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A COMPARATIVE STUDY OF FUZZY AND PROBABILISTIC MEASURES OF INFORMATION

Mathematics

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ABSTRACT

Different type of measures such as measures of entropy , measures of directed divergence and measures of symmetric divergence have been defined and discussed for probability distributions in previous years. Recently these measures have also been defined for fuzzy sets in similar manner. But two sets of measures have different interpretations. A comparative study of these two sets of measures has been done in this paper.

KEYWORDS

Fuzzy, sets, probability, Measures, Entropy

INTRODUCTION :

Probabilistic measures of information for the probability distribution of the type

$P=(p_1,p_2,p_3 \;......,p_n)$ ……………………(1)

is defined where

$0\le p_i\le 1$, $i=1,2,......n$

And $\sum_{i=1}^{n} p_i=1$ …………….(2)

The probabilities are thus dependents and one can not be changed without changing one or more of the others.

Where as fuzzy measures of information are defined for fuzzy sets with fuzzy vectors of the type

$\{\mu_A(x_1),\mu_A(x_2),\mu_A(x_3),\ldots\ldots\ldots\mu_A(x_n)\}$ …………………………(3)

Where $\mu_A(x_i)$ denotes the degree of membership of the element x_i; of the set A.

If $\mu_A(x)=0, x\notin A$

If $\mu_A(x)=1$, x definitely belongs to A.

and $0\le\mu_A(x_i)\le 1$, for $i=1,2,3,\ldots,n$………………………………(4)

The sum $\sum\mu_A(x_i)$ is not the same for all fuzzy sets. The values of $\mu_A(x_i)$ are independent and any one of them can be changed without affecting the others.

While mathematical forms of two sets of measures seem to be similar, yet they are quite different.

Measures of Entropy:

Among many measures defined for probability entropy and for fuzzy entropy, we take here the simplest measures

$$E(P) = -\sum_{i=1}^{n} p_i \log p_i - \sum_{i=1}^{n}(1-p_i)\log(1-p_i) \;.............(a)$$

$$\text{And } E(A) = -\sum_{i=1}^{n} \mu_A(x_i)\log\{\mu_A(x_i)\} - \sum_{i=1}^{n}\{1-\mu_A(x_i)\}\log\{1-\mu_A(x_i)\}$$

………………………..(b)

respectively for the purpose of comparison.

(1) E(P) is maximum when $p_1=p_2=p_3=\ldots\ldots=p_n=1/n$.
E(A) is maximum when $\mu_A(x_1)=\mu_A(x_2)=\mu_A(x_3)=\cdots\ldots\ldots=\mu_A(x_n)=1/2$
(2) E (P) is minimum when one of the p_i's is unity and others are zero just as for
$D_1=(1,0,0,\ldots\ldots\ldots..,0),D_2=(0,1,0,\ldots\ldots\ldots..,0),\ldots\ldots\ldots\ldots\ldots\ldots..,$
$D_n=(0,0,0,\ldots\ldots\ldots..,1)$. and $E(P)\ge 0$

E (P) is zero for n points while E (A) is zero for 2n points in characterizing function space.

(3) The maximum value of E (P) is l/n [n log n - (l-n) log (1 -n)] The maximum value of E (A) is n log 2.
(4) E (P) and E (A) both are concave continuous permutationally symmetric functions of the arguments.
(5) E (P) is a measure of degree of equality of $p_1,p_2,p_3,\ldots\ldots\ldots.,p_n$.

It is maximum when probabilities are maximally equal and it is minimum when probabilities are maximally unequal E(A) does not measure equality of $\mu_A(x_1),\mu_A(x_2),\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

It measures the total fuzziness of the n support elements.

