ORIGINAL RESEARCH PAPER

Volume - 15 | Issue - 08 | August - 2026 | PRINT ISSN No. 2277 - 8179 | DOI : 10.36106/ijsr

INTERNATIONAL JOURNAL OF SCIENTIFIC RESEARCH

AN OPTIMAL REPLACEMENT POLICY FOR A TWO-UNIT COLD STANDBY REPAIRABLE SYSTEM USING MONOTONE PROCESSES EXPOSING TO EXPONENTIAL FAILURE LAW

Statistics

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ABSTRACT

This paper investigates an optimal replacement policy N for cold standby repairable system composed of two identical components maintained by a single repairman. The system assumes that repaired components are not restored to an "as good as new" condition. Consequently, the successive working times follow a decreasing -series process, while the successive repair times follow an increasing geometric process. Both processes are governed by an exponential failure distribution. Under these assumptions, we derive an explicit formula for the long-run average cost per unit time associated with the replacement policy. By analysing this cost function, the optimal replacement number N^*is obtained, ensuring minimal expected operational cost over an infinite time horizon. Numerical examples are presented to illustrate the influence of various model parameters on system performance and to validate the applicability of the derived results. Graphical representations are also provided wherever necessary to offer a clearer understanding of the behaviour of the cost function and the impact of monotone process characteristics.

KEYWORDS

Renewal Process, Cold Standby System, Optimal Replacement Policy, Monotone Processes, Exponential Failure Distribution, Long-run Average Cost, Maintenance Optimization.

INTRODUCTION

Most repairable systems assume that the system after repair is "as good as new".This leads to a perfect repair model. But it is not always true for deteriorating systems due to ageing and accumulated wear. On these lines, Barlow and Hunter [1] investigated a minimal repair model in which the minimal repair does not change the age of the system. Brown and Proschan [2,3] studied an imperfect repair model under which the repair will be perfect repair with probability'p' and with probability '(1-p)' as a minimal repair. It is reasonable to assume that the successive working times of the deteriorating systems after repair will become shorter and shorter, while the consecutive repair time of the system will become longer and longer. Finally, neither it works nor repaired any more.

Furthermore, to model such a deteriorating repairable system Lam [5,6] proposed a geometric process repair model in which he studied two kinds of replacement policies, one based on the working age T of the system and the other based on the failure number N of the system. He provided an explicit expression for the long run average cost per unit time under these two kinds of policies and also proved optimal policy N* is better than the optimal policy T*. Stadje and Zuckerman [8] presented a general monotone process to generalize Lam's work. Further much research work has been carried out by using geometric process to generalize Lam's work and corresponding optimal replacement policies were developed by Wang and Zhang [10,11]. Zhang at.al [12] determined an optimal replacement policy for a deteriorating production system with preventive maintenance by generalizing Lam's [5,6] works. Many optimal replacement policies were also developed for cold standby repairable systems using geometric processes.

Zhang et.al [13] considered a cold standby repairable system consisting of two identical components and one repairman. He developed two kinds of repair replacement policies namely **replacement policy T**, under which the system is replaced when working age of **component 1** reaches T and **replacement policy N** under which the system is replaced when the failure number of **component 1** reaches N. He derived an explicit expression for long-run average cost per unit time of the system under these two kinds of policies. However the geometric process is more useful model for deteriorating system, Braun et .al [4] introduced an alternative model, the α-series process, which contributes these characteristics. Furthermore, Braun et al [4] explained the increasing geometric process grows at most logarithmically, while the decreasing geometric process is almost certain to have a time of explosion. The -series process grows either as a polynomial or exponential in time. Further it is also noted that the geometric process does not satisfy a central limit theorem, while the -series process does.

Furthermore, Braun et.al [4] also presented that both the increasing geometric process and the -series process have a finite first moment under certain general conditions. However the decreasing geometric process usually has an infinite first moment under certain conditions. Thus, the decreasing -series process may be more appropriate for modeling system working times while the increasing geometric process is more suitable for modeling repair times of the system.

