1, 0, 1, 2, 2, 2\} = 3$

$e(8) = \max\{d(8, u) \mid u \text{ in } V(G)\} = \max\{4, 4, 4, 3, 3, 2, 1, 0, 1, 1, 1\} = 4$

$e(9) = \max\{d(9, u) \mid u \text{ in } V(G)\} = \max\{5, 5, 5, 4, 4, 3, 2, 1, 0, 1, 1\} = 5$

$e(10) = \max\{d(10, u) \mid u \text{ in } V(G)\} = \max\{5, 5, 5, 4, 4, 3, 2, 1, 1, 0, 2\} = 5$

$e(11) = \max\{d(11, u) \mid u \text{ in } V(G)\} = \max\{5, 5, 5, 4, 4, 3, 2, 1, 1, 2, 0\} = 5$

Radius rad G of a connected graph G is defined as rad G = min $\{e(v) \mid v$ in V(G)$\}$

rad G = min $\{e(1), e(2), e(3), e(4), e(5), e(6), e(7), e(8), e(9), e(10), e(11)\}$

= min {5, 5, 5, 4, 4, 3, 3, 4, 5, 5, 5}

= 3

Efficient domination number and Radius of G are 2 and 3 respectively

Efficient domination number of G is less than that of Radius of G 2<3

We proved that Efficient domination number of a graph G is less than Radius of G.

Theorem 2: If T = {T1, T2, . . . , Tn } be a trapezoid family and TG is a trapezoid graph corresponding to a trapezoid family 'T'. Then the Independent domination number is greater than the radius of a trapezoid graph TG

Proof: Independent sets play an important role in Graph Theory and other areas like discrete optimization. They appear in matching theory, coloring of graphs and in trees. A set of vertices in a graph G is said to be an independent set or an internally stable set if no two vertices in the set are adjacent. An independent set S is said to be maximal independent set if S U {v} is not an independent set for every vertex v not in S. The independence number is the, maximum cardinality of an independent set in G. It is denoted by β0(G). A set with minimum cardinality among all the maximal independent set of G is called minimum independent dominating set of G or just i set of G. The cardinality of a minimum independent dominating set is called independent domination number of the graph G and it is denoted by i(G). Note: A maximal independent set is a dominating set of G.

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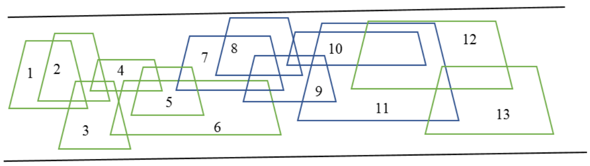
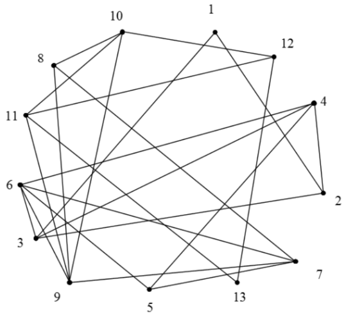
Experimental Problem





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Vertex v_9 is adjacent to exactly one vertex v_8, Vertex v_{10} is adjacent to exactly one vertex v_8,

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S={v_4, v_8} is an independent set and each vertex not in S is adjacent to exactly one vertex in S. so S is an efficient dominating set of G.

Efficient dominating set of G {v_4, v_8}

Efficient domination number of G is 2

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d(1,2)=1	d(2,2)= 0	d(3,2)=2
d(1,3)=2	d(2,3)=2	d(3,3)=0
d(1,4)=1	d(2,4)=1	d(3,4)=1
d(1,5)=2	d(2,5)=1	d(3,5)=2
d(1,6)=2	d(2,6)=2	d(3,6)=2
d(1,7)=3	d(2,7)=3	d(3,7)=3
d(1,8)=4	d(2,8)=4	d(3,8)=4
d(1,9)=5	d(2,9)=5	d(3,9)=5
d(1,10)=5	d(2,10)=5	d(3,10)=5
d(1,11)=5	d(2,11)=5	d(3,11)=5

