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EXPLORING CELESTIAL MECHANICS USING TRADITIONAL METHODS AND NEURAL NETWORK

Computer Science

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ABSTRACT

We have studied the rise of celestial mechanics and its integral part in astronomy. The primary objective of this paper is to compare between the traditional way of solving Circular Restricted Three Body problem using Newton's laws with Kepler's foundational laws and Physics Informed Neural Network (PINN). In the future PINN can be used to predict, determine and analyze the stability of orbits.

KEYWORDS

Celestial mechanics, Kepler's laws, Physics Informed Neural Network, Circular Restricted Three Body Problem

INTRODUCTION

Celestial mechanics deals with the application of classical mechanics to motion of celestial bodies influenced by several types of forces. Celestial Bodies mostly experience their mutual gravitational attraction which is the most important force as far as we are concerned. But other external forces can also affect the system, such as atmospheric drag on artificial satellites, and electromagnetic forces if they are electrically charged and moving in a magnetic field.

The term celestial mechanics is mainly used to refer to the analysis developed for the motion of point mass particles moving under their mutual gravitational attractions, such as the orbital motions of solar system bodies. The celestial mechanics of artificial satellite motion is called astrodynamics.

Dynamic astronomy is a very broad term, which, in addition to celestial mechanics and astrodynamics, includes all aspects of celestial body motion (e.g., rotation, mass and mass distribution determinations for stars and galaxies, and so forth).

Celestial mechanics emerged in the dawn of the 16^{th} century with works of Nicolaus Copernicus (1473 - 1543), Johannes Kepler (1572 - 1630), Galileo Galilei (1564 - 1642) and Isaac Newton (1642 - 1727). In the following centuries, many scientists added important contributions to celestial mechanics, such as Leonhard Euler (1707 - 1783), Pierre Laplace (1749 - 1827), Henri Poincaré (1854 - 1912), and many more. The cumulative work of these scientists expanded the field of celestial mechanics more than ever before.

Until 1957 - the launch of sputnik, objects of study in celestial mechanics were only natural celestial bodies - large and small planets of the solar system, their satellites, comets and meteors, stars and star systems etc. Now, we must include - artificial satellites, space stations and space shuttles in the family of celestial bodies. Therefore, we must distinguish natural and artificial bodies.

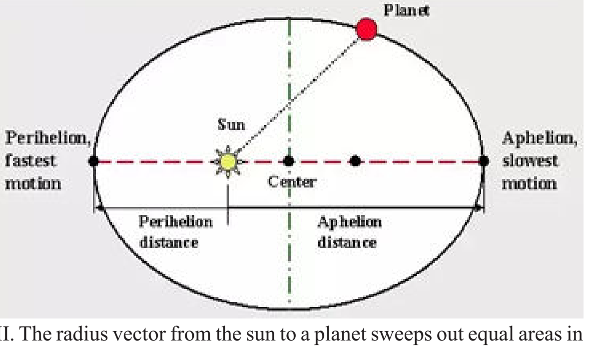
A "natural celestial body" refers to an object in space that occurs naturally, like a planet, star, moon, or asteroid, while an "artificial celestial body" is a man-made object orbiting a celestial body, such as a satellite launched by humans.

Mathematically, celestial mechanics is concerned with the solution of Newton's equations. From Newton's time until 20^{th} century, solutions were obtained only with immense labor and by means of genius mathematicians. The digital computer and the dawn of the space program profoundly affected celestial mechanics. The James web telescope is right now in space finding exoplanets and new planetary systems, and thereby providing new challenges for celestial mechanics.