Based on this understanding, the present paper investigates a cold standby repairable system consisting of two identical components namely **component 1, component 2** and with one repairman. Assume that each component after repair is not 'as good as new' and also the successive working times form a decreasing -series process, the successive repair time's form an increasing geometric process and both the processes are governed by exponential failure law. Under these assumptions, an optimal replacement policy N is considered under which it replaces the system when the number of failures of component 1 reaches N. it can be determined an optimal repair replacement policy N* such that the long run average cost per unit time is minimized. It can also be derived an explicit expression of the long-run average cost and the corresponding optimal replacement policy N* can be determined analytically. Numerical results are provided to support the theoretical results and the results obtained are represented through graphs at appropriate places.

In modeling these deteriorating systems, the definitions according to Lam [6] are considered.

THE MODEL

This section deals with a development of a model for two component cold standby repairable system with one repairman using geometric process and exposing to exponential failure law in such a way that the long-run average loss is minimized with the following assumptions.

ASSUMPTIONS

1) At the beginning, both the components are new and component 1 is in working state while the other is in cold standby state.
2) The two components appear alternatively in the system. i.e., when the working component fails immediately the standby component begins to work and the failed one is repaired by the repairman. Whenever the repair of the failed one is completed, it becomes cold standby. If one fails and the other is still under repair, it must wait for repair and the system breaks down.
3) A component in the system is replaced some time by an identical one and the replacement time is negligible.
4) The components after repair are not as good as new. The time interval between the completion of the (n-1)th repair and the completion of the nth repair on component i is called nth cycle of component i for i=1, 2 and n=1,2,… .
5) A component in the system can't produce the working reward during cold standby and no cost is incurred during the waiting period for repair.
6) Let $X_n^{(i)}$ *and* $Y_n^{(i)}$, for i=1, 2 and n=1, 2 …are all S-independent.
7) Let $X_n^{(i)}$ be working time which follows decreasing α −series processes exposing

7) Let $X_n^{(i)}$ be working time which follows decreasing α −series processes exposing to exponential failure law and $Y_n^{(i)}$ be the repair time which follows an increasing geometric processes exposing to exponential failure law of component i in the n^{th} cycle , for i=1, 2 and n=1, 2, …. .

8) Let $E\left(X_1^{(i)}\right)=\lambda$ and $E\left(Y_1^{(i)}\right)=\mu$, for i=1, 2.

9) Let $F(k^{\alpha} x)=F_n(x)$ and $G(b^{n-1} y)=G_n(y)$ be the distribution functions of $X_n^{(i)}$ and $Y_n^{(i)}$ respectively for i=1,2 and n=1,2,3,… where α>0 and 0<b<1.

10) Let the repair cost rate of each component is C_r, the working reward per unit time of each component is C_W and the replacement cost of the system is C.

THE LONG- RUN AVERAGE COST RATE UNDER POLICY N

This section dwells upon an optimal replacement policy N based on the number of failures of component 1. As two components appears alternatively in the system, component 2 may be in the repair state of the (N-1)th cycle or in the cold standby state in the Nth cycle. Naturally, a reasonable replacement policy N should be that component 1 can not be repaired any more when the number of its failures reaches N and component 2 works until failure in the Nth cycle.

According to the renewal reward theorem Ross [7], the long-run average cost rate of the system under policy N is:

$$C(N)=\frac{\text{The expected cost incurred in a renewal cycle}}{\text{The expected length of renewal cycle}}, \tag{3.1}$$

Where the length of a system in a renewal cycle under policy N is:

$$L=\sum_{k=1}^{N} X_k^{(1)}+\sum_{k=1}^{N-1} Y_k^{(1)}+\sum_{k=2}^{N}(Y_{k-1}^{(2)}-X_k^{(1)}) I_{\{Y_{k-1}^{(2)}-X_k^{(1)}>0\}}+\sum_{k=1}^{N-1}(X_k^{(2)}-Y_k^{(1)}) I_{\{X_k^{(2)}-Y_k^{(1)}>0\}}+X_N^{(2)}, \tag{3.2}$$

where the first, second, third, fourth and the fifth terms refers to working age, repair time, waiting for repair time, cold standby time of component 1 and working age of component 2 in the Nth cycle respectively.

