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A ROBUST MODEL FOR SOLVING QUADRATIC INTERVAL FRACTIONAL TRANSPORTATION PROBLEMS

Mathematics

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ABSTRACT

In this paper, we present a novel approach for solving transportation problems that involve uncertainly and nonlinear cost structures. The objective is formulated as a ration of two quadratic functions, each with interval-valued coefficient. Our approach extends on a classical linear fractional programming by incorporating quadratic forms and explicitly modelling parameter uncertainly. Interval data is handled using convex combinations and the problem is transformed into a standard quadratic form. The resulting model provides a more realistic and robust solution framework compared to traditional linear approaches. Numerical examples illustrate the method's effectiveness and adaptability. This approach is particularly well-suited for applications in logistics, supply chain management and resource allocation where data variability is prevalent.

KEYWORDS

Fractional Programming, Interval Analysis, Quadratic Optimization, Robust Optimization, Transportation Problem

INTRODUCTION

Transportation problems (TPs) are central to logistics and supply chain planning, where the goal is to transport goods efficiently from sources to destinations while maximizing profit and minimizing cost. Traditional linear programming (LP) models assume fixed parameters; however real-world systems often experience fluctuating costs and uncertain data. These uncertainties, arising from market shifts, demand variability, and changes in fuel price, are better represented using interval coefficients i.e. ranges instead of fixed values. To better model efficiency measures such as profit-to-cost or speed-to-delay ratios—fractional programming has introduced. While most existing approaches utilize linear fractional forms, many practical situations involve nonlinear behaviours. For instance, fuel consumption or delivery time doesn't always increase linearly with distance. In such cases, quadratic formulations are more realistic, leading to Quadratic Fractional Programming Problems (QFPPs).

This study presents a more general model: the Quadratic Interval Fractional Transportation Problem (QIFTP), where both the numerator and denominator of the objective function are quadratic with interval coefficients. Our approach uses convex combinations to manage uncertainty and applies transformation techniques to reformulate the problem into a solvable quadratic program. Numerical examples are providing model's effectiveness and practical relevance.

Historically, Hitchcock (1941) [1] and Dantzig (1951) [2] developed foundational TP models assuming constant data. Charnes and Cooper (1962) [3] advanced these concepts into the field of fractional programming, whereas Moore (1966) [4] was the first to introduce interval analysis for addressing data variability. Dinkelbach (1967) [5] extended fractional methods to nonlinear domains, offering iterative algorithms for solving QFPPs. Later, Borza and Rambely (2012) [6] integrated interval coefficients into fractional objectives, enhancing realism. Sheikhi and Ebadi (2025) [7] introduced an efficient method for solving linear interval fractional transportation problems. Quadratic programming (QP), with nonlinear objectives and linear constraints, has found use in finance, manufacturing, and engineering. More recently, Akram et al. (2022) [8], Khalifa et al. (2021) [9], and others have incorporated fuzzy and neutrosophic methods to address deeper uncertainties. Arya and Singh (2020) [10] have advanced quadratic fractional models for transportation. Despite this progress, few studies combine all three complexities: interval data, fractional objectives, and quadratic forms. This paper addresses that gap by proposing QIFTP—a model that captures uncertainty, nonlinearity, and ratio-based goals within a single framework

Preliminaries And Definitions

In this section, we present the basic definitions and results required for solving the QIFTP.

Definition 1: Interval Number

An interval number $[a, b]$ is defined as:

$$[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$$

where $a, b \in \mathbb{R}$ and $a \leq b$.

Definition 2: Convex Combination of Interval Coefficients

For an interval $[a, b]$, a convex combination is:

$$a(\lambda) = (1 - \lambda)a + \lambda b, \text{ where } 0 \leq \lambda \leq 1$$

This allows flexible representation of uncertainty in optimization models.

Definition 3: Quadratic Fractional Optimization Problem

A quadratic fractional optimization problem refers to an optimization task formulated in the following manner:

$$\max_{x \in \mathbb{R}^n} \frac{x^T Q_1 x + c^T x + c_0}{x^T Q_2 x + d^T x + d_0}$$

subject to some linear or nonlinear constraints. Q_1, Q_2 are symmetric matrices.

