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ON FUZZY EPIGRAPH AND CONCAVITY OF FUZZY SETS

Mathematics

Trupti Mohite K.m.c. College, Khopoli. Tal. Khalapur, Dist. Raigad Pin Code.410203

ABSTRACT

The main purpose of this paper is to study the concavity of fuzzy sets and fuzzy Relations defined on the R and Cartesian product of the set of real numbers respectively. We use the concept of α-level sets to prove these results.

KEYWORDS

Fuzzy sets, Concave Fuzzy sets, Fuzzy Relations, Concave Fuzzy Relations, α−level sets.

INTRODUCTION

A key concept in the study of many concepts in the pure and practical sciences is convexity. Concavity is the complement of convexity, hence studying concavity of function is equally significant. Studying the features of the functions and isolating them is made simple by their convexity. The phrases fuzzy set, convex fuzzy set, fuzzy relation, and α-cut were initially used by Prof. Zadeh in 1965. Fuzzy set M is a function $M : R \rightarrow [0, 1]$, called as membership function and M(x) is called membership grade at x in M; $x \in R$; values of membership grades lie in [0,1]. Fuzzy relation K is a fuzzy set defined on the Cartesian product of the set of real numbers. B.B.Choudhuri[10] introduced the notion of a concave fuzzy set while studying different shapes of fuzzy sets.The idea was expanded upon to convex and concave fuzzy mappings by Yu-Ru Syau [11]. Concavo-convex fuzzy sets were introduced by Sarkar [12], who also established some intriguing properties of this particular kind of fuzzy set. Ban [13, 14] provided insightful commentary on convex intuitionistic fuzzy sets and created Convex temporal intuitionistic fuzzy sets. The generalized features of the aggregation of convex intuitionistic fuzzy sets were thoroughly examined and characterized by Díiaz et al. [15].

Closed and convex fuzzy sets were presented, and their relationship was studied, by Xinmin Yang [2] and Syau [5]. Nadaban and Dzitac[4] distinguished the convex fuzzy relation and included examples of specific types of fuzzy relations in their research. Chen-Wei-Xu[6] created new fuzzy relations based on earlier ones and gave convexity results for fuzzy relations. The concept of α-cut is used in [1] to provide a definition of the convexity of fuzzy sets. The fuzzy set F is concave, and its α-cut, F, is convex; ∀α∈ (0, 1). This paper will investigate its properties.We will extend the concavity of a fuzzy Set with respect to α −cut to Fuzzy Relation and try to prove that fuzzy relation T is concave then α −cut of the fuzzy relation, □ is convex; (0, 1].We will study that cut of α ∀α∈ α − a fuzzy set is an interval if F is a concave fuzzy set and study its connection with a strongly concave fuzzy set and strictly concave fuzzy set.

Preliminaries

Throughout this paper, M denotes fuzzy set defined on T denotes fuzzy relation defined on R Here are some definitions that will be useful in this paper.

Definition[6]:
A fuzzy set M defined on R is a function; $M: R \rightarrow [0, 1]$ is called as membership function and $M(x)$ is called membership grade of M at x.

Definition[7]:
A Fuzzy relation T is a fuzzy set defined on Cartesian product of crisp sets where tuples $Y1 \times Y2 \times Y3 \times \ldots\ldots \times y\,n$ (y that may have varying degrees of membership value is usually represented by a real number for $Y1 \times Y2 \times Y3$, ….. , $y\,n$) closed intervals[0,1] and indicate the strength of the present relation between elements of the topic. Consider $T: X \times Y \rightarrow [0, 1]$ then the fuzzy relation on $X \times Y$ denoted by T or $T(x, y)$ is defined as the set $T(X, Y) = \{((x, y), T(x, y))/(x, y) \in X, Y\}$. where $T(x, y)$ is the strength of the relation in two variables called membership function. It gives the degree of membership of the ordered pair (x, y) in $X \times Y$ a real number in the interval [0,1].

Definition[3]:
T be fuzzy relation on $X \times Y$. Then T is concave if and only if $T(\mu(x1, y1)) + (1-\mu)(x2, y2)) \leq \max[T(x1, y1) \wedge T(x2, y2)]$; $\forall (x1, y1), (x2, y2) \in X \times Y$ and $\mu \in [0, 1]$.