The expected length of a renewal cycle is:

$$\begin{aligned}E(L)=&\sum_{k=1}^{N} E\left(X_k^{(1)}\right)+\sum_{k=1}^{N} E\left(Y_k^{(1)}\right)+\sum_{k=2}^{N} E\left[(Y_{k-1}^{(2)}-X_k^{(1)}) I_{\{Y_{k-1}^{(2)}-X_k^{(1)}>0\}}\right] \\ &+\sum_{k=1}^{N-1} E\left[(X_k^{(2)}-Y_k^{(1)}) I_{\{X_k^{(2)}-Y_k^{(1)}>0\}}\right]+E(X_N^{(2)})\end{aligned} \tag{3.3}$$

Where I is the indicator function, such that

$$I_A=\begin{cases}1 & \text{if event A occurs} \\ 0 & \text{if event A does't occurs.}\end{cases}$$

According to the assumptions of the model, definition of probability density function, convolution and Jacobian transformations, the probability density function of $Y_{k-1}^{(2)}-X_k^{(1)}$ and $X_k^{(2)}-Y_k^{(1)}$ are respectively.

$$g(u)=\int_0^{\infty} f(v, u+v)\, dv, \tag{3.4}$$

Where $X_k^{(1)}=v,\ Y_{k-1}^{(2)}=u+v$, such that $u=Y_{k-1}^{(2)}-X_k^{(1)}$,

And

$$g(v)=\int_0^{\infty} f(u+v, u)\, du, \tag{3.5}$$

$$\text{where } X_k^{(2)}=u+v;\ Y_k^{(1)}=u \text{ such that } v=X_k^{(2)}-Y_k^{(1)}. \tag{3.6}$$

Therefore:

$$E\left[Y_{k-1}^{(2)}-X_k^{(1)} I_{\{Y_{k-1}^{(2)}-X_k^{(1)}>0\}}\right]=\int_0^{\infty} u . g(u)\, du . \tag{3.7}$$

$$E\left[X_k^{(2)}-Y_k^{(1)} I_{\{X_k^{(2)}-Y_k^{(1)}>0\}}\right]=\int_0^{\infty} v g(v)\, dv . \tag{3.8}$$

Now the expected length of working time can be obtained as follows:

Let $X_k^{(i)} \sim \exp(\lambda)$ for $k=1,2,3,\ldots,$ and $i=1,2$.

Then the distribution function of $X_k^{(i)}$, for k=1,2,3,….and i=1,2 is :

$$F_k(x)=F(k^{\alpha} x)=1-e^{-\left(\frac{k^{\alpha} x}{\lambda}\right)} ; x>0, \lambda>0 \tag{3.9}$$

By definition the expected length of working time is :

$$E\left(X_k^{(i)}\right)=\int_0^{\infty} x dF\left(k^{\alpha} x\right), \quad i=1,2. \tag{3.10}$$

$$=\frac{\lambda}{k^{\alpha}}, \text{ where } i=1,2 \tag{3.11}$$

The expected length of repair time of component 1 can be obtained as follows:

Let $Y_k^{(i)} \sim \exp(\mu)$ then the distribution function of $Y_k^{(i)}$ for i=1,2, and k=1, 2, 3, …., is :

$$F_k(y)=F(b^{k-1} y)=1-e^{-\left(\frac{b^{k-1} y}{\mu}\right)} ; y>0, \mu>0 \tag{3.12}$$

By definition, the expected length of repair time is:

$$E\left(Y_x^{(i)}\right)=\int_0^{\infty} y dF(b^{k-1} y) \quad i=1,2.$$

$$=\frac{\mu}{a^{k-1}}, \quad i=1,2. \tag{3.13}$$

The expected length of waiting time for repair can be computed as follows:

Let g(u) be the probability density function of $u=Y_{k-1}^{(2)}-X_k^{(1)}$, then by definition of probability density function and using Jacobian transformation we have:

$$g(u)=\int_0^{\infty} f(v, u+v)\, dv,$$

$$\text{where } X_k^{(1)}=v,\ Y_{k-1}^{(2)}=u+v, \text{ such that } u=Y_{k-1}^{(2)}-X_k^{(1)}. \tag{3.14}$$