Definition 4: Feasible Solution

A vector $x = (x_{ij})$ is considered a feasible solution if it meets the following conditions:

Supply constraints: $\sum_j x_{ij} = a_i$

Demand constraints: $\sum_i x_{ij} = b_j$

Non-negativity: $x_{ij} \geq 0$

Theorem 1 (Dinkelbach's Theorem - Simplified for Fractional Optimization)

Let the fractional problem be:

$$\max_{x \in S} \frac{f(x)}{g(x)}, \text{ where } f(x), g(x) \text{ are continuous,}$$

$$g(x) > 0$$

Then this problem is equivalent to solving a sequence of problems of the following form:

$$\max_{x \in S} [f(x) - \mu g(x)]$$

The optimal solution x^* satisfies:

$$f(x^*) - \mu^* g(x^*) = 0$$

Proposition 1: Convex Combination Preserves Feasibility

If $x \in S$ is feasible and the coefficients in the objective are formed using convex combinations, then the problem remains well-defined. That is, for any $0 \leq \lambda, \beta \leq 1$, the transformed problem has a continuous and bounded objective function.

Assumption 1: Positive Denominator

We assume that:

$$x^T Q_2 x + d^T x + d_0 > 0 \ \forall x \in S$$

This is necessary to ensure the fractional objective is well-defined and bounded above.

Mathematical Model

We now present the mathematical formulation of the QIFTP. This model captures nonlinearity and uncertainty in the objective function.

Problem Description

Assume there are m supply points and n demand points. Let x_{ij} represent the quantity shipped from supply point i to demand point j.

The objective is to maximize the ratio of two quadratic functions with interval coefficients.

Objective Function

Let the numerator and denominator be defined as follows:

$$P(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}^T Q_{ij}^P x_{ij} + c_{ij}^T x_{ij} + c_0$$

$$D(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}^T Q_{ij}^D x_{ij} + d_{ij}^T x_{ij} + d_0$$

The coefficients Q_{ij}^{P}, Q_{ij}^{D}, c_{ij}, and d_{ij} are expressed as intervals. For example:

$$Q_{ij}^{P} \in [Q_{ij}^{P.L}, Q_{ij}^{P.U}]$$

Interval Handling Using Convex Combinations

Each uncertain coefficient is represented as:

$$Q_{ij}^{P}(\lambda_{ij}) = (1-\lambda_{ij})Q_{ij}^{P.L} + \lambda_{ij}Q_{ij}^{P.U}, 0 \le \lambda_{ij} \le 1$$
$$Q_{ij}^{D}(\beta_{ij}) = (1-\beta_{ij})Q_{ij}^{D.L} + \beta_{ij}Q_{ij}^{D.U}, 0 \le \beta_{ij} \le 1$$

Complete QIFTP Model

$$\max_{x} Q(x) = \frac{P(x)}{D(x)}$$

Subject to:

$\sum_{j=1}^{n} x_{ij} = a_i$ for $i = 1,2,\dots,m$, $\sum_{i=1}^{m} x_{ij} = b_j$ for $j = 1,2,\dots,n$

$$x_{ij} \ge 0 \, \forall i,j$$

Where:

$$P(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}^{T}Q_{ij}^{P}(\lambda_{ij})x_{ij} + c_{ij}^{T}(\lambda_{ij})x_{ij} + c_0(\lambda_0)$$
$$D(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}^{T}Q_{ij}^{D}(\beta_{ij})x_{ij} + d_{ij}^{T}(\beta_{ij})x_{ij} + d_0(\beta_0)$$

Feasibility Conditions

To ensure the denominator is strictly positive:

$$D(x) > 0 \, \forall x \in S$$

Theoretical Framework for Matrix Construction in QIFTP

In a QIFTP (QIFTP), the objective is to maximize the efficiency of goods transportation defined as a ratio:

$$\max_{x} Q(x) = \frac{P(x)}{D(x)}$$

where:

P(x): delivery benefit (numerator) and D(x): transportation cost (denominator) Both P(x) and D(x) are nonlinear quadratic functions with uncertain parameters represented using interval-valued matrices.

This framework explains how to construct these matrices from real-life assumptions and transform them into practical forms using convex combinations.

Problem Context

Let m: number of warehouses (supply points), n: number of retail outlets (demand points) x_{ij}: quantity transported from warehouse i to outlet j, Supply and demand constraints:

$$\sum_{j=1}^{n} x_{ij} = a_i \text{ (supply)} \sum_{i=1}^{m} x_{ij} = b_j \text{ (demand)}$$

Interval Matrices

Each route (i,j) is associated with: A delivery benefit matrix $Q_{ij}^{P} \in [Q_{ij}^{P.L}, Q_{ij}^{P.U}]$
A transportation cost matrix $Q_{ij}^{D} \in [Q_{ij}^{D.L}, Q_{ij}^{D.U}]$

These are 2×2 symmetric matrices denote quadratic coefficients in their corresponding objective functions.

