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FRACTIONAL DERIVATIVE MODEL ON RUMORS CAUSING STRESS WITHIN A FAMILY

Mathematics

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ABSTRACT

The act of spreading rumors within a society can have detrimental effects on individuals, including stress, hypertension, and anxiety. Given the complexity of life as a dynamic system, it is crucial to address this chaos with utmost attention. The proposed fractional derivative epidemic model for transmitting rumors in social networks highlights the importance of effective counseling and medical intervention in reducing the severity of rumors. The fractional order parameter ρ is crucial in determining the stability of individuals recovering from the stress caused by rumor attacks. This model incorporates Holling type II recovery rate to observe the effects of measures taken by the susceptible and infectious populations. Our observations indicate that a decrease in the fractional order leads to an increase in stress and a decrease in recovery, while an increase in the fractional order results in a decrease in stress and an increase in recovery. The simulation results indicate that the increase in the recovery rate can be attributed to the successful implementation of counseling and medical interventions. It is recommended to prioritize addressing lower values of ρ and take measures to identify and address individuals who spread rumors to reduce stress and anxiety within society.

KEYWORDS

rumors; fractional derivative; epidemic model; stress; simulation

INTRODUCTION

Family is an intimate and relatively everlasting domestic institution connected with the aid of blood, marriage, or adoption that lives together and shares social and financial duties and is the smallest fundamental social unit and the most vital number one institution discovered in any society. In each circle of relatives or society, we find gossipers. A gossiper is someone who talks eagerly and casually approximately different human beings. If you want to spread rumors and hear ultra-modern information about your pals, you might be a gossiper. Whilst you gossip, you talk enthusiastically about different humans' information or business. To do this regularly is to be a gossiper. Gossipers are acknowledged for his or her behaviour of chattering, filling their pals in on details of humans' lives, a few proper and others based simply on rumors or guesses [1].

Lack of communication among the family members is the strongest initial predictor of such rumor problems. It has been observed that in a joint family, the lack of communication among family members and rumors always lead to distraction. This distraction led to have a strong impact on society. Rumors play a vital role in break-up of family relations, very specifically, domestic violence between wife and husband resulting in divorce. Suicide one of the reasons of stress, depression, anxiety in family members is also one of the severe outputs of rumor problems.

Life is a complex dynamical system and rumors one of the critical parameters of this complexity leads to serious social chaos. It needs proper attention by the scientists to come out with a concrete solution and try to understand the rate of transmission of these spreads in the family and society. As rumors are highly epidemic in nature, globally, this rumor phenomenon can be expressed with the help of the theory of derivatives and integrals with fractional order. Fractional-time chaotic systems have been shown to exhibit richer dynamics and feature added degree of freedom, as in most cases the dynamics heavily depend on the fractional order [2]. As opposed to ordinary derivative, which is a local operator, the fractional order derivative has the main property called the memory effect. More precisely, the next state of fractional derivative for any given function f depends not only on their current state, but also upon all their historical states [3]. For this realistic property, the usage of fractional-order systems is becoming popular to model the behaviour of real systems. It is to be noted that the present states of any real-life dynamic system are dependent upon the history of its past states. In addition, when fitting data, the fractional model has one more degree of freedom than the integer order model. In recent years, the dynamic behaviours of fractional-order differential systems have received increasing attention.

The definition of the fractional differentiation of the Riemann Liouville type played an important role in the development of the theory of fractional derivatives and integrals and for its applications in pure mathematics (solution of integer-order differential equations, definitions of new function classes, summation of series, etc.) [4]. In this paper, our approach is through using Caputo-type fractional derivative. The main advantage of Caputo's approach is that the initial conditions for fractional differential equations with Caputo derivatives take the same form as for integer-order differential equations, i.e., contain limit values of integer-order derivatives of unknown functions at lower terminal [4].

This paper discusses the epidemic SIRS (Susceptible- Infectious- Recovered-Susceptible) model of fractional order ρ , $0 < \rho \leq 1$, using Caputo fractional order derivative, for the transmission of rumors in social networks.

