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SOLITARY TRANSMISSION OF NEURONAL SIGNALS APPROACHED VIA HE'S SEMI INVERSE METHOD

Physics

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ABSTRACT

An investigation of neuronal signal transmission is intended in this paper through the establishment of solitary wave solutions for the improved Heimburg Jackson model governing the propagation of the mechanical wave in biomembranes. The computation of soliton solutions is carried out employing He's semi inverse variational principle. The role of nonlinearity and dispersive effects in the solitonic propation is correlated to the role of compressibility, elasticity, and inertia over the neuronal signal transmission in the unilamellar DPPC vesicles at T = 45o. The study reveals that He's semi inverse method is a direct and effective algebraic method to study the experimental features of the nerve pulse in the biomembranes.

KEYWORDS

soliton, nerve signal, semi inverse method, variational principle

INTRODUCTION

Quite a large number of theoretical methods have attempted to explain the signal transmission in nerve membranes. It is the Hodgin-Huxley model, proposed way back in 1952, the pioneer in the neuronal models, which spoke of the signals to be traveling along the cell membrane in the form of action potentials essentially in an electrical manner [1]. Then arrived a few simplified versions of it namely the FitzHugh-Nagumo model [2,3], which allowed for both cancellation and penetration of pulses depending on various crucial parameters. In this pathway, there came a striking proposal by a recent model developed in 2005 by Thomas Heimburg and Andrew D. Jackson [4] where the nerve pulse was treated as electromechanical density pulses called solitons. The word 'Soliton' has become an inevitable term in the description of physical phenomena of every realm of nonlinear sciences. In soliton theory, a soliton is a self-reinforcing wave packet maintaining its shape and velocity under the prerequisite of the balance of dispersive and nonlinear effects. The improved Heimburg and Jackson model is arrived to account for the propagation of mechanical waves in biomembranes with an inherent feature of lipid melting of membranes that are found to display nonlinearity and dispersion during the melting transition. Hence as a direct consequence, this model should lead to the attainment of solitary wave solutions. Extensive experimental analysis has been carried out by Heimburg and Jackson both experimental and theoretical, of how membranes undergo phase transitions to create solitons[5,6,7].

On the theoretical front, there is a multitude of methods utilized to obtain soliton solutions to nonlinear equations, to quote a few, inverse scattering method [8], B¨acklund transformation method [9], homogeneous balance method [10], Hirota's bilinear scheme [11], pseudo-spectral method, Jacobi elliptic method, the standard truncated Painlev´e expansion [12], the tanh method [13], multilinear variable separation approach [14], F-expansion method [15], etc..

The soliton model representing the action potential of nerves is the solution of partial differential equations falling into the general class of Boussinesq approximation of water waves for solitons [16] in water canals. Studies of stability and collision of moving solitons have been carried out in biomembranes and nerves [17] through mathematical simulations under the Good -Boussinesq approximation [18]. Applying the ansatz method, 1 soliton solution has been obtained for the solitonic neuronal model, and Lie symmetry analysis is subsequently applied to obtain the conservation laws for the model [19]. After some reduction procedure made on the nonlinear evolution equation for nerve pulses, based on thermodynamic principles, new classic and non-classic traveling solutions, the bell and compacton like solutions were obtained[20].

Here in this paper, we have utilized He's semi inverse method to obtain solitary wave solutions to the improved Heimburg and Jackson model. This approach involves a variational principle algorithm and has proved itself to be a powerful mathematical tool for a variety of physical problems and it provides accurate physical insight into the nature of the solution of the problem. The paper is structured as follows. In section II the governing model is described and the physically transparent straightforward step by step procedure of the He's semi inverse method is offered to yield soliton solutions. In section III the plotted results of solitary wave solutions are discussed. In section IV, conclusions are made.

THEORETICAL MODEL GOVERNING THE SOLITARY SIGNAL TRANSMISSION IN BIOMEMBRANES

For our solitary signal transmission studies on biomembranes, the governing model chosen is the improved Heimburg Jackson model for the mechanical wave in biomembranes as

$$y_{tt} = [(c^2 + py + qy^2)y_x]_x - r_1 y_{xxxx} + r_2 y_{xxtt} \quad (1)$$

Where $y = \Delta_A$ is the longitudinal density change, r_1 and r_2 are the dispersion coefficient, p and q are the nonlinear coefficients, c is the velocity of the mechanical wave. The term $(c^2+py+ qy^2)$ leading to nonlinearity in this equation is actually related to the compressibility of the circular lipid biomembrane. The constant c is the velocity of the small-amplitude sound, p and q are nonlinear constants which are determined from experiments. The dispersive constants r_1 and r_2 are related to the elasticity of the biomembrane and inertia of the lipids in the biomembranes respectively. This equation is of the famous Boussinesq-type with nonlinearities of the amplitude-dependent kind and the fourth-order dispersive terms up to fourth-order.This equation is further reduced to a second-order ordinary differential equation through the following steps.

