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# UNSTEADY FLOW AND HEAT TRANSFER OF A COUPLES STRESS BIOLOGICAL FLUIDS IN A VERTICAL CHANNEL

Mathematics

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## ABSTRACT

The problem of unsteady couple stress fluid flow in a vertical channel with equal and different wall temperature has been considered. The partial differential equations governing the flow and heat transfer are transformed to ordinary differential equations and are solved analytically using regular perturbation technique valid for small values of N considering only real part of the exponential series. Graphs are drawn for different values of Prandtl number, couple stress parameter and periodic frequency parameter on the flow and heat transfer.

## KEYWORDS

couple stress fluid, unsteady flow, heat transfer, vertical channel

## INTRODUCTION

Heat transfer by natural convection is frequently encountered in our environment and in engineering device. Natural convection arises from the buoyancy force induced by density differences in a fluid. Ostrich [1] has investigated the effect of viscous dissipation and heat source in the fluid on the fully developed laminar free convection flow between two vertical parallel plates when the latter are kept at either constant or linearly varying temperature along the plate.

The analyses of flow properties of non-Newtonian fluids are very important in the fields of fluid dynamics and heat transfer. But due to the complex stress-strain rate relationships of the non-Newtonian fluids, not many investigators have studied the flow behavior of the fluids in various flow fields. It has great importance in engineering and industry, such as liquid crystals, fluid film lubrication, and the entrance region of the pipe flow and reduction of the drag force. In the category of non-Newtonian fluids, couple stress fluid has distinct features such as polar effects in addition to possessing large viscosity. The consideration of couples stress in addition to Cauchy stress has led to the recent development of several theories (Erigin [2], Umavathi[3]) of fluid micro-continua. This new branch of fluid mechanism has attracted a growing interest during recent years mainly because it possesses the mechanism to describe such rheologically complex fluids as liquid crystals, polymeric suspension of animal blood for which the Navier-Stokes theory is inadequate. One such couples stress theory of fluids was developed by Stokes [4] on the channel flow. Recently Abdul Mateen [5] considered oscillatory flow and heat transfer in a vertical channel with equal wall temperature.

In this paper we have extended the work of Abdul Mateen [6] by consider the real part of the . The object of this paper is to study the unsteady couples stress fluid flow and heat transfer in a vertical channel with equal wall temperatures.

### MATHEMATICAL FORMULATION

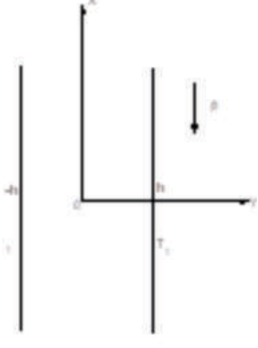


Fig. 1 Physical configuration

It is considered an incompressible, unsteady couple stress fluid with constant properties and using the conditions of the equilibrium state of the fluid and further assuming that the density of the fluid is a function of temperature alone. Therefore, the governing equations and boundary conditions for this flow geometry in dimensionless form given by Abdul Mateen [6]:

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\theta+\frac{\partial^2 u}{dy}-\frac{1}{a^2}\frac{\partial^4 u}{\partial y^4} \tag{1}$$

$$\frac{\partial \theta}{\partial t}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\left[\frac{\partial^2 \theta}{\partial y^2}+N\left(\frac{\partial u}{\partial y}\right)^2\right] \tag{2}$$

the transformed boundary conditions are

$$u=\frac{d^2u}{dy^2}=0 \qquad at\ y=\pm 1 \tag{3}$$

$$\theta=1 \qquad at\ y=\pm 1 \tag{4}$$

Where $a^2=\frac{\mu h^2}{\eta}$ , $N=\frac{\rho g^2\beta^2 h^4(T_1-T_0)}{v\,k}$

### SOLUTIONS

The above governing equations are solved subject to the boundary conditions. These equations are coupled partial differential equations that cannot be solved in closed form. However, it can be reduced to ordinary differential equations by assuming

$$u(y,t)=u_0(y)+\varepsilon\, e^{i\omega t}\, u_1(y)+---\tag{5}$$

$$\theta\,(y,t)=\theta_0(y)+\varepsilon\, e^{i\omega t}\,\theta_1(y)+---\tag{6}$$

This is a valid assumption because of the choice of v as defined in equation (9) that the amplitude $\varepsilon A<<1$.