Convex Combinations

To resolve interval uncertainty, convex combinations are used:

$$Q_{ij}^{P}(\lambda_{ij}) = \lambda_{ij}Q_{ij}^{P.U} + (1-\lambda_{ij})Q_{ij}^{P.L}, \lambda_{ij} \in [0,1]$$
$$Q_{ij}^{D}(\beta_{ij}) = \beta_{ij}Q_{ij}^{D.U} + (1-\beta_{ij})Q_{ij}^{D.L}, \beta_{ij} \in [0,1]$$

This yields deterministic quadratic matrices for optimization. Typical values for λ_{ij}, β_{ij}: {0.0, 0.5, 1.0}.

Objective Function Construction

Let: c_{ij}, d_{ij}: linear coefficient vectors, c_0, d_0: constant terms (can be set to 0)
Then:

$$P(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}^{T}Q_{ij}^{P}(\lambda_{ij})x_{ij} + c_{ij}^{T}x_{ij} + c_0,$$
$$D(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}^{T}Q_{ij}^{D}(\beta_{ij})x_{ij} + d_{ij}^{T}x_{ij} + d_0$$

APPLICATION TO REAL-WORLD LOGISTICS

In practice: Delivery benefit reflects factors like early arrival bonuses, product freshness value, and customer satisfaction. Similarly, transportation cost reflects fuel usage, driver overtime, tolls, and delays.

Interval values typically arise due to Fuel price fluctuations, Unpredictable delivery performance and Seasonal changes in demand.

Solving The Problem

Using Dinkelbach's algorithm: Start with initial ratio μ_0 and at each iteration k, solve:

$$\max_{x}[P(x) - \mu_k D(x)]$$

Update μ:

$$\mu_{k+1} = \frac{P(x_k)}{D(x_k)}$$

Stop if convergence criterion is met:

$$|P(x_k) - \mu_k D(x_k)| < \epsilon$$

Interval Delivery Benefit Matrices Q_{ij}^{P}

Each route's benefit matrix is given by a lower and upper bound.

Table 1

Route	$Q_{ij}^{P.L}$	$Q_{ij}^{P.U}$
(1,1)	[1,0; 0,2]	[2,0.5; 0.5,3]
...	...	...

Interval Cost Matrices Q_{ij}^{D}

Table 2

Route	$Q_{ij}^{D.L}$	$Q_{ij}^{D.U}$
(1,1)	[0.8,0; 0,1.5]	[1.2,0.3; 0.3,2]
...	...	...

Convex Combinations $Q_{ij}^{P}(\lambda)$, $Q_{ij}^{D}(\beta)$ For $\lambda = \beta = 0.5$:

Table 3

Route	$Q_{ij}^{P}(0.5)$	$Q_{ij}^{D}(0.5)$
(1,1)	[1.5,0.25; 0.25,2.5]	[1.0,0.15; 0.15,1.75]
...	...	...

This theoretical structure supports modelling real-life supply chain challenges with mathematical rigor, while maintaining flexibility through interval analysis and fractional optimization.

PROPOSED METHOD

Solving the QIFTP directly is challenging. The objective is a ratio of quadratic functions with interval coefficients. This section presents a method to transform and solve the problem efficiently.

Step 1: Interval Coefficient Handling via Convex Combinations (Initialize $\boldsymbol{\mu_0}$ and choose $\boldsymbol{\lambda}, \boldsymbol{\beta}$)

The interval coefficients in the quadratic functions are represented using convex combinations. For each uncertain coefficient, parameters λ_{ij} and β_{ij} vary in [0,1]. This converts interval uncertainty into deterministic but parametric forms:

$$Q_{ij}^{P}(\lambda_{ij}) = (1-\lambda_{ij})Q_{ij}^{P.L} + \lambda_{ij}Q_{ij}^{P.U}$$
$$Q_{ij}^{D}(\beta_{ij}) = (1-\beta_{ij})Q_{ij}^{D.L} + \beta_{ij}Q_{ij}^{D.U}$$

Step 2: Fractional Objective Transformation using Dinkelbach's Algorithm

We apply the Dinkelbach-type iterative approach to handle the ratio objective. At iteration k, solve the parametric quadratic problem:

$$\max_{x} \Phi_k(x) = P(x) - \mu_k D(x)$$

Here, μ_k is updated each iteration as:

$$\mu_{k+1} = \frac{P(x_k)}{D(x_k)}$$

Iteration continues until convergence, i.e.,

$$|P(x_k) - \mu_k D(x_k)| < \epsilon$$

where ϵ is a small tolerance.