Measures of Directed Divergence: Among many measures for probabilistic directed divergence of distribution $P=(p_1,p_2,p_3 \;......,p_n)$ from another probability distribution $Q=(q_1,q_2,p_3 \;......,q)$ we take the simplex due to Kulback Liebler.

$$D(P,Q) = \sum_{i=1}^{n} p_i \log\frac{p_i}{q_i} + \sum_{i=1}^{n}(1-p_i)\log\frac{1-p_i}{1-q_i} \;.............(a)$$

And for fuzzy sets A and B, we take

$$D(A:B) = \sum_{i=1}^{n} \mu_A(x_i)\log\left\{\frac{\mu_A(x_i)}{\mu_B(x_i)}\right\} + \sum_{i=1}^{n}\{1-\mu_A(x_i)\}\log\left\{\frac{1-\mu_A(x_i)}{1-\mu_B(x_i)}\right\} \;.........(b)$$

(1) $D(P:Q)\ge 0$, and vanishes iff P=Q.
(2) D(P: Q) is a convex function of both P and Q. Its minimum value is zero.
(3) $D(A:B)\ge 0$, and vanishes iff A=B.
(4) D(A:B) is a convex function of both $\mu_A(x_i)$ and $\mu_B(x_i)$.

Its minimum value need not is zero.

Measures of Inaccuracy:

For probability distribution P and Q inaccuracy is defined by

$$I(P,Q) = -\sum_{i=1}^{n} p_i \log q_i - \sum_{i=1}^{n}(1-p_i)\log(1-q_i)$$

$$= -\sum_{i=1}^{n} p_i \log p_i + \sum_{i=1}^{n} p_i \log\frac{p_i}{q_i} - \sum_{i=1}^{n}(1-p_i)\log(1-p_i)$$

$$+\sum_{i=1}^{n}(1-p_i)\log\left(\frac{1-p_i}{1-q_i}\right)$$

= Measure of entropy of P+ Measure of Directed divergence of P from Q.

For two fuzzy sets A and B, the corresponding measure of inaccuracy is defined by

I(A:B)= Measure of Fuzzy entropy of A+Measure of fuzzy directed divergence of A from B.

I(A:B) is minimum when $\mu_A(x_i)=\mu_B(x_i)$ for all values of i and the minimum value in that case is the fuzzy entropy of the set A.

Relation Between Fuzzy Entropy and Fuzzy Directed Divergence:

If U is the set of which every element has the maximum fuzziness value i.e. ½, then

$$D(A:U) = \sum_{i=1}^{n} \mu_A(x_i)\log\left\{\frac{\mu_A(x_i)}{(1/2)}\right\} + \sum_{i=1}^{n}\{1-\mu_A(x_i)\}\log\left\{\frac{1-\mu_A(x_i)}{1-1/2}\right\}$$

$$= \sum_{i=1}^{n} \mu_A(x_i)\log\{\mu_A(x_i)\} + \sum_{i=1}^{n} \mu_A(x_i)\log 2 + \sum_{i=1}^{n}\{1-\mu_A(x_i)\}\log\{1-\mu_A(x_i)\}$$

$$+\sum_{i=1}^{n}\{1-\mu_A(x_i)\}\log 2$$

$$= n\log 2 + \sum_{i=1}^{n}\mu_A(x_i)\log\{\mu_A(x_i)\} + \sum_{i=1}^{n}\{1-\mu_A(x_i)\}\log\{1-\mu_A(x_i)\}$$

$$\text{i.e. } D[A:U] = n\log 2 - E(A)$$

E(A) is therefore a monotonic decreasing function of the directed divergence of A from the most fuzzy set U and that is why it is measure of fuzziness of the set A.

Maximum Fuzziness:

To find the maximum fuzziness we maximize

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$$-\sum_{i=1}^{n}\mu_A(x_i)\log\{\mu_A(x_i)\} - \sum_{i=1}^{n}\{1-\mu_A(x_i)\}\log\{1-\mu_A(x_i)\}$$

Subject to $\sum_{i=1}^{n}\mu_A(x_i) = \alpha_0$

Emax= $n \log 2$, and this arises when α_0=n/2
i.e. When each $\mu_A(x_i)$=1/2.
i.e. when the set is most fuzzy set.

CONCLUSION:

In above discussion, by quoting one measure of entropy and directed divergence we have cleared the conception of entropy measure and measure of directed divergence. In a similar fashion they can be discussed for other measures of entropy and directed divergence.

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