MATHEMATICAL MODEL

Hypothesis

Society under study is placed in the susceptible class S which includes the smallest unit family. Rumors spreads from one member to another and then group of people spread the rumors about individual, family, or families at a rate β and make the proportion of susceptible population S in a doubtful stage. The doubt among the spouse, and other family members make them exposed towards these rumors and are placed in E class. The deadly infectious rumor which created doubt among the members after rumormonger attack get highly infected and get transferred in the compartment I. Family or family's members who have gone under depression or stress, or hypertension or anxiety may after proper counselling or medical treatment recover from this deadly rumor infection and are placed in R class. Family members may commit suicide due to stress caused by the deadly infection rumor at a constant rate α. Individuals who get recovered from this infected rumor may again be trapped or get effected by the rumors in future and thus they again become susceptible.

Nomenclature

μ : Recruitment rate of susceptibles which is constant

ρ : fractional order $(0 < \rho \leq 1)$

β: Per capita contact rate

λ: Natural mortality rate

ϵ: Rate at which recovered population goes to susceptible class.

α: Infection mortality rate (due to rumors) in infectious class

σ: Rate at which exposed population (Grand parents, spouse) goes to infectious class

δ: Rate at which infectious population (after rumormonger attack family)goes to recovered class

Model

A Caputo-fractional order SEIRS model based on our hypothesis is represented here by the following non-linear fractional differential equations:

$$
\begin{aligned}
{}_0^C D_t^{\rho} S(t) &= \mu - \beta S(t)I(t) - \lambda S(t) + \epsilon R(t)\\
{}_0^C D_t^{\rho} E(t) &= \beta S(t)I(t) - (\sigma + \lambda)E(t)\\
{}_0^C D_t^{\rho} I(t) &= \sigma E(t) - (\lambda + \alpha + \delta)I(t)\\
{}_0^C D_t^{\rho} R(t) &= \delta I(t) - (\lambda + \epsilon)R(t) \qquad (1)
\end{aligned}
$$

with initial conditions $S(0) = S_0 > 0$, $E(0) = E_0 \geq 0$ $I(0) = I_0 \geq 0$ $R(0) = R_0 \geq 0$.

${}_0^C D_t^{\rho}$ is the Caputo fractional operator of order $0 < \rho \leq 1$.

Total population is given by

$$N(t) = S(t) + E(t) + I(t) + R(t) \qquad (2)$$

Then,

$${}_0^C D_t^{\rho} N(t) = \mu - \lambda N(t) - \alpha I(t)$$

In absence of disease, $I(t) = 0$, we have,

$${}_0^C D_t^{\rho} N(t) = \mu - \lambda N(t) \qquad (3)$$

wich implies $N(t) \to \frac{\mu}{\lambda}$ $as\ t \to \infty$.

We study the dynamics of the fractional order SEIRS model in the biologically feasible set

$$\Omega = \left\{ (S, E, I, R) \in \mathbb{R}^4 \middle| N(t) \leq \frac{\mu}{\lambda} \right\}. \qquad (4)$$

Considering (3) as Initial Value Problem(IVP) with initial condition $N(t)|_{t=0} = N(0)$.

Applying Laplace transform to (3) [4], we get,

$$L\ [{}_0^C D_t^{\rho} N(t)] = L[\mu - \lambda N(t)]$$

$$or, \quad s^{\rho} L[N(t)] - s^{\rho-1} N(0) = \frac{\mu}{s} - \lambda L[N(t)]$$

$$or, \quad L[N(t)] = \frac{s^{\rho-1}}{s^{\rho} + \lambda} N(0) + \frac{\mu s^{-1}}{s^{\rho} + \lambda}$$

Applying inverse Laplace transform[4] to the above equation, we get,

$$N(t) = N(0).E_{\rho,1}(-\lambda t^{\rho}) + \mu t^{\rho} E_{\rho,\rho+1}(-\lambda t^{\rho}) \qquad (5)$$

According to the properties of Mittag-Leffler function,

$$E_{\rho,\alpha}(z) = z.E_{\rho,\rho+\alpha}(z) + \frac{1}{\Gamma(\alpha)}$$

We get from (4),

$$N(t) = \left(N(0) - \frac{\mu}{\lambda}\right) E_{\rho,1}(-\lambda t^{\rho}) + \frac{\mu}{\lambda}$$

Thus, $\lim_{t\to\infty} Sup\, N(t) \le \frac{\mu}{\lambda}$

Hence, the model is bounded above and S(t), E(t), I(t), R(t) are all non-negative and the model is non-negative invariant.