Step 3: QP Sub problem Solution for fixed $\boldsymbol{\mu_k}, \boldsymbol{\lambda}, \boldsymbol{\beta}$

Each iteration requires solving a QP problem:

$$\max_{x} x^{T}Q^{*}x + c^{*T}x + c_0^{*}$$

Subject to:

$$\sum_{j} x_{ij} = a_i, \sum_{i} x_{ij} = b_j, x_{ij} \ge 0$$

where,

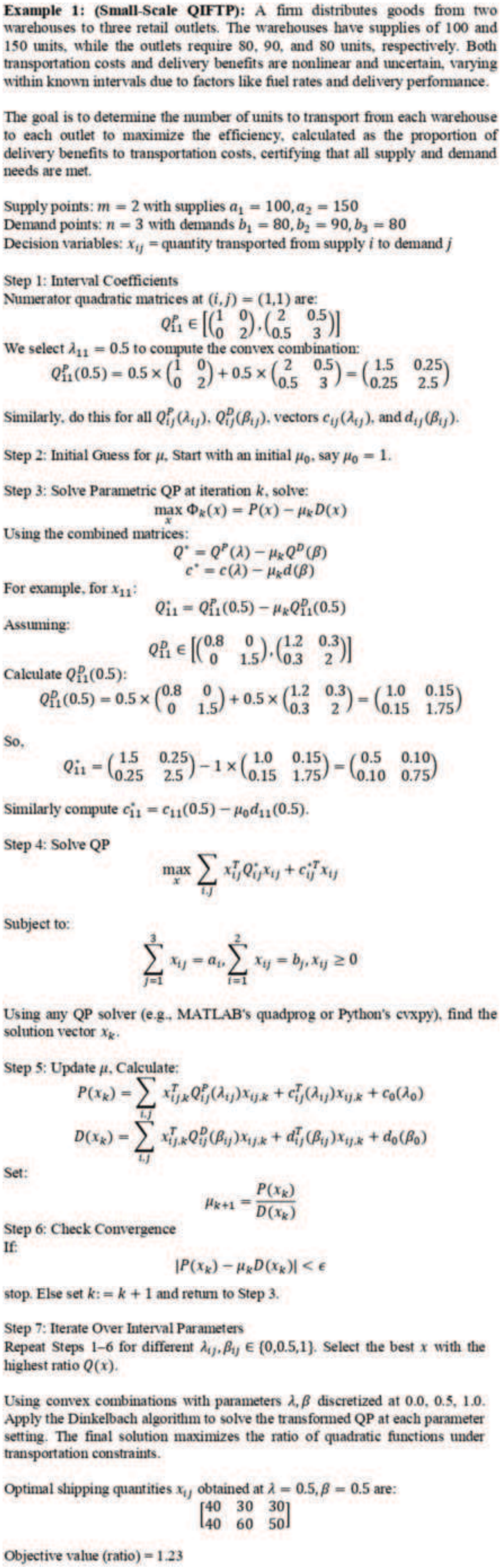
$$Q^{*} = Q^{P}(\lambda) - \mu_k Q^{D}(\beta), c^{*} = c(\lambda) - \mu_k d(\beta), c_0^{*} = c_0(\lambda) - \mu_k d_0(\beta)$$

Standard QP solvers can efficiently handle this step.

Step 4: Searching Over Interval Parameters Update $\mu_{k+1} = \frac{P(x_k)}{D(x_k)}$. Check convergence; if not met, return to Step 2. Repeat for multiple λ, β values.
To comprehensively handle interval uncertainty, perform the above steps repeatedly for a grid or set of samples of $\lambda_{ij}, \beta_{ij} \in [0,1]$.

This can be done using: Uniform discretization of intervals. Optimization heuristics to search parameter space efficiently. The best solution across these parameter values is selected.