d(4,1)=1	d(5,1)=2	d(6,1)=2
d(4,2)=1	d(5,2)=1	d(6,2)=2
d(4,3)=1	d(5,3)=2	d(6,3)=2
d(4,4)=0	d(5,4)=1	d(6,4)=1
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=1	d(5,6)=1	d(6,6)=0
d(4,7)=2	d(5,7)=2	d(6,7)=1
d(4,8)=3	d(5,8)=3	d(6,8)=2
d(4,9)=4	d(5,9)=4	d(6,9)=3
d(4,10)=4	d(5,10)=4	d(6,10)=3
d(4,11)=4	d(5,11)=4	d(6,11)=3

d(7,1)=3	d(8,1)=4	d(9,1)=5
d(7,2)=3	d(8,2)=4	d(9,2)=5
d(7,3)=3	d(8,3)=4	d(9,3)=5
d(7,4)=2	d(8,4)=3	d(9,4)=4
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d(7,6)=1	d(8,6)=2	d(9,6)=3
d(7,7)=0	d(8,7)=1	d(9,7)=2
d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=2	d(8,9)=1	d(9,9)=0
d(7,10)=2	d(8,10)=1	d(9,10)=1
d(7,11)=2	d(8,11)=1	d(9,11)=1

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d(10,4)=4	d(11,4)=4
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d(10,7)=2	d(11,7)=2
d(10,8)=1	d(11,8)=1
d(10,9)=1	d(11,9)=1
d(10,10)=0	d(11,10)=2
d(10,11)=2	d(11,11)=0

Find the eccentricity, e(v) of G

$e(v)$ = max {d(v, u) | u in V(G)}.
$e(1)$ = max {d(1, u) | u in V(G)}= max{0, 1, 2, 1, 2, 2, 3, 4, 5, 5, 5}=5
$e(2)$ = max {d(2, u) | u in V(G)}= max{1, 0, 2, 1, 1, 2, 3, 4, 5, 5, 5}=5
$e(3)$ = max {d(3, u) | u in V(G)}= max{2, 2, 0, 1, 2, 2, 3, 4, 5, 5, 5}=5
$e(4)$ = max {d(4, u) | u in V(G)}= max{1, 1, 1, 0, 1, 1, 2, 3, 4, 4, 4}=4
$e(5)$ = max {d(5, u) | u in V(G)}= max{2, 1, 2, 1, 0, 1, 2, 3, 4, 4, 4}=4
$e(6)$ = max {d(6, u) | u in V(G)}= max{2, 2, 2, 1, 1, 0, 1, 2, 3, 3, 3}=3
$e(7)$ = max {d(7, u) | u in V(G)}= max{3, 3, 3, 2, 2, 1, 0, 1, 2, 2, 2}=3
$e(8)$ = max {d(8, u) | u in V(G)}= max{4, 4, 4, 3, 3, 2, 1, 0, 1, 1, 1}=4
$e(9)$ = max {d(9, u) | u in V(G)}= max{5, 5, 5, 4, 4, 3, 2, 1, 0, 1, 1}=5
$e(10)$ = max {d(10, u) | u in V(G)}= max{5, 5, 5, 4, 4, 3, 2, 1, 1, 0, 2}=5
$e(11)$ = max {d(11, u) | u in V(G)}= max{5, 5, 5, 4, 4, 3, 2, 1, 1, 2, 0}=5

Radius rad G of a connected graph G is defined as rad G = min {$e(v)$ | v in V(G)}

rad G = min {$e(1)$, $e(2)$, $e(3)$, $e(4)$, $e(5)$, $e(6)$, $e(7)$, $e(8)$, $e(9)$, $e(10)$, $e(11)$}
= min {5, 5, 5, 4, 4, 3, 3, 4, 5, 5, 5}
= 3

Efficient domination number and Radius of G are 2 and 3 respectively
Efficient domination number of G is less than that of Radius of G 2<3
We proved that Efficient domination number of a graph G is less than Radius of G.

Theorem 3: If T = {T_1, T_2, . . . , T_n} be a trapezoid family and TG is a trapezoid graph corresponding to a trapezoid family 'T'. Then the Connected domination number is equal to the radius of a trapezoid graph TG

Proof: A connected dominating set D to be a dominating set D whose induced sub graph $\langle D\rangle$ is connected. The connected domination number (G) of a connected graph G is the minimum cardinality of a connected dominating set of G.