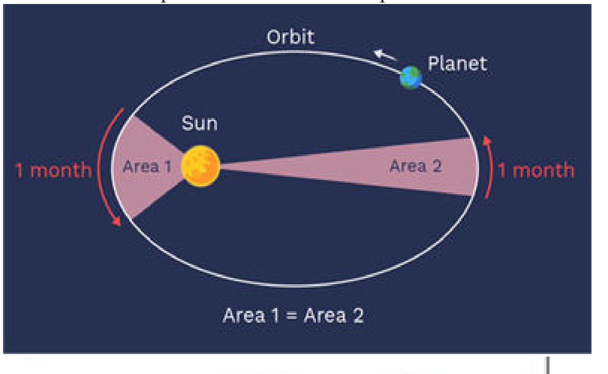
Johannes Kepler discovered laws of planetary motion around 1600, and the explanation of these laws was given by Isaac Newton around 1685. It was an astonishing leap in the understanding of the universe.

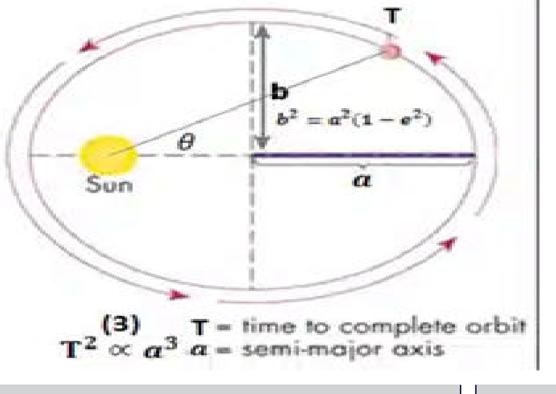
Kepler's Laws of motion

I. Each planet's orbit about the Sun is an ellipse. The Sun's center is always located at one focus of the orbital ellipse. The planet traces the ellipse in its orbit, meaning that the planet to Sun distance is constantly changing as the planet goes around its orbit.



II. The radius vector from the sun to a planet sweeps out equal areas in equal time.
Actually, the speed of planets varies and they do not move with constant speed along their orbits. The point of nearest approach of the planet to the Sun is called perihelion. The point of greatest separation is called aphelion. Hence by Kepler's Second Law, a planet is moving fastest when it is at perihelion and slowest at aphelion.





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III. The period of revolution T of a planet about the sun is related to the semi-major axis a of the ellipse by $T^2=ka^3$ where k is the same for all planets. The squares of the orbital periods of the planets are directly proportional to the cubes of the semi-major axes of their orbits. Kepler's Third Law implies that the period for a planet to orbit the Sun increases rapidly with the radius of its orbit. Thus, we find that Mercury, the innermost planet, takes only 88 days to orbit the Sun. The earth takes 365 days, while Saturn requires 10,759 days. Though Kepler didn't know about gravitation when he came up with his three laws, they were foundational for Isaac Newton deriving his theory of universal gravitation, which explains the unknown force behind Kepler's Third Law. Kepler and his theories were crucial in the better understanding of dynamics of our solar system and acted as a starting point for newer theories that can more accurately approximate our planetary orbits.

The third law was published in 1619, and efforts to discover and solve the equation of motion of the planets generated four hundred years of mathematical and scientific discovery. In his honor, this problem has been named the Kepler Problem.

The Inverse Square Law

Newton computed the acceleration of a planet moving according to Kepler's first and second laws.

The magnitude of the acceleration is inversely proportional to the square of the planet's distance from the Sun.

Newton's Law of Gravitation

Newton presented the first formulation of gravitational force, which was published in his book entitled Phylosophiae Naturalis Principia Mathematica.

$$F=\frac{Gm_1m_2}{r^2} \qquad (1)$$

Where F is the force of gravity, is the mass of one object, is the mass of second object

R is the distance between the objects, and $G = 6.672\times10^{-11}\ Nm^2/kg^2$ is a constant called Newton's Universal Gravitational Constant.

For example: The Motion of the Star SO-2 around the Black Hole at the Galactic Center

The UCLA Galactic Center Group, headed by Dr. Andrea Ghez, measured the orbits of many stars within 0.8″ × 0.8″ of the galactic center. The orbits of six of those stars are shown in Figure 4.