Considering the real part of $e^{i\omega t}$ equations (5) and (6) becomes

$$u\ (y,t)=u_0(y)+\varepsilon\ \cos\omega t\ u_1(y)+--- \tag{7}$$

$$\theta(y,t)=\theta_0(y)+\varepsilon\ \cos\omega t\ \theta_1(y)+--- \tag{8}$$

Substituting equations (7) and (8) into equations (1) to (2) the harmonic and non-harmonic terms and neglecting the higher order terms of $\varepsilon^2$ , one obtains the following of equations

#### Non-periodic Coefficients

$$\frac{d^4u_0}{dy^4}-a^2\frac{d^2u_0}{dy^2}+a^2\frac{du_0}{dy}=a^2\theta_0 \tag{9}$$

$$\frac{d^2\theta_0}{dy^2}-\Pr\frac{d\theta_0}{\partial y}=-N\left(\frac{\partial u_0}{\partial y}\right)^2 \tag{10}$$

#### Periodic Coefficients

$$\frac{d^4u_1}{dy^4}-a^2\frac{d^2u_1}{dy^2}+a^2\frac{du_1}{dy}-a^2\omega\tan\omega t u_1 =a^2\theta_1-a^2A\frac{du_0}{dy} \tag{11}$$

$$\frac{d^2\theta_1}{dy^2}-\Pr\frac{d\theta_1}{\partial y}+\Pr\omega\tan\omega t\theta_1 =\Pr A\frac{d\theta_0}{dy}-2N\frac{du_0}{dy}\frac{du_1}{dy} \tag{12}$$

Corresponding boundary conditions becomes

**Non-periodic Coefficients**

$$u_0 = \frac{d^2 u_0}{dy^2} = 0 \quad at\ y = \pm 1 \tag{13}$$

$$\theta_0 = 1 \quad at\ y = \pm 1 \tag{14}$$

**Periodic Coefficients**

$$u_1 = \frac{d^2 u_1}{dy^2} = 0 \quad at\ y = \pm 1 \tag{15}$$

$$\theta_1 = 0 \quad at\ y = \pm 1 \tag{16}$$

In order to solve the equations (9) to (12) subjected to the boundary conditions (13) to (14) using the regular perturbation technique considering N<<1. we express u, θ in powers of N as

$$(u_0, \theta_0) = (u_{00}, \theta_{00}) + N(u_{01}, \theta_{01}) + - - - \tag{17}$$

$$(u_1, \theta_1) = (u_{10}, \theta_{10}) + N(u_{11}, \theta_{11}) + - - - \tag{18}$$

Substituting equations (17) and (18) in equations (9) to (16) and equating the co-efficient of like powers of N, we get

**Non-periodic coefficients**
**Zeroth Order equations**

$$\frac{d^4 u_{00}}{dy^4} - a^2 \frac{d^2 u_{00}}{dy^2} + a^2 \frac{du_{00}}{dy} = a^2 \theta_{00} \tag{19}$$

$$\frac{d^2 \theta_{00}}{dy^2} - \Pr \frac{d\theta_{00}}{\partial y} = 0 \tag{20}$$

**First Order Equations**

$$\frac{d^4 u_{01}}{dy^4} - a^2 \frac{d^2 u_{01}}{dy^2} + a^2 \frac{du_{01}}{dy} = a^2 \theta_{01} \tag{21}$$

$$\frac{d^2 \theta_{01}}{dy^2} - \Pr \frac{d\theta_{01}}{\partial y} = -\left(\frac{du_{00}}{dy}\right)^2 \tag{22}$$

corresponding boundary conditions becomes

**Zeroth Order Equations**

$$u_{00} = \frac{d^2 u_{00}}{dy^2} = 0 \quad at\ y = \pm 1 \tag{23}$$

$$\theta_{00} = 0 \quad at\ y = \pm 1 \tag{24}$$

**First Order Equations**

$$u_{01} = \frac{d^2 u_{01}}{dy^2} = 0 \quad at\ y = \pm 1 \tag{25}$$

$$\theta_{01} = 0 \quad at\ y = \pm 1 \tag{26}$$

**Periodic coefficients**
**Zeroth Order equations**

$$\frac{d^4 u_{10}}{dy^4} - a^2 \frac{d^2 u_{10}}{dy^2} + a^2 \frac{du_{10}}{dy} - a^2 \omega \tan \omega t\, u_{10} = a^2 \theta_{10} - a^2 A \frac{du_{00}}{dy} \tag{27}$$

$$\frac{d^2 \theta_{10}}{dy^2} - \Pr \frac{d\theta_{10}}{\partial y} + \Pr \omega \tan \omega t\, \theta_{10} = \Pr A \frac{d\theta_{00}}{dy} \tag{28}$$