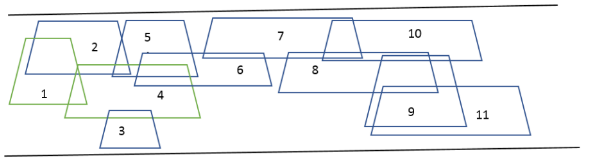
Distance between Two Vertices in a graph G is number of edges in a shortest path between Vertex u and Vertex v. If there are multiple paths connecting two vertices, then the shortest path is considered as the distance between the two vertices. Denoted as d(u,v).

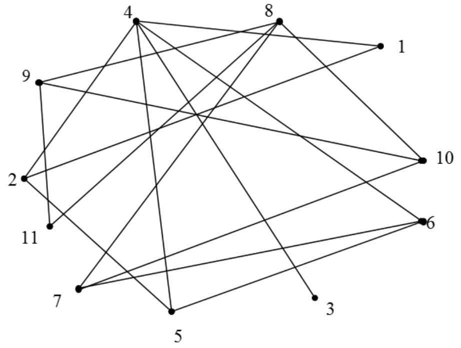
Eccentricity of a Vertex is the maximum distance between a vertex to all other vertices is considered as the eccentricity of vertex. Denoted as e(V). The distance from a particular vertex to all other vertices in the graph is taken and among those distances, the eccentricity is the highest of distances.

Radius of a Connected Graph G is the minimum eccentricity from all the vertices is considered as the radius of the Graph G. The minimum among all the maximum distances between a vertex to all other vertices is considered as the radius of the Graph G. denoted as r(G). From all the eccentricities of the vertices in a graph, the radius of the connected graph is the minimum of all those eccentricities.

Let G be a graph and v be a vertex of G. The eccentricity of the vertex v is the maximum distance from v to any vertex. That is, e(v)=max{d(v, w):w in V(G)}. The radius of G is the minimum eccentricity among the vertices of G. Therefore, radius(G)=min{e(v):v in V(G)}.

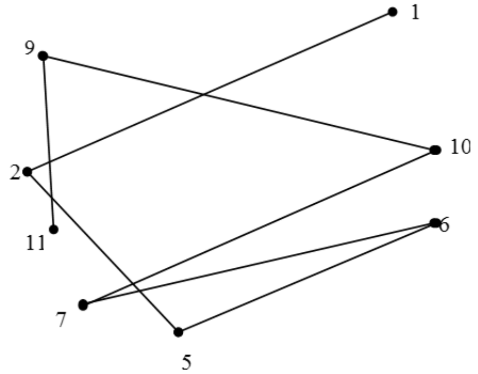
Experimental Problem





Find the connected domination number of G

Connected dominating set of G is $\{v_3, v_4, v_8\}$
Connected domination number of G is 3
A connected dominating set D to be a dominating set D whose induced subgraph ⟨V-*D*⟩ is connected.



The induced sub graph (V-D) is connected so D is connected dominating set.

Find the radius of G

Find the distance between u ,v such that u ,v are in V

d(1,1)=0	d(2,1)= 1	d(3,1)=2
d(1,2)=1	d(2,2)= 0	d(3,2)=2
d(1,3)=2	d(2,3)=2	d(3,3)=0
d(1,4)=1	d(2,4)=1	d(3,4)=1
d(1,5)=2	d(2,5)=1	d(3,5)=2
d(1,6)=2	d(2,6)=2	d(3,6)=2
d(1,7)=3	d(2,7)=3	d(3,7)=3
d(1,8)=4	d(2,8)=4	d(3,8)=4
d(1,9)=5	d(2,9)=5	d(3,9)=5
d(1,10)=5	d(2,10)=5	d(3,10)=5
d(1,11)=5	d(2,11)=5	d(3,11)=5

d(4,1)=1	d(5,1)=2	d(6,1)=2
d(4,2)=1	d(5,2)=1	d(6,2)=2
d(4,3)=1	d(5,3)=2	d(6,3)=2
d(4,4)=0	d(5,4)=1	d(6,4)=1
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=1	d(5,6)=1	d(6,6)=0
d(4,7)=2	d(5,7)=2	d(6,7)=1
d(4,8)=3	d(5,8)=3	d(6,8)=2
d(4,9)=4	d(5,9)=4	d(6,9)=3
d(4,10)=4	d(5,10)=4	d(6,10)=3
d(4,11)=4	d(5,11)=4	d(6,11)=3