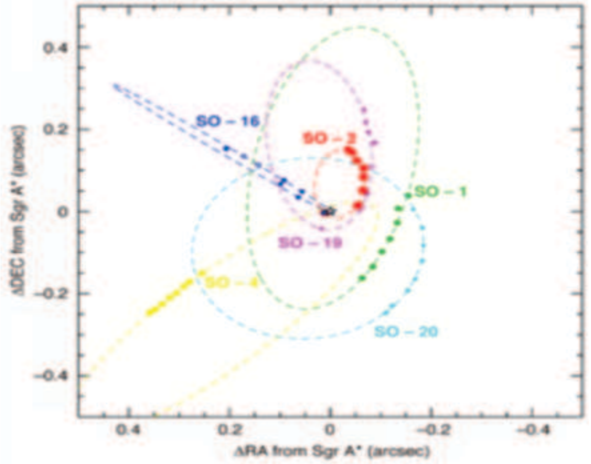

Figure 4. Orbits Of Six Stars Near Black Hole At Center Of Milky Way Galaxy.

Table 1. Orbital Properties of S0-2

Star	Period (yrs)	Eccentricity	Semi-major axis (10^{-3} arc sec)	Periapse (au)	Apoapse (au)
S0-2	15.2 (0.68/0.76)	0.8763 (0.0063)	120.7 (4.5)	119.5 (3.9)	1812 (73)

Circular Restricted Three Body Problem (CR3BP) Dynamics

Here, we start the analysis of formulation of satellite motion under the gravitational influence of the Earth and Moon within the cislunar volume using Circular Restricted Three-Body Problem (CR3BP) dynamics.

Under CR3BP dynamics, the spacecraft's motion is due to gravitational influence of the Earth and Moon that are approximated as point masses. In order to simplify equations, the spacecraft's state is expressed within a non-inertial or rotating coordinate system that rotates with angular velocity of magnitude equal to the rate of rotation of the primary masses moving in circular orbits about their bary-center. The time-scales, distances, and masses of the three bodies are adimensionalized to further simplify the equations of motion. This results in the following equations for the non-maneuvering acceleration of the satellite within the Earth-Moon rotating frame as a function of its non-inertial, adimensional position (r = [x, y, z]).

$$\ddot{x} = -\frac{1-\mu}{\rho_1^3}(x+\mu) - \frac{\mu}{\rho_2^3}(x-1+\mu) + 2\dot{x} + \dot{x}, \qquad (2)$$

$$\ddot{y} = -\frac{1-\mu}{\rho_1^3}y - \frac{\mu}{\rho_2^3}y - 2\dot{x} + \dot{y}, \qquad (3)$$

$$\ddot{z} = -\frac{1-\mu}{\rho_1^3}z - \frac{\mu}{\rho_2^3}z, \qquad (4)$$

where the mass ratio is used to adimensionalize the *SI* masses of the Earth(m_1) and Moon (m_2) ; ρ_1 is the distance of the satellite from the Earth at fixed position (R_1) such that $\rho_1=|r-R_1|$; and ρ_2 is the distance of the satellite from the Moon at fixed position (R_2) such that $\rho_2=|r-R_2|$.

Neural Networks

A neural network is a machine learning program, or model, that makes decisions in a manner similar to the human brain, by using processes that mimic the way biological neurons work together to identify phenomena, weigh options and arrive at conclusions.

Neural networks can be applied to celestial mechanics, they have advantages such as identifying intricate patterns, dealing with highly non-linear dynamics, allowing for more accurate predictions of celestial bodies, especially in scenarios with multiple interacting bodies or chaotic systems, where traditional methods might struggle.

Key applications of neural networks in celestial mechanics include Modelling complex gravitational fields, Neural networks can be trained on observational data to represent the irregular gravitational fields of celestial bodies, particularly asteroids and comets, which can have complex shapes and mass distributions, leading to more precise trajectory predictions. By learning from observational data, neural networks can be used to improve the accuracy of orbit determination for satellites and spacecraft, especially in scenarios with limited data or complex perturbations.