Reproduction Number:

The next generation method will be used for basic reproduction number. It is defined as the average number of secondary infections when one infective is extended into totally susceptible population.

The reproduction number for this model is found by next generation matrix method and is obtained as

$$R_0 = \frac{\beta\left(\frac{\mu}{\lambda}\right)}{(\lambda + \alpha + \delta)}$$

STABILITY ANALYSIS

Equilibrium Points

The disease free equilibrium(DFE) is found by equating all equations of system (1) to zero and

$S(0) = S_0$, $E(0) = 0$, $I(0) = 0$, $R(0) = 0$, we get,

DFE: $(S_0, 0, 0, 0) = \left(\frac{\mu}{\lambda}, 0, 0, 0\right)$

Showing no illness in the environment and all the people are susceptibles only.

Theorem: The disease free equilibrium (DFE) is locally asymptotically stable if $R_0 < 1$, otherwise not stable.

Proof: For the disease free equilibrium(DFE), $\left(\frac{\mu}{\lambda}, 0, 0, 0\right)$, the Jacobian matrix for the system (1) is given as-

$$J_{DFE} = \begin{pmatrix} -\lambda & 0 & -\beta\frac{\mu}{\lambda} & \varepsilon \\ 0 & -(\lambda+\sigma) & \beta\frac{\mu}{\lambda} & 0 \\ 0 & \sigma & -(\lambda+\alpha+\sigma) & 0 \\ 0 & 0 & \delta & -(\lambda+\varepsilon) \end{pmatrix}$$

The eigenvalues are here: $q_1 = -\lambda; q_2 = -(\lambda+\sigma);\ q_3 = -(\lambda+\alpha+\sigma);\ q_4 = -(\lambda+\varepsilon)$.

All the above eigenvalues are negative. Hence, by Fractional Routh-Hurwitz criteria [5], all the roots follow-

$$|\arg(q_i)| > \frac{\rho\pi}{2}; i = 1,2,3,4 \ \ and \ \ 0 < \rho < 1.$$

Hence, the disease free equilibrium is locally asymptotically stable when $R_0 < 1$.

Endemic Equilibrium

Endemic equilibrium (EE) is given by equating all equations of system (1) to zero and

$S(t) = S^*$, $E(t) = E^*$, $I(t) = I^*$, $R(t) = R^*$, where $S^*, E^*, I^*, R^* \in \mathbb{R}^+$.

$S^*=\frac{(\sigma+\lambda)(\lambda+\alpha+\delta)}{\beta\sigma}$

$E^* = \frac{(\lambda+\alpha+\delta)}{\sigma} I^*$

$I^* = \frac{\lambda S^*-\mu}{\left[\frac{\varepsilon\delta}{\lambda+\varepsilon}-\beta S^*\right]}$

$R^* = \frac{\delta}{\lambda+\varepsilon} I^*$

Locally asymptotically stability of Endemic equilibrium

For the endemic equilibrium, the Jacobian for the system (1) is given as

$$J = \begin{pmatrix} -\beta I^* - \lambda & 0 & -\beta S^* & \varepsilon \\ \beta I^* & -(\sigma+\lambda) & \beta S^* & 0 \\ 0 & -\sigma & -(\lambda+\alpha+\delta) & 0 \\ 0 & 0 & \delta & -(\lambda+\varepsilon) \end{pmatrix}$$

Let us suppose that $\beta I^* + \lambda = a$, $(\sigma+\lambda) = b$, $(\lambda+\alpha+\delta) = c$, $(\lambda+\varepsilon) = d$, then this gives rise to the characteristec equation

$$P(x) = x^4 + A_1x^3 + A_2x^2 + A_3x + A_4 = 0 \quad (6)$$

Where,

$A_1 = (a+b+c+d)$,

$A_2 = ab + ac + ab + ac + bd + cd$,

$A_3 = abc + acd + abd + bcd + \beta^2\sigma S^* I^*$,

$A_4 = \beta^2\sigma b S^* I^* + \beta\varepsilon\delta\sigma I^*$.