Here the first step is to introduce a transformation to seek a travelling wave solitary solution, y(x,t)=V() with =x +c t; upon substitution of this second order differential equation is obtained as

$$V'' - Z_1 V - Z_2 V^2 - Z_3 V^3 + a + b = 0 \quad (2)$$

Where $Z_1 = (1-C^2)/(R_1 - R_2 C^2)$, $Z_2 = P/2(R_1 - R_2C^2)$ and $Z_3 = Q/3(R_1 - R_2C^2)$

with $P = pA/c^2$; $Q = pA^2/c^2$; $R_1 = r_1/c^2l^2$ and $R_2 = r_2/l^2$; with l being the diameter of the axon and a and b the constants of integration. By He's a semi-inverse method the variational formulation is established for the second-order differential equation $V'' - Z_1 V - Z_2 V^2 - Z_3 V^3 = 0$, with constants of integration conveniently taken as a null value. According to He's a semi-inverse method, we construct the following trial-functional $J(V) = R\,L\,d\xi$, (8) where L is an unknown function of V and its derivatives. There exist alternative approaches to the construction of the trial-functionals. Thus, the variational integral obtained is of the form

A solitary wave solution is sorted out By the Ritz-like method by taking up the following form of solution.

$$V = m\,\mathrm{sech}^2(n) \quad (4)$$

where m and n are constants to be determined. Upon substituting the above form into Eq. (3),

we get the resultant J as

$J = 2m^2n^2 \operatorname{sech}^4(n)\tanh^2(n) + Z_1/2\,(m^2\operatorname{sech}^4(n)) + Z_2/3\,(m^3\operatorname{sech}^6(n)) + Z_3/4\,(m^4\operatorname{sech}^8(n))\,d$ (5)

The value of this integral is obtained using hyperbolic integration of each part

$J = 4m^2n/15 + Z_1 m^2/3\,n + 8Z_2 m^3/45\,n + 16Z_3 m^4/35$ (6)

in $J/m = 0$, and $J/n = 0$.

HENCE MAKING J STATIONARY WITH RESPECT TO M AND N RESULTS

$J/m = 8mn/15 + 2Z_1 m/3\,n + 24Z_2 m^2/45\,n + 64Z_3 m^3/35\,n = 0$

$J/n = 4m^2/15 - Z_1 m^2/3\,n^2 - 8Z_2 m^3/45\,n^2 - 16Z_3 m^4/35\,n^2 = 0$ (7)

Upon solving these two equations m and n are easily obtained as following relations

$n = (1/18\,6) \times Z_3^{-1}\,[-35Z_2^2 - 810Z_1Z_3 + 35Z_2(35b^2 - 648Z_1Z_3)^{1/2}]^{1/2}$

$m = -35b + 35(35b^2 - 648Z_1Z_3)^{1/2}/216Z_3$

The Solitary solution is thus obtained as

$V = m\operatorname{sech}^2(n) =$

$-35b + 35(35b^2 - 648Z_1Z_3)^{1/2}/216Z_3\ \operatorname{Sech}^2((1/18\,6) \times Z_3^{-1}\,[-35Z_2^2 - 810Z_1Z_3 + 35Z_2(35b^2 - 648Z_1Z_3)^{1/2}]^{1/2})$

Such a solitary wave profile obtained through He's semi inverse method of variational principle would have a constant profile with no dissipation of energy as depicted in figure1.

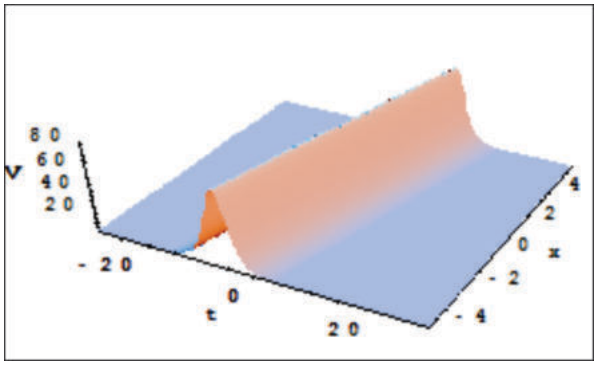


FIGURE 1. DEPICTION OF SOLITARY WAVE SOLUTION TO THE IMPROVED HEIMBURG JACKSON MODEL THROUGH HE'S VARIATIONAL METHOD.