Since $X_k^{(i)}$ and $Y_k^{(i)}$ are all independent, for i=1,2 and k=1,2,3,…..,n.

$$g(u)=\int_0^{\infty} f(v) . f(u+v)\, dv. \tag{3.15}$$

From equations (3.9) and (3.12) we have:

$$g(u)=\frac{k^{\alpha} b^{k-2} \lambda \mu}{k^{\alpha} \lambda+b^{k-2} \mu} e^{-b^{k-2} \mu u} \quad \text{for } u \geq 0 . \tag{3.16}$$

$$\text{Let } E\left[Y_{k-1}^{(2)}-X_k^{(1)} I_{\{Y_{k-1}^{(2)}-X_k^{(1)}>0\}}\right]=\int_0^{\infty} u g(u)\, du \tag{3.17}$$

$$=\int_0^{\infty} u \frac{k^{\alpha} b^{k-2} \lambda \mu}{k^{\alpha} \lambda+b^{k-2} \mu} e^{-b^{k-2} \mu u} du .$$

$$=\frac{k^{\alpha} \lambda}{b^{k-2} \mu\left(k^{\alpha} \lambda+b^{k-2} \mu\right)}, \quad \text{For } k>2. \tag{3.18}$$

Similarly, the expected length of cold standby time can be computed as follows:

$$E\left[X_k^{(2)}-Y_k^{(1)} I_{\{X_k^{(2)}-Y_k^{(1)}>0\}}\right]=\int_0^{\infty} v g(v)\, dv . \tag{3.19}$$

Where $g(v)$ be the p.d.f of $v=X_k^{(2)}-Y_k^{(1)}$. By definition of p.d.f and using Jacobean Transformation we have:

$$g(v)=\int_0^{\infty} f(u+v, u)\, du . \tag{3.20}$$

$$\text{where } X_k^{(2)}=u+v,\ Y_k^{(1)}=u \text{ such that } v=X_k^{(2)}-Y_k^{(1)}. \tag{3.21}$$

Since $X_k^{(i)}$ and $Y_k^{(i)}$, for i=1, 2 are all independent and form a geometric process,

$$g(v)=\int_0^{\infty} f(u+v) . f(u) du. \tag{3.22}$$

Using equations (3. 9) and (3.12), we get:

$$g(v)=\frac{k^{\alpha} b^{k-2} \lambda \mu}{k^{\alpha} \lambda+b^{k-1} \mu} e^{-k^{\alpha} \lambda v}, \quad \text{for } v \geq 0 \tag{3.23}$$

$$\text{Let } E\left[X_k^{(2)}-Y_k^{(1)} I_{\{X_k^{(2)}-Y_k^{(1)}>0\}}\right]=\int_0^{\infty} v g(v)\, dv$$

Using equations (3.15) and (3.23), we have:

$$=\frac{b^{k-1} \mu}{k^{\alpha} \lambda\left(k^{\alpha} \lambda+b^{k-1} \mu\right)} \tag{3.24}$$

Using the equations (3.11), (3.13), (3.18) and (3.24) ,equation (3.3) becomes:

$$C(N)=\frac{C_r E\left[\sum_{k=1}^{N-1}\left(Y_k^{(1)}+Y_k^{(2)}\right)\right]+C-C_w E\left[\sum_{k=1}^{N}\left(X_k^{(1)}+X_k^{(2)}\right)\right]}{E(L)}$$

$$C(N)=\frac{2 C_r \sum_{k=1}^{N-1} \frac{\mu}{b^{k-1}}+C-C_w 2 \sum_{k=1}^{N} \frac{\lambda}{k^{\alpha}}}{\sum_{k=1}^{N} \frac{\lambda}{k^{\alpha}}+\sum_{k=1}^{N-1} \frac{\mu}{b^{k-1}}+\sum_{k=1}^{N} \frac{k^{\alpha} \lambda}{b^{k-2} \mu\left(k^{\alpha} \lambda+b^{k-2} \mu\right)}+\sum_{k=1}^{N-1} \frac{b^{k-1} \mu}{k^{\alpha} \lambda\left(k^{\alpha} \lambda+b^{k-1} \mu\right)}+\frac{\lambda}{N^{\alpha}}} \tag{3.26}$$

 PRINT ISSN No. 2277 - 8179 | DOI : 10.36106/ijsr

This is the long-run average cost function under policy N.