By Routh-Hurwitz criteria for fractional order system [5], let $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ are the Routh- Hurwitz determinants,

$\Delta_1 = A_1$, $\Delta_2 = \begin{vmatrix} A_1 & 1 \\ A_3 & A_2 \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} A_1 & 1 & 0 \\ A_3 & A_2 & A_2 \\ 0 & A_4 & A_3 \end{vmatrix}$,

So, endemic equilibrium is locally asymptotically stable for $D(P) < 0$, where D(P) denotes the determinant of equation (6) which is defined in [8]. $A_1 > 0, A_2 > 0, A_3 > 0, A_4 > 0$, and $A_2 = \frac{A_1A_4}{A_3} + \frac{A_3}{A_1}$, for all $\rho \in (0,1)$.

Global Stability Of Endemic Equilibrium

Let us construct a Lyapunov function as

$$L(t) = E(t) - E^* - E^* \ln\frac{E(t)}{E^*} + I(t) - I^* - I^* \ln\frac{I(t)}{I^*} \quad (7)$$

Taking derivative both sides of equation (7), we get,

${}_0^C D_t^\rho L(t) \le \left(1 - \frac{E^*}{E(t)}\right) {}_0^C D_t^\rho E(t) + \left(1 - \frac{I^*}{I(t)}\right) {}_0^C D_t^\rho I(t)$,

$or,\ {}_0^C D_t^\rho L(t) \le \left(1 - \frac{E^*}{E(t)}\right) [\beta(S(t)I(t) - S^*I^*) - (\sigma+\lambda)(E(t) - E^*)] + \left(1 - \frac{I^*}{I(t)}\right) [\sigma(E(t) -$

$$E^*) - (\lambda + \alpha + \delta)(I(t) - I^*)].$$

$$or, {}_0^cD_t^\rho L(t) \leq -\frac{(E(t)-E^*)^2}{E(t)}\left[(\sigma+\lambda) - \beta\frac{(S(t)I(t)-S^*I^*)}{(E(t)-E^*)}\right] - \frac{(I(t)-I^*)^2}{I(t)}\left[(\lambda+\alpha+\delta) - \sigma\frac{(E(t)-E^*)}{(I(t)-I^*)}\right].$$

(8)

$If\ R_0 > 1, then,\ {}_0^cD_t^\rho L(t) < 0, from$ (8). Therefore, $EE(S^*, E^*, I^*, R^*)$ is globally asymptotically stable, according to LaSalle's invariance principle[5-8].

Fractional Forward Euler Method

Consider the Caputo type differential equation,

$$\left.\begin{aligned} &{}_0^cD_t^\rho\ u(t) = f(t, u(t)), \quad m-1 < \rho < m \in \mathbb{Z}^+ \\ &u^{(j)}(0) = u_0^j,\ \ j = 1,2,3, \dots\dots\dots, (m-1). \end{aligned}\right\} \qquad (9)$$

If we apply ${}_0^cD_t^{-\rho}$ on the both sides of (8), we can obtain the following equivalent Volterra integral equation-

$$u(t) = \sum_{j=0}^{m-1}\frac{t^j}{j!}u_0^{(j)} + \frac{1}{\Gamma(\rho)}.\int_0^t (t-s)^{(\rho-1)}.f(s,u(s))ds$$

$$or, \qquad u(t) = \textstyle\sum_{j=0}^{m-1}\frac{t^j}{j!}u_0^{(j)} + {}_0^cD_t^{-\rho} f(t, u(t)) \qquad (10)$$

In the above equation (9), the approximation of $\left[{}_0^cD_t^{-\rho} f(t,u(t))\right]_{t=t_{n+1}}$ is done by Left fractional rectangular formula [9-11]

$$u_{n+1} = \textstyle\sum_{j=0}^{m-1}\frac{t_{n+1}^j}{j!}u_0^{(j)} + (\Delta t)^\rho \sum_{j=0}^{n} b_{j,n+1}\ f(t_j, u_j).$$