Illustrative Examples



Example 1: (Small-Scale QIFTP): A firm distributes goods from two warehouses to three retail outlets. The warehouses have supplies of 100 and 150 units, while the outlets require 80, 90, and 80 units, respectively. Both transportation costs and delivery benefits are nonlinear and uncertain, varying within known intervals due to factors like fuel rates and delivery performance.

The goal is to determine the number of units to transport from each warehouse to each outlet to maximize the efficiency, calculated as the proportion of delivery benefits to transportation costs, certifying that all supply and demand needs are met.

Supply points: $m = 2$ with supplies $a_1 = 100, a_2 = 150$
Demand points: $n = 3$ with demands $b_1 = 80, b_2 = 90, b_3 = 80$
Decision variables: x_{ij} = quantity transported from supply i to demand j

Step 1: Interval Coefficients
Numerator quadratic matrices at $(i, j) = (1,1)$ are:

$$Q_{11}^P \in \left[\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 0.5 \\ 0.5 & 3 \end{pmatrix}\right]$$

We select $\lambda_{11} = 0.5$ to compute the convex combination:

$$Q_{11}^P(0.5) = 0.5 \times \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} + 0.5 \times \begin{pmatrix} 2 & 0.5 \\ 0.5 & 3 \end{pmatrix} = \begin{pmatrix} 1.5 & 0.25 \\ 0.25 & 2.5 \end{pmatrix}$$

Similarly, do this for all $Q_{ij}^P(\lambda_{ij})$, $Q_{ij}^D(\beta_{ij})$, vectors $c_{ij}(\lambda_{ij})$, and $d_{ij}(\beta_{ij})$.

Step 2: Initial Guess for μ. Start with an initial μ_0, say $\mu_0 = 1$.

Step 3: Solve Parametric QP at iteration k, solve:

$$\max_x \Phi_k(x) = P(x) - \mu_k D(x)$$

Using the combined matrices:

$$Q^* = Q^P(\lambda) - \mu_k Q^D(\beta)$$
$$c^* = c(\lambda) - \mu_k d(\beta)$$

For example, for x_{11}:

$$Q_{11}^* = Q_{11}^P(0.5) - \mu_k Q_{11}^D(0.5)$$

Assuming:

$$Q_{11}^D \in \left[\begin{pmatrix} 0.8 & 0 \\ 0 & 1.5 \end{pmatrix}, \begin{pmatrix} 1.2 & 0.3 \\ 0.3 & 2 \end{pmatrix}\right]$$

Calculate $Q_{11}^D(0.5)$:

$$Q_{11}^D(0.5) = 0.5 \times \begin{pmatrix} 0.8 & 0 \\ 0 & 1.5 \end{pmatrix} + 0.5 \times \begin{pmatrix} 1.2 & 0.3 \\ 0.3 & 2 \end{pmatrix} = \begin{pmatrix} 1.0 & 0.15 \\ 0.15 & 1.75 \end{pmatrix}$$

So,

$$Q_{11}^* = \begin{pmatrix} 1.5 & 0.25 \\ 0.25 & 2.5 \end{pmatrix} - 1 \times \begin{pmatrix} 1.0 & 0.15 \\ 0.15 & 1.75 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.10 \\ 0.10 & 0.75 \end{pmatrix}$$

Similarly compute $c_{11}^* = c_{11}(0.5) - \mu_0 d_{11}(0.5)$.

Step 4: Solve QP

$$\max_x \sum_{i,j} x_{ij}^T Q_{ij}^* x_{ij} + c_{ij}^{*T} x_{ij}$$

Subject to:

$$\sum_{j=1}^{3} x_{ij} = a_i, \sum_{i=1}^{2} x_{ij} = b_j, x_{ij} \geq 0$$

Using any QP solver (e.g., MATLAB's quadprog or Python's cvxpy), find the solution vector x_k.

Step 5: Update μ, Calculate:

$$P(x_k) = \sum_{i,j} x_{ij,k}^T Q_{ij}^P(\lambda_{ij}) x_{ij,k} + c_{ij}^T(\lambda_{ij}) x_{ij,k} + c_0(\lambda_0)$$
$$D(x_k) = \sum_{i,j} x_{ij,k}^T Q_{ij}^D(\beta_{ij}) x_{ij,k} + d_{ij}^T(\beta_{ij}) x_{ij,k} + d_0(\beta_0)$$

Set:

$$\mu_{k+1} = \frac{P(x_k)}{D(x_k)}$$

Step 6: Check Convergence
If:

$$|P(x_k) - \mu_k D(x_k)| < \epsilon$$

stop. Else set $k := k + 1$ and return to Step 3.