RESULTS AND DISCUSSION

The primary building block of biological membranes is the lipid membrane, consisting of principally a large number of versatile lipids and proteins with a composition specific to the particular membrane under consideration. Such lipids of biological membranes display chain melting transitions in the proximal temperatures of physiological interest. During this transition, the heat capacity, volume and area compressibilities, and relaxation times all attain the maximal point. Compressibilities thus could be very well seen as nonlinear functions of temperature and pressure in the vicinity of the melting transition. This would eventually lead to the possibility of soliton propagation in such membranes.

Such a possibility of soliton propagation in unilamellar DPPC vesicles at T = 45° C in the solitary wave solution obtained from the He's variational approach.

The material parameters are taken from the literature [21]

i.e., c = 176.6 m/s; $\rho_A = 4.107 \cdot 10^{-3}$ g/m^2 ; $p = -16.6\,c^2/\rho_A$; $q = 79.5\,c^2/(\rho_A)^2$; & r1 = 2.25 m^4/s^2 ;r2 = 10^{-6} m^2 ; These values correspond roughly to unilamellar DPPC vesicles at T = 45°C and the value of l used for the transforming into dimensionless form is 10^{-3} m. The application of these values results in the following depiction of solitary wave propagation through nerve membranes as depicted in Fig. 2 and the corresponding Density plot in Fig. 3 also shows the solitary wave propagation in the spatiotemporal regime.

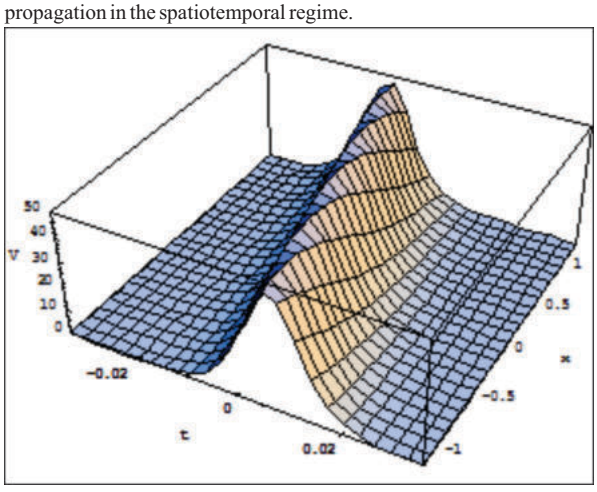
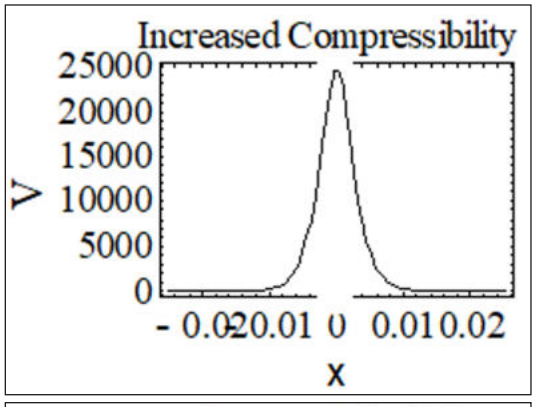


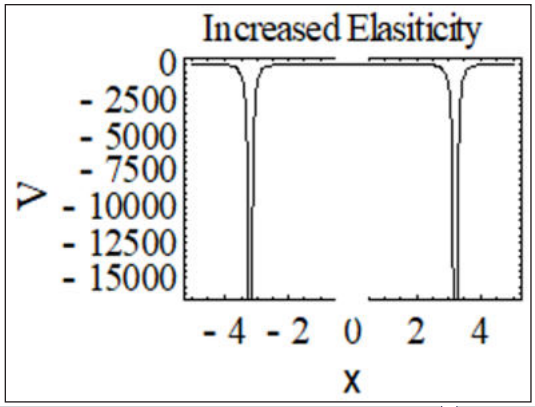
FIG. 2. THREE DIMENSIONAL PROPAGATION OF NERVE PULSE IN LIPID BIOMEMBRANE OF UNILAMELLAR DPPC VESICLES AT T = 45° C

We intend to further investigate the role of nonlinearities and dispersive terms in the propagation of solitary signal transmission. When the nonlinearity is increased via the term p or q, we expect an increase in the amplitude, and it happens at the cost of the

width as expected. While mathematically it is the nonlinearity, physiologically it is the compressibility of lipid biomembrane, and as the compressibility is increased, the neuronal solitary profile shows a steep rise in amplitude at the cost of its width. This is shown in Fig. 3(a) for an increase of compressibility of order 10^2.