Where,

$$b_{j,n+1} = \frac{1}{\Gamma(\rho+1)}[(n-j+1)^\rho - (n-j)^\rho].$$

We have by fractional forward Euler method [12] for the system (1),

$$S_{n+1} = S_0 + \frac{h^\rho}{\Gamma(\rho+1)}\sum_{j=1}^{n}[(n-j+1)^\rho - (n-j)^\rho]\left[\mu - \beta\frac{S_jI_j}{1+\beta_1S_j+\beta_2I_j} - \lambda S_j + \varepsilon R_j\right]$$

$$I_{n+1} = I_0 + \frac{h^\rho}{\Gamma(\rho+1)}\sum_{j=1}^{n}[(n-j+1)^\rho - (n-j)^\rho] \times$$

$$\left[\frac{\beta S_jI_j}{1+\beta_1S_j+\beta_2I_j} - (\lambda+\alpha+\gamma)I_j - \frac{\sigma I_j}{1+\delta I_j}\right]$$

$$R_{n+1} = R_0 + \frac{h^\rho}{\Gamma(\rho+1)}\sum_{j=1}^{n}[(n-j+1)^\rho - (n-j)^\rho]\left[\frac{\sigma I_j}{1+\delta I_j} + \gamma I_j - (\lambda+\varepsilon)R_j\right]$$

NUMERICAL METHODS AND DISCUSSION

In this section, experiments have been performed to verify the analytical results obtained in section 2. Simulations have been carried out to depict local and global stability. Some examples are mentioned below for simulating the models either for $R_0 < 1$ or $R_0 > 1$.

Example 1: The local stability of the infection-free equilibrium point has been numerically simulated and depicted in Figure 1 when $\rho = 0.55$. It is observed that the infection-free equilibrium point turns out to be stable for $R_0 < 1$.

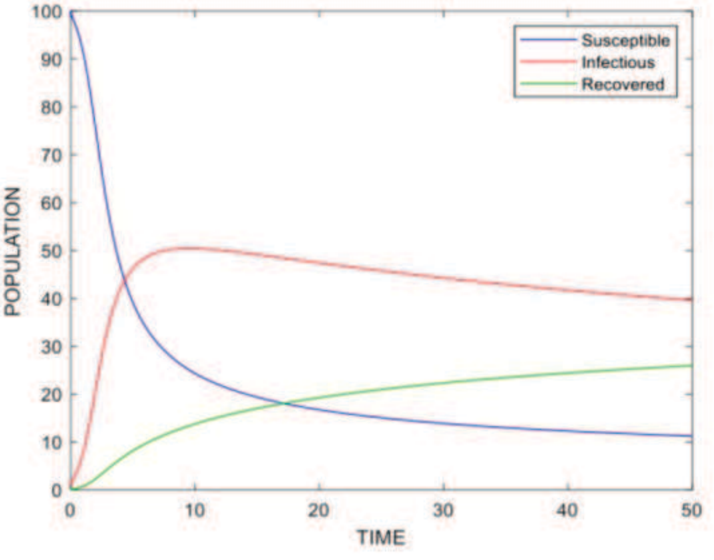

Figure 1: Dynamical behavior of populations with respect to time, $\rho = 0.55$

Example 2: For $(R_0 < 1)$, the behaviour of the system (1) is studied by considering susceptible populations versus the infectious population concerning time when $\rho = 0.55$. Figure 2 reflects that the infectious population is under control.