Physics-Informed Neural Network (PINN)

Physics-Informed Neural Network (PINN) are a new class of neural networks. PINNs are neural networks which explicitly incorporate known differential constraints into their cost function. During training, the networks are forced to learn solutions which obey the underlying physics of the system.

Recently it has been shown that PINNs can be applied to orbit determination problems. The output of neural network is the position of the cislunar space object (u = r = [x, y, z]) and the neural network input is the normalized time of the observation. No spatial information is input to the neural network such that the output is solely a function of input time.

The observation determination problem's goal is to estimate the satellite's position within the CR3BP frame using a line of sight from the telescope's position to the satellite. The line of sight vector is denoted by right ascension (α) and declination (δ).

$$\alpha = \arctan\left(\frac{y-y_s}{x-x_s}\right);\ \delta = \arctan\left(\frac{z-z_s}{\sqrt{(x-x_s)^2+(y-y_s)^2+(z-z_s)^2}}\right) \qquad (5)$$

Where denotes the known Earth-fixed location of the telescope. The measurement line of sight vector is taken to be I = [cos α, sin α, sin δ]

Actually, the position of the satellite is never directly observed, only the line of sight along which the satellite lies is observed. Due to this abstraction, there is no direct observability of the satellite's position

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over the time of observation. The loss function for the measurements, MSEu, then takes on the form

$$MSE_u = \frac{1}{N_u}\sum_{i_u=1}^{N_u} \left|los(\hat{r}(t_{i_u}); r_{si_u}) - l_{iu}\right|^2$$

Where los is the line of sight transformation function of that accepts the predicted satellite position by the deep learning model (^r) and the known telescope location (r_{siu}) at timestep, t_{iu} and l_{iu} is the measured line of sight at timestep t_{iu}.

Now to express the dynamical loss, we utilize the CR3BP dynamics that were introduced in equations (2), (3) and (4). Specifically, we use automatic differentiation to approximate the velocity $(\partial\hat{r}/\partial t)$ and acceleration $(\partial^2\hat{r}/\partial t^2)$ from the satellite position that is predicted by the PINN. This results in the following loss terms that must be minimized to satisfy the laws of motion within the three-body frame.

$$L_{d_x} = \frac{1}{N_f}\sum_{i}^{N_f}\left(\frac{\partial^2\hat{x}}{\partial t^2} - 2\frac{\partial\hat{y}}{\partial t} + \hat{x} + \left(\frac{1-\mu}{\rho_1^3}\right)(\hat{x}+\mu) + \left(\frac{\mu}{\rho_2^3}\right)(\hat{x}-1+\mu)\right)^2$$

$$L_{d_y} = \frac{1}{N_f}\sum_{i}^{N_f}\left(\frac{\partial^2\hat{y}}{\partial t^2} + 2\frac{\partial\hat{x}}{\partial t} - \hat{y} + \hat{y}\left(\frac{1-\mu}{\rho_1^3} + \frac{\mu}{\rho_2^3}\right)\right)^2$$

$$L_{d_z} = \frac{1}{N_f}\sum_{i}^{N_f}\left(\frac{\partial^2\hat{z}}{\partial t^2} + \hat{z}\left(\frac{1-\mu}{\rho_1^3} + \frac{\mu}{\rho_2^3}\right)\right)^2$$