Definition[6]:
Let T be a fuzzy relation defined on $X \times Y$ and α be such that $0 < \alpha \leq 1$.Thenα −cut of T is denoted by T is defined by $T\alpha = \{(x, y) \in X \times Y / T(x, y) \leq \alpha\}$.

Definition [1]:
A be a fuzzy set defined on R and α be such that $0 < \alpha \leq 1$.Then α −level of M, is denoted by M and defined by α M is a crisp set. $\alpha = \{x \in R / M(x) \leq \alpha\}$.

Definition [1]:
M be a fuzzy set defined on R. Then M is concave if and only if $M(\mu x1 + (1-\mu)x2) \leq \max[M(x1), M(x2)]$; $\forall x1, x2 \in R$ and $\mu \in (0, 1]$.

Definition[2]:
A fuzzy set M on R is said to be strongly convex fuzzy set if $M(\mu x1 + (1-\mu)x2) < \max[M(x1), M(x2)]$ $\forall x1, x2 \in R, x1 \neq x2$ $\mu \in (0, 1)$.

2.8 Definition [2]:
A fuzzy set M on is said to be strictly concave fuzzy set if $M(\mu x1 + (1-\mu)2) < \max[M(x1), M(x2)]$; $M(x1) \neq M(x2)$, $\forall x1, x2 \in R$ and $\mu \in (0, 1)$.

III. Main Results

3.1 Theorem

M be a fuzzy set defined on R then M is concave then if for every $\alpha \in (0, 1]$, $M\alpha$ is convex.

Proof: Suppose M is a concave fuzzy set defined on R.
to prove that $M\alpha$ is convex, $\forall \alpha \in (0, 1]$.
Let, if possible, for some $\alpha \in (0, 1]$. $M\alpha$ is not convex.
That is there exist $x, y \in M$ such that,
$\mu x + (1-\mu) \notin M.\alpha$; $\forall \mu \in [0, 1]$.
Implies that, $(\mu x + (1-\mu)y) > \alpha$.
Since, $x \in M\alpha$, $M(x) \leq \alpha$.
and $y \in M\alpha$, $M(y) \leq \alpha$.
Given M is a concave fuzzy set.
Therefore $(\mu x + (1-\mu)y) < \max(M(x), M(y))$.
$< \max(\alpha, \alpha)$.
$= \alpha$.
By definition of α −level set of fuzzy set,
$\mu x + (1-\mu) \in M$.
A contradiction to our assumption that $M\alpha$ is not convex, for some $\alpha \in (0, 1]$.
Therefore, $M\alpha$ is convex, $\forall \alpha \in (0, 1]$.

3.2 Corollary:

M be a strongly (strictly) concave fuzzy set defined on R then $M\alpha+$ is convex; for all $\alpha \in (0, 1]$, where $M\alpha+$ is strong α −level set of M.

3.3 Theorem

T be a fuzzy relation defined on $R \times R$ then T is concave then for every $\alpha \in (0, 1]$; $T\alpha$ is convex.

Proof.
Suppose that T is convex fuzzy relation defined on $R \times$R.
to prove that T_α is convex.
Let if possible, for some $\alpha \in (0, 1]$. T_α is not convex.

Then $\exists(x_1, y_1), (x_2, y_2) \in T_\alpha$ such that

$\mu(x_1, y_1) + (1 - \mu)(x_2, y_2) \notin T_\alpha$; $\mu \in [0, 1]$. Implies that $(\mu(x_1, y_1) + (1 - \mu)(x_2, y_2)) > \alpha$.

Since, $(x_1, y_1) \in T_\alpha; T(x_1, y_1) \leq \alpha$. (1)

and $(x_2, y_2) \in T_\alpha$; $T(x_2, y_2) \leq \alpha$.

Since T is concave fuzzy relation.Consider,

$T(\mu(x_1, y_1) + (1-\mu)(x_2, y_2)) < \max(T(x_1, y_1), T(x_2, y_2))$.

$< mi(\alpha, \alpha)$. (by 1)

$= \alpha$.

By definition of α –level set of fuzzy relation,

$\mu(x_1, y_1) + (1-\mu)(x_2, y_2) \in T_\alpha$.

A contradiction.

T_α is convex set, for every $\alpha \in (0, 1]$.

3.4 Theorem

T be a strongly concave fuzzy relation defined on R^2 then there exist unique element (x_1, x_2) such that $T(x_1, x_2) = \min\{T(y_1, y_2)/(y_1, y_2) \in R^2\}$.

Proof.