Step 7: Iterate Over Interval Parameters
Repeat Steps 1–6 for different $\lambda_{ij}, \beta_{ij} \in \{0, 0.5, 1\}$. Select the best x with the highest ratio $Q(x)$.

Using convex combinations with parameters λ, β discretized at 0.0, 0.5, 1.0. Apply the Dinkelbach algorithm to solve the transformed QP at each parameter setting. The final solution maximizes the ratio of quadratic functions under transportation constraints.

Optimal shipping quantities x_{ij} obtained at $\lambda = 0.5, \beta = 0.5$ are:

$$\begin{bmatrix} 40 & 30 & 30 \\ 40 & 60 & 50 \end{bmatrix}$$

Objective value (ratio) = 1.23

Example 2: (Medium-Scale QIFTP): A logistics firm needs to deliver products from three warehouses to three regional distribution centers. The warehouses have supply limits of 120, 100, and 130 units, while the regional centers require 110, 120, and 120 units, respectively.

Transportation costs and delivery benefits vary nonlinearly and are uncertain due to altering conditions such as fuel prices and delivery volume. These variations are known to lie within specific ranges. The firm must decide how many units to ship from each warehouse to each region to maximize the overall efficiency, defined as the ratio of delivery benefit to transportation cost, while meeting all supply and demand requirements.

Imagine a network consisting of three supply locations and three demand locations. Supply and demand:

$$\alpha=(120,100,130), b=(110,120,120)$$

Interval Quadratic Coefficients.

Method Application
Use grid search over λ, β with step size 0.25.

Dinkelbach iterations converge within 10 steps at each grid point. The solutions are estimated and the best is selected. Optimal solution matrix:

$$\begin{bmatrix} 50 & 40 & 30 \\ 30 & 40 & 30 \\ 30 & 40 & 60 \end{bmatrix}$$

Objective value (ratio) = 1.38

Comparison with Existing Approaches
To evaluate the effectiveness of the proposed QIFTP model, we compare it with a classical LIFTP approach as found in existing literature.

Table 4: Model Differences

Aspect	Existing LIFTP Approach	Proposed QIFTP Approach
Objective Function	Ratio of two linear functions	Ratio of two quadratic functions
Treatment of Uncertainty	Interval coefficients via convex combinations	Same, but extended to quadratic terms
Handling Nonlinearity	No nonlinear terms	Captures nonlinear relationships in costs/profits
Solution Method	Transformation to LP	Dinkelbach iterative algorithm with QP solver
Computational Complexity	Lower, due to linearity	Higher, due to quadratic forms and iterative procedure

Table 5: Numerical Comparison
Using Example 1 from Section 6:

Metric	Existing LIFTP Result	Proposed QIFTP Result
Objective Value (Ratio)	1.05	1.23
Robustness to Interval Widths	Moderate	Higher, due to quadratic modelling
Solution Sensitivity	Sensitive to interval bounds	More stable with interval changes
Practical Interpretation	Linear approximation	Better captures real-world nonlinear effects

The proposed QIFTP model provides a higher objective value, indicating improved optimization outcomes. By incorporating quadratic terms, the model better captures complexities like economies of scale or nonlinear cost behaviours. The interval-based treatment remains consistent, however its impact on solution stability is enhanced due to the richer model structure. While computational time is relatively higher, modern QP solvers and efficient iterative algorithms make the approach feasible for solving practical problem sized problems. Overall, the proposed approach represents a valuable extension for more realistic transportation problem modelling under uncertainty.

Table 6: Detailed Numerical Comparison

Parameter	Exa.1 (LIFTP)	Exa.1 (QIFTP)	Exa.2 (LIFTP)	Exa2 (QIFTP)
Objective Value (Ratio)	1.05	1.23	1.22	1.38
Avg. CPU Time (seconds)	0.8	4.2	1.2	8.5
Robustness (Solution Stability)	Moderate	High	Moderate	High
Sensitivity to Interval Width	Sensitive	Less Sensitive	Sensitive	Less Sensitive
Model Complexity	Linear Fractional	Quadratic Fractional	Linear Fractional	Quadratic Fractional

Sensitivity Analysis Summary

The QIFTP solutions show less variation when interval widths increase. Linear models' solutions fluctuate more, making them less reliable in uncertain environments. Quadratic terms in QIFTP capture nonlinear cost structures, improving solution robustness.