**First Order Equations**

$$\frac{d^4 u_{11}}{dy^4} - a^2 \frac{d^2 u_{11}}{dy^2} + a^2 \frac{du_{11}}{dy} - a^2 \omega \tan \omega t\, u_{11} = a^2 \theta_{11} - a^2 A \frac{du_{10}}{dy} \tag{29}$$

$$\frac{d^2 \theta_{11}}{dy^2} - \Pr \frac{d\theta_{11}}{\partial y} + \Pr \omega \tan \omega t\, \theta_{11} = \Pr A \frac{d\theta_{01}}{dy} - 2 \frac{du_{00}}{dy} \frac{du_{10}}{dy} \tag{30}$$

corresponding boundary conditions becomes

**Zeroth Order Equations**

$$u_{10} = \frac{d^2 u_{10}}{dy^2} = 0 \quad at\ y = \pm 1 \tag{31}$$

$$\theta_{10} = 0 \quad at\ y = \pm 1 \tag{32}$$

**First Order Equations**

$$u_{11} = \frac{d^2 u_{11}}{dy^2} = 0 \quad at\ y = \pm 1 \tag{33}$$

$$\theta_{11} = 0 \quad at\ y = \pm 1 \tag{34}$$

## RESULTS AND DISCUSSION

Unsteady couple stress fluid flow in a vertical channel with equal wall temperatures has been studied. The partial differential equations governing the flow and heat transfer are transformed to ordinary nonlinear differential equations considering real part of exponential series. The nonlinear differential equations are then solved using regular perturbation method valid for small values of buoyancy parameter N. Solutions are depicted graphically for zeroth, first and total solutions in Figs. 2 to 5, 6 to 10 and 11 to 16 respectively for different values of couple stress parameter, Prandtl number and periodic frequency parameter.

Figure 2 shows the velocity profiles for different values of couple stress parameter 'a'. As couple stress parameter 'a' increases the velocity increases upto a=4.355 and then decreases continuously. Form Fig. 3 we observe that as couple stress parameter 'a' increases the temperature increases upto a=6.0 and then decreases.

Figure 4 show the effect of Prandtl number on velocity. For Pr<1, velocity increases and decreases when Pr>1. The temperature decreases continuously as Prandtl number increases which is evident from Fig. 5. Figures 2 to 5 are for zeroth order solutions i.e., for steady flow and in the presence of buoyancy forces.

Figures 6 to 10 are drawn for first order solutions i.e., for unsteady flow and in the presence of buoyancy forces. Effect of couples stress parameter 'a' on velocity is shown in Fig. 6. As couple stress parameter 'a' increases velocity increases upto 4.0 and reversal flow accurse from a=6.0. It is interesting to note that velocity becomes zero (stagnation point) at y=0.2242 and 0.1147 for a=6 and 8 respectively. We observe the similar effect on temperature as shown in Fig. 7, i.e, temperature increases upto 4.0 and then decreases. As Prandtl number increases temperature also increases as observed in Fig. 8. The effect of periodic frequency parameter $t\omega$on velocity and temperature is shown in Figs. 9 and 10 respectively. Velocity increases as increases. We observe in Fig. 10 that temperature increases for 0θ> and decrease for0θ<.

Figures 11 to 16 display the effect of couple stress parameter, Prandtl number and periodic frequency parameter on the flow in the presence of cosine of periodic frequency parameter and the buoyancy force. It is seen that velocity and temperature increases as couple stress parameter 'a' increases upto a=4.0 and then decreases continuously. Figures 13 and 14 show that as Prandtl number increases velocity decreases whereas temperature increases. As the periodic frequency parameter increases velocity decreases upto 1.0476 and remains constant as seen in Fig. 15. The temperature also increases upto 1.047 and then remains almost constant upto 1.5539 as seen in Fig. 16.

Thus one can conclude that unsteadiness will result in fluctuations of the velocity and temperature fields.