In the next section, an optimal replacement policy N* is determined such that the long-run average cost rate is minimum and also provide numerical work to highlight the obtained theoretical results.

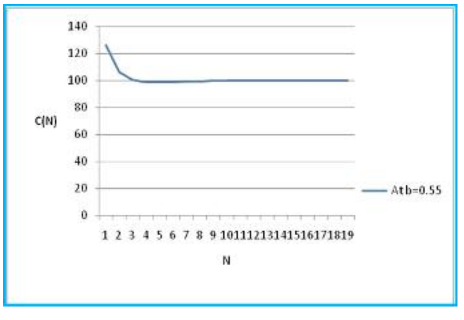
NUMERICAL RESULTS AND CONCLUSIONS

For the given hypothetical values of the parameters of λ, μ, α, b, Cw, C, and Cr the values of long-run average cost per unit time are calculated from the expression (3.26) as follows:

Table 4.1: Values of the long run average cost Rate under policy N. λ =10, μ=20, , α=0.95, C=3000, Cw=10 Cr=50

	b=0.55	**b=0.65**
N	C(N)	C(N)
2	126.2512	126.2512
3	105.8242	106.2895
4	100.1968	100.2799
5	98.72032	98.34467
6	98.568	97.89295
7	98.79244	97.99933
8	99.0675	98.28836
9	99.29496	98.60374
10	99.45876	98.88481
11	99.56867	99.11361
12	99.63936	99.29054
13	99.68355	99.42291
14	99.71063	99.5197
15	99.72698	99.5893
16	99.73675	99.6387
17	99.74252	99.67343
18	99.74592	99.69763
19	99.7479	99.714
20	99.74904	99.72577

Graph: 4.1 Plot of N v/s C(N

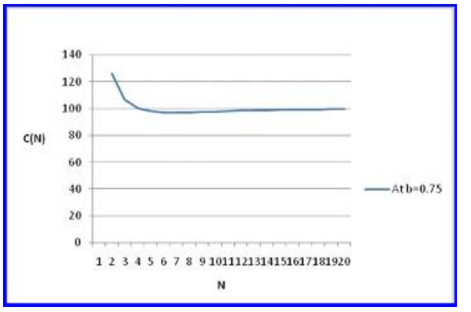


For the given hypothetical values of the parameters of λ, μ, α, b, Cw, C, and Cr the values of long-run average cost per unit time are calculated from the expression (3.26) as follows:

Table 4.2: Values of the Long Run Average Cost Rate under Policy N. λ =10, μ=20, , α=0.95, C=3000, C w=10 Cr=50

	b=0.75	**b=0.85**
N	C(N)	C(N)
2	126.2512	126.2512
3	106.6783	107.0081
4	100.3526	100.4155
5	97.97114	97.61592
6	97.1396	96.36694
7	96.98982	95.85723
8	97.14627	95.73054
9	97.43005	95.81027
10	97.75205	96.00138
11	98.06785	96.24986
12	98.3565	96.52387
13	98.60963	96.80423
14	98.82569	97.0794
15	99.00659	97.34249
16	99.1559	97.58961
17	99.27777	97.81879
18	99.37638	98.02928
19	99.45557	98.22117
20	99.51709	98.38516

Graph: 4.2- Plot of N v/s C(N)



CONCLUSIONS

- From the table 4.1 and graph 4.1, we can observe that the long-run average cost per unit time at the time C (6) = **98.5680** is minimum at b=0.55**.** We should replace the system at the time of 6th failure.
- From the table 4.2 and graph 4.2, we examine that the long-run average cost per unit time at the time C (6) = **97.89295** is minimum at b=0.65**.** We should replace the system at the time of 6th failure.
- If the repairman experiences with repair then the successive repair times form a decreasing geometric process, while the consecutive working times form a an increasing alpha process. Thus this model can also be applied for an improved model.

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