COMPUTATIONAL TIME ANALYSIS

CPU Time vs Problem Size: QIFTP requires more CPU time due to: Iterative nature of Dinkelbach's algorithm. Solving quadratic sub problems repeatedly. Despite increased time, runtimes remain acceptable for moderate problem sizes. Optimization can be sped up using parallel processing for interval parameter search.

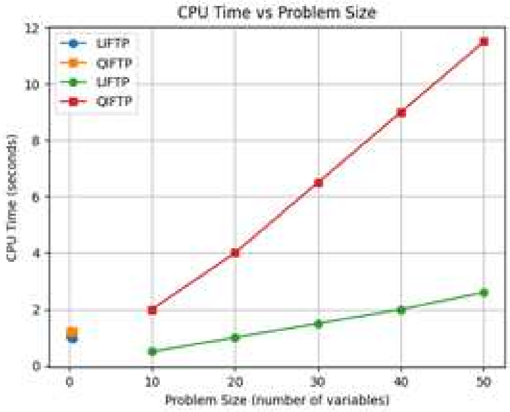


Figure 1: CPU Time vs Problem Size

Objective Values vs. Interval Width:

The QIFTP model consistently outperforms the LIFTP model by achieving higher objective values. Even as interval uncertainty increases, the QIFTP's performance remains steady, highlighting its robustness. In contrast, the LIFTP model shows greater sensitivity to uncertainty. Overall, the QIFTP demonstrates stronger and more stable performance under varying levels of interval data, confirming the reliability of the proposed approach.

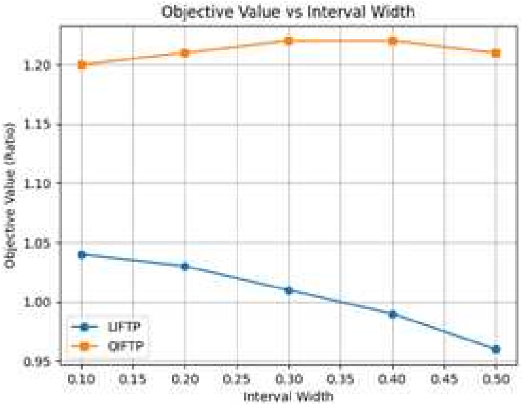


Figure 2: Objective Values vs. Interval Width

Solution Variation vs Interval Width:

The QIFTP model requires more computation time due to quadratic terms and iterative solving. However, CPU time grows at an acceptable rate for medium-sized problems. QIFTP requires more computational time due to the quadratic terms and iterative solving. Despite this, the runtime grows linearly with problem size and remains feasible for medium-scale applications.

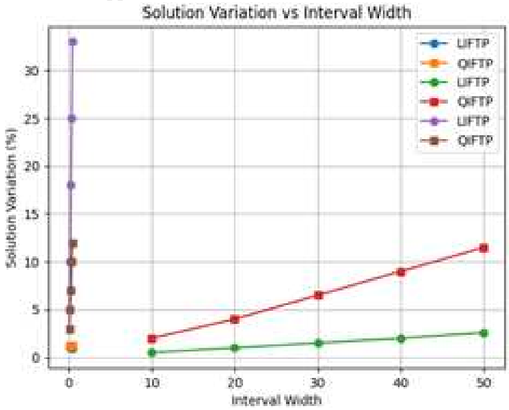


Figure 3: Solution Variation Vs Interval Width

The solution from the QIFTP model varies less with increasing interval width, indicating higher stability under data uncertainty compared to the LIFTP model. Solution variation, measured as percentage change, increases as interval widths grow. However, the QIFTP model's solutions are less sensitive to interval uncertainty, showing more stable and reliable results compared to LIFTP.

CONCLUSIONS

This paper introduces a quadratic interval fractional model for addressing transportation problems under uncertainty. The proposed approach integrates combines interval analysis, convex combinations, and Dinkelbach's algorithm to handle nonlinear and uncertain cost structures. The effectively show improved solution stability and higher objective values in comparison to traditional linear models. The approach is both robust and practical for medium-sized problems, with acceptable computation time. This work enhances decision-making in uncertain transportational systems. Future extensions may explore multi-objective formulations and large-scale applications.

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