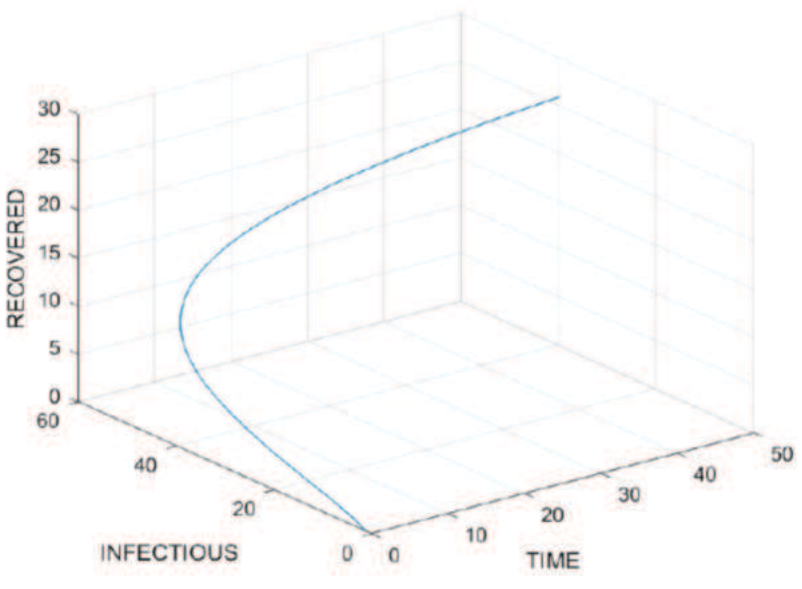

Figure 2: Time -series analysis of Infectious class versus Recovered class, when $\rho = 0.55$

Example 3: For $(R_0 < 1)$, the behaviour of the system (1) is studied by considering susceptible populations versus the infectious population concerning time when ρ = 0.95. Figure 3 reflects that the infectious population is under control with high recovery rate.

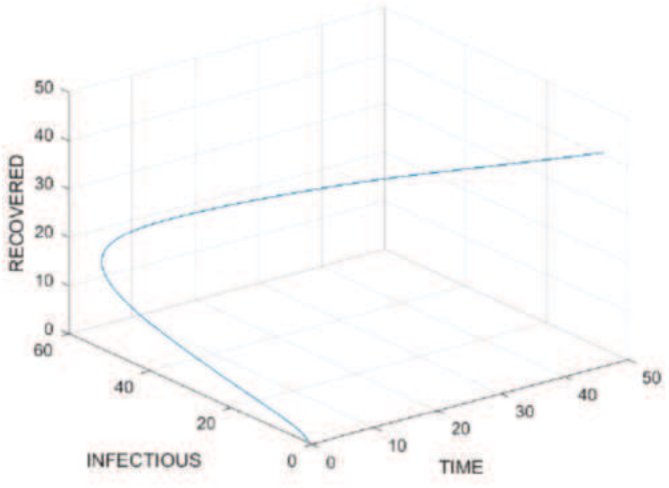

Figure 3: Time-series analysis of Infectious class versus Recovery class, when ρ = 0.95

Example 2 and 3 under different values of fractional order when simulated under parametric values depicted in figure 2 and 3 respectively clearly indicates the increasing rate of recovery which may be due to proper counselling and medical treatment. The value of ρ decides the increase or decrease of infection in the population indicating the stability, that is, recovery of persons from stress or hypertension due to rumor attack. Lower the value of ρ, high will be infectivity, and there will be minimum recovery, as shown in figure 1, and figure 2. Higher the value of ρ, low will be infectivity, and there will be maximumn recovery as shown in figure 3.

CONCLUSION

Rumors in society are highly epidemic in nature. Fractional derivative epidemic model SIRS (Susceptible- Infectious- Recovered-Susceptible) of fractional order ρ, $0 < \rho \leq 1$, using Caputo fractional order derivative, for the transmission of rumors in social networks is proposed. Analytically we have established the local and global asymptotic stability under different conditions of reproduction number for our proposed model. The Fractional Forward Euler Method is used to numerically establish the results and the severity of rumors for different values of fractional order, and its rate of recovery is clearly observed in the simulated results. The simulation outcome demonstrates that the rise in recovery rate can be attributed to effective counseling and medical intervention. The parameter ρ plays a crucial role in determining whether the infection in the population will increase or decrease, thereby indicating the stability of individuals recovering from stress or hypertension caused by a rumor attack. Adequate attention must be given to the lower value of the fractional order, and it is imperative for family members to identify individuals who are spreading rumors and causing doubts. This will undoubtedly alleviate stress, anxiety, and hypertension.

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