These are the loss terms of position (L_{tp}) and velocity(L_{tv}),

$$L_{t_p} = \frac{1}{N_f-1}\sum_{i}^{N_f-1}\left(\hat{x}_{i-1} + \Delta t\left.\frac{\partial\hat{x}}{\partial t}\right|_{i-1} + \frac{1}{2}\Delta t^2\left.\frac{\partial^2\hat{x}}{\partial t^2}\right|_{i-1} - \hat{x}_i\right)^2 + \left(\hat{y}_{i-1} + \Delta t\left.\frac{\partial\hat{y}}{\partial t}\right|_{i-1} + \frac{1}{2}\Delta t^2\left.\frac{\partial^2\hat{y}}{\partial t^2}\right|_{i-1} - \hat{y}_i\right)^2 + \left(\hat{z}_{i-1} + \Delta t\left.\frac{\partial\hat{z}}{\partial t}\right|_{i-1} + \frac{1}{2}\Delta t^2\left.\frac{\partial^2\hat{z}}{\partial t^2}\right|_{i-1} - \hat{z}_i\right)^2$$

$$L_{t_v} = \frac{1}{N_f-1}\sum_{i}^{N_f-1}\left(\left.\frac{\partial\hat{x}}{\partial t}\right|_{i} + \Delta t\left.\frac{\partial^2\hat{x}}{\partial t^2}\right|_{i} - \left.\frac{\partial\hat{x}}{\partial t}\right|_{i+1}\right)^2 + \left(\left.\frac{\partial\hat{y}}{\partial t}\right|_{i} + \Delta t\left.\frac{\partial^2\hat{y}}{\partial t^2}\right|_{i} - \left.\frac{\partial\hat{y}}{\partial t}\right|_{i+1}\right)^2 + \left(\left.\frac{\partial\hat{z}}{\partial t}\right|_{i} + \Delta t\left.\frac{\partial^2\hat{z}}{\partial t^2}\right|_{i} - \left.\frac{\partial\hat{z}}{\partial t}\right|_{i+1}\right)^2$$

The final loss term is given by,

$$MSE_f = L_{d_x} + L_{d_y} + L_{d_z} + L_{t_p} + L_{t_v}$$

RESULTS AND DISCUSSION

Kepler's laws alone cannot solve the Circular Restricted Three-Body Problem (CR3BP) because Kepler's laws only apply to the two-body problem, while the CR3BP involves three bodies, making it significantly more complex and with no general closed-form solution. In this problem, the small body's motion is significantly influenced by the gravitational perturbations from the two larger bodies, making it difficult to find analytical solutions.

Physics Informed Neural Network (PINN) can be used to solve the Circular Restricted 3-Body Problem (CR3BP), as it can effectively approximate solutions to the complex differential equations governing the motion of a test particle under the gravitational influence of two larger bodies in a circular orbit, even when analytical solutions are not readily available;
PINNs can be trained to learn these equations and predict the trajectory of the test particle.

PINN can handle the nonlinearity and complexity of the CR3BP, which can be challenging for traditional numerical methods and with sufficient training data PINN can provide highly accurate solutions to the CR3BP. PINN can be used to explore a wide range of initial conditions and parameter variations within the CR3BP.

CONCLUSION

We conclude that PINN can produce fast and accurate solutions to the computationally challenging circular restricted three-body problem. Despite the simplifications, the particle trajectories regularly undergo a series of complex interactions and the PINN captures this behavior, matching predictions at a fraction of the computational time cost.

PINN can be used to discover new periodic orbits in the CR3BP, which are important for understanding the long-term dynamics of spacecraft in the Earth-Moon system. By predicting the evolution of trajectories over time, PINN can be used to assess the stability of various locations within the CR3BP. PINN can help in designing optimal trajectories for spacecraft, such as finding fuel-efficient paths around Lagrange points. Due to the chaotic nature of the CR3BP, PINNs may sometimes get trapped in unstable regions of the solution space during training, requiring careful design of the loss function and regularization techniques. Selecting appropriate initial conditions for training is crucial to ensure the PINN converges to a physically meaningful solution.

We are encouraged that other problems of similar complexity can be solved more effectively by replacing traditional methods with machine learning algorithms trained on the underlying physical processes. In the near future, PINN may be trained on more complex systems such as the 4 and 5-body problems, reducing the computational load more than ever before.

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