As the role of dispersive effects via the terms r_1 and r_2 representing the elasticity and inertia in the biomembrane is investigated, it is seen that while increased r_1 results in the steeper increase in the amplitude in the reverse axis, increased r_2 results in a decrease of both the amplitude and width. So this result could be physiologically interpreted as while an increase in the elasticity leads to dark solitary wave profile kind of neuronal signal transmission, increased inertia decreases the amplitude and width of the solitary neuronal profile. This is depicted in Fig. 3(b) and3(c).





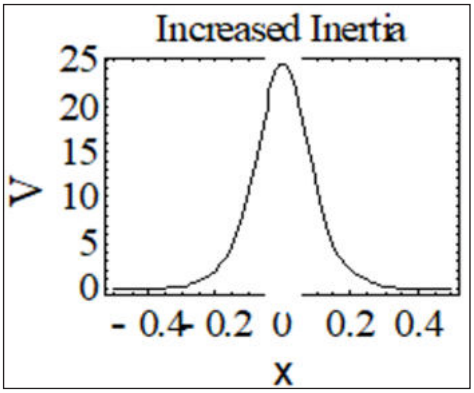


FIG. 3. ROLE OF INCREASED COMPRESSIBILITY (A), ELASTICITY(B) AND INERTIA (C) AS COMPARED TO THE ORIGINAL SOLITARY NEURONAL PROFILE.

As we tend to peek in further the spatiotemporal progression of solitary wave profile, it is noted that as nonlinearity is increased via p and q by an order 10^3, a different phenomenon peaks up in the spatial progression. This is depicted in Fig. 4 where dramatically a kind of asymmetry is introduced in the spatial progression of the solitary wave profile. Note that an increase of nonlinearity implies the increase of compressibility in the biomembrane wall made of lipids and proteins.

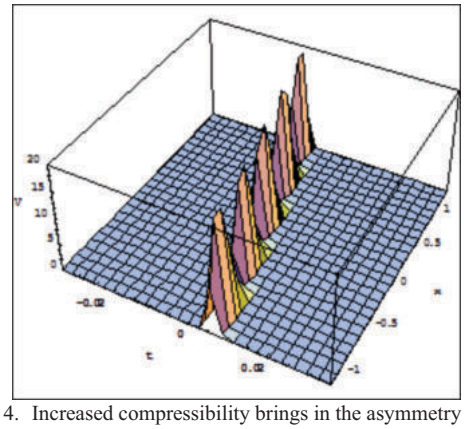


Figure 4. Increased compressibility brings in the asymmetry in three dimensional propagation of solitary nerve pulse in lipid biomembrane of unilamellar DPPC vesicles

Usually, in the nonlinear optical solitary wave studies, analysis is also carried out to study the impact of positive and negative values of dispersion. Similarly, the role of dispersive effects is analyzed for the positive and negative values of r_1 and r_2 here. The former doesn't show any crucial change in solitary behavior. Varying does not modify the shape of the solution, however, it alters the length scale of the pulse i.e. its width. Now, considering the higher-order mixed derivative dispersive term r_2, as it is changed from positive to negative values, the solitary pattern shows a distinct change to a kind of periodic wave train. Fig. 5 shows the emerging of periodic wave trains from the solitary wave profile. In general, we find that dispersion effects in biomembranes should be related to the structural characteristics of lipids like it is done for microstructured materials. Negative values of r_2 implying negative values of inertia might imply a kind of heterogeneity in the biomembrane structure.

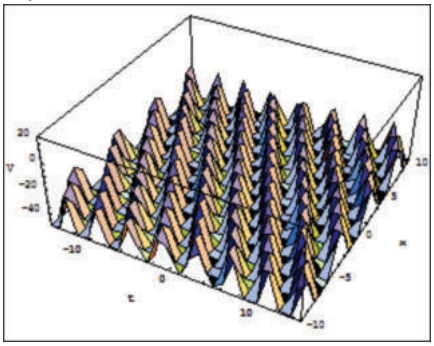


FIG. 5. PERIODIC WAVE TRAIN ARISING IN THE PRESENCE OF NEGATIVE R_2 IN THE BIOMEMBRANE OF UNILAMELLAR DPPC VESICLES

CONCLUSIONS

The improved Heimburg Jackson model describing the mechanical wave propagation was treated with He's semi inverse method where the variational formulation is established and Ritz like method is utilized to obtain solitary wave solutions. The solution obtained out of this mathematical approach is treated for examining the possibility of soliton propagation in unilamellar DPPC vesicles at T = 45° C. The role of nonlinearities and dispersive terms in the solitary wave profile is studied. This, in turn, hints the role of compressibility, elasticity, and inertia of lipid biomembrane over solitary signal transmission in neuronal biomembranes. This investigation is very crucial as it would throw light on a particularly important feature of biomembranes, the propagation e of voltage signals in the axons of neurons which allows cells to communicate quickly over long distances, an ability that is vital for higher lifeforms such as animals.

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