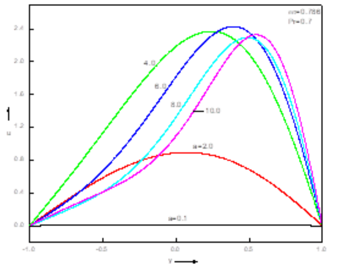


Fig.2 Velocity profiles for different values of 'a'

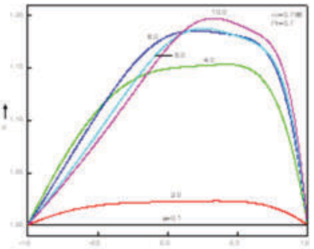

Fig.3 Temperature profiles for different values of 'a'

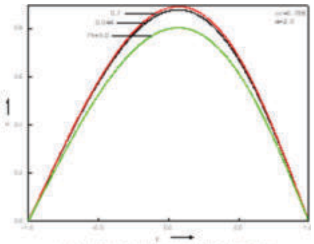

Fig.4 Velocity profiles for different values of Prandtl number Pr

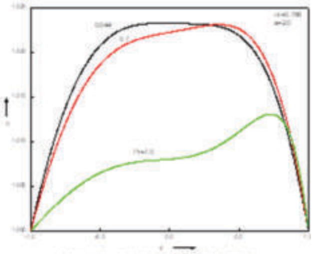

Fig.5 Temperature profiles for different values of Prandtl number Pr

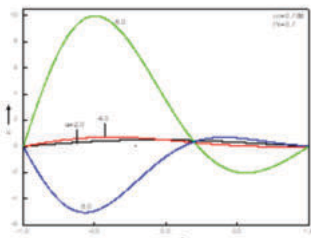

Fig.6 Velocity profiles for different values of 'a'

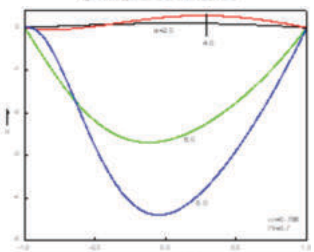

Fig. 7 Temperature profiles for different values of 'a'

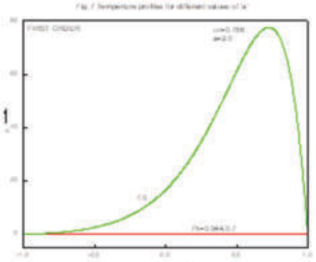

Fig.9 Temperature profiles for different values of Prandtl number Pr

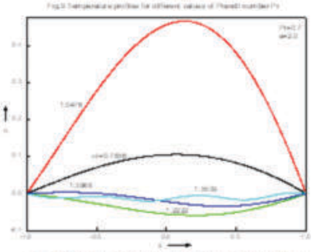

Fig.10 Temperature profiles for different values of periodic frequency parameter ω

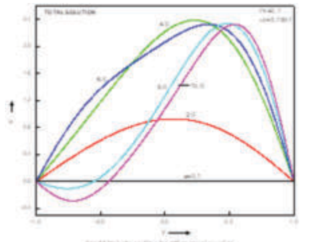

Fig.11 Velocity profiles for different values of 'a'

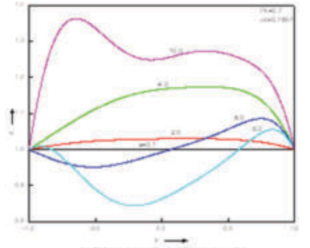

Fig.12 Temperature profiles for different values of 'a'

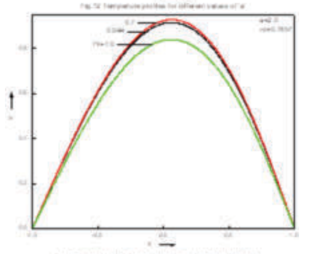

Fig.13 Velocity profiles for different values of Prandtl number Pr

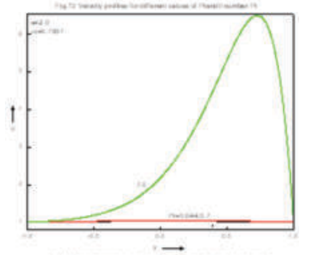

Fig.14 Temperature profiles for different values of Prandtl number Pr

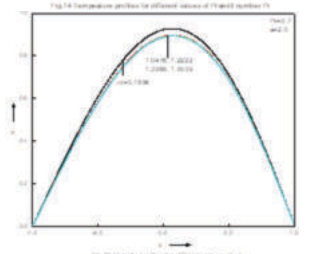

Fig.15 Velocity profiles for different values of ωt

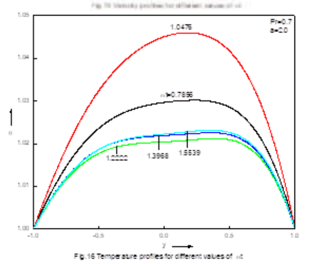


Fig.16 Temperature profiles for different values of ωt

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