d(7,1)=3	d(8,1)=4	d(9,1)=5
d(7,2)=3	d(8,2)=4	d(9,2)=5
d(7,3)=3	d(8,3)=4	d(9,3)=5
d(7,4)=2	d(8,4)=3	d(9,4)=4
d(7,5)=2	d(8,5)=3	d(9,5)=4
d(7,6)=1	d(8,6)=2	d(9,6)=3
d(7,7)=0	d(8,7)=1	d(9,7)=2
d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=2	d(8,9)=1	d(9,9)=0
d(7,10)=2	d(8,10)=1	d(9,10)=1
d(7,11)=2	d(8,11)=1	d(9,11)=1

d(10,1)=5	d(11,1)=5
d(10,2)=5	d(11,2)=5
d(10,3)=5	d(11,3)=5
d(10,4)=4	d(11,4)=4
d(10,5)=4	d(11,5)=4
d(10,6)=3	d(11,6)=3
d(10,7)=2	d(11,7)=2
d(10,8)=1	d(11,8)=1
d(10,9)=1	d(11,9)=1
d(10,10)=0	d(11,10)=2
d(10,11)=2	d(11,11)=0

Find the eccentricity, $e(v)$ of G
$e(v) = \max \{d(v, u) \mid u \text{ in } V(G)\}$.
$e(1) = \max \{d(1, u) \mid u \text{ in } V(G)\} = \max\{0, 1, 2, 1, 2, 2, 3, 4, 5, 5, 5\} = 5$
$e(2) = \max \{d(2, u) \mid u \text{ in } V(G)\} = \max\{1, 0, 2, 1, 1, 2, 3, 4, 5, 5, 5\} = 5$
$e(3) = \max \{d(3, u) \mid u \text{ in } V(G)\} = \max\{2, 2, 0, 1, 2, 2, 3, 4, 5, 5, 5\} = 5$
$e(4) = \max \{d(4, u) \mid u \text{ in } V(G)\} = \max\{1, 1, 1, 0, 1, 1, 2, 3, 4, 4, 4\} = 4$
$e(5) = \max \{d(5, u) \mid u \text{ in } V(G)\} = \max\{2, 1, 2, 1, 0, 1, 2, 3, 4, 4, 4\} = 4$
$e(6) = \max \{d(6, u) \mid u \text{ in } V(G)\} = \max\{2, 2, 2, 1, 1, 0, 1, 2, 3, 3, 3\} = 3$
$e(7) = \max \{d(7, u) \mid u \text{ in } V(G)\} = \max\{3, 3, 3, 2, 2, 1, 0, 1, 2, 2, 2\} = 3$
$e(8) = \max \{d(8, u) \mid u \text{ in } V(G)\} = \max\{4, 4, 4, 3, 3, 2, 1, 0, 1, 1, 1\} = 4$
$e(9) = \max \{d(9, u) \mid u \text{ in } V(G)\} = \max\{5, 5, 5, 4, 4, 3, 2, 1, 0, 1, 1\} = 5$
$e(10) = \max \{d(10, u) \mid u \text{ in } V(G)\} = \max\{5, 5, 5, 4, 4, 3, 2, 1, 1, 0, 2\} = 5$
$e(11) = \max \{d(11, u) \mid u \text{ in } V(G)\} = \max\{5, 5, 5, 4, 4, 3, 2, 1, 1, 2, 0\} = 5$

Radius rad G of a connected graph G is defined as rad G = min $\{e(v) \mid v \text{ in } V(G)\}$

rad G = min $\{e(1), e(2), e(3), e(4), e(5), e(6), e(7), e(8), e(9), e(10), e(11)\}$
= min {5, 5, 5, 4, 4, 3, 3, 4, 5, 5, 5}
= 3

Connected domination number and Radius of G are 3 and 3 respectively
Connected domination number of G is equal to Radius of G 3=3
We proved that Connected domination number of a graph G is equal to Radius of G.

CONCLUSION:

We proved that Efficient domination number of a graph G is less than Radius of G.
We proved that Independent domination number of a graph G is greater than Radius of G.
We proved that Connected domination number of a graph G is equal to Radius of G.
By this we concluded that Independent Domination Number of a graph G is greater than Radius of G, Efficient Domination Number of a graph G is less than Radius of G, and Connected Domination Number of a graph G is equal to Radius of G.

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