Suppose that there are two elements (x_1, x_2) *and* (z_1, z_2) in R^2 such that

$\alpha = T(x_1, x_2) = T((z_1, z_2)) = \min\{T(y_1, y_2)/(y_1, y_2) \in R^2\}$.

$(x_1, x_2) \neq (z_1, z_2)$.

$S_\alpha = \{(x_1, x_2), (z_1, z_2)\}$

Given that T is strongly concave fuzzy relation on R^2 therefore, T_α is convex; $\alpha \in (0, 1)$.

As $(x_1, x_2) \neq (z_1, z_2)$.

We have

$\mu(x_1, y_1) + (1-\mu)(x_2, y_2) \notin T_\alpha$.

as, T_α *contains only two elements* ; for all $\alpha \in (0, 1)$ and $\mu \in (0, 1)$.

Therefore, T_α is not convex. A contradiction.

$\therefore (x_1, x_2) = (z_1, z_2)$.

Therefore, there exist unique element (x_1, x_2) $T(x_1, x_2) = \min\{T(y_1, y_2)/(y_1, y_2) \in R^2\}$.

Conversely, part of the above theorem is not true in general; we may take non concave fuzzy set and find a unique minimum element.

3.5 Theorem

T is concave fuzzy relation defined on R^2 then for every $\alpha \in (0, 1]$, T_α is closed and connected.

Proof.

First, we prove that T_α is a closed set.

Let (x, y) be any limit point of T_α then there exists sequence $(x_n, y_n) \in T_\alpha$.

$(x_n, y_n) \to (x, y) \forall n \geq N$.

As $(x_n, y_n) \in T_\alpha$, we have $T(x_n, y_n) < \alpha$.therefore $\lim_{n\to\infty} T(x_n, y_n) < \alpha$.

(x, y) is limit point of T_α ; (x, y) is either an interior point or boundary point of T_α.

Case 1:(x, y) is an interior point of T_α then clearly $T(x, y) \leq \alpha$.

Case 2:(x, y) is boundary point of T_α. We can find $\in > 0$ such that $(x-\in, x+\in)$ is an interior point of T_α

Given that T is concave fuzzy relation on R^2.

Then by definition of concavity of fuzzy relation, $T_\alpha$is not convex, for all $\alpha \in (0, 1]$.

For, (x_n, y_n), $(x-\in, x+\in) \in T_\alpha, \mu(x_n, y_n) + (1-\mu)(x-\in, x+\in) \in T_\alpha$

$\lim \mu(x_n, y_n) + (1-\mu)(x-\in, x+\in) \in T_\alpha$.

$\lim_{n\to\infty} \mu(x_n, y_n) + \lim (1-\mu)(x-\in, x+\in) \in T_\alpha$.

$\mu(x, y) + (1-\mu)(x-\in, y-\in) \in T_\alpha$

for $\mu = 1, (x, y) \in T_\alpha$

Therefore $M(x, y) \leq \alpha$.

Therefore T_α is closed.

Now to prove that T_α *is* connected.

Let, if possible, it is not connected. Then there exist two non-empty, disjoint, open sets A and B such that

$T_\alpha = A \cup B$ and $A \cap B = \emptyset$.

We can choose $(x, y) \in A$ *and* $(x', y') \in B$.

Then $\mu(x, y) + (1-\mu)(x', y') \notin T_\alpha$.

T_α is not a concave set. Contradiction. Therefore T_α is connected.

Converse of the above theorem is not true in general.

$$T(x, y) = 1 \; if \; (x, y) \in \{(x, y) \in R^2 \; / x^2 + y^2 \geq 4 \; and \; x^2 + y^2 \leq 9\}.$$

$$= \frac{1}{2} \; if \; (x, y) \in \{R^2 / x^2 + y^2 < 1 \; and \; x^2 + y^2 > 4\}.$$

Then $T_1 = (x, y) \in \{(x, y) \in \frac{R^2}{} x^2 + y^2 \geq 4 \; and \; x^2 + y^2 \leq 9\}$.

which is closed and connected but not convex.

IV. CONCLUSION

We demonstrated the relationship between concave fuzzy set (concave fuzzy relation) and fuzzy epigraph fuzzy set. α −level set works as a bridge between crisp sets and fuzzy sets. Because concavity has tremendous applications in various fields, it's crucial to investigate it using a fuzzy method at several levels. We have used it to look into the concavity of fuzzy sets and concave fuzzy relations.

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