Advances In Materials And Manufacturing Technology 2015

Volume : 4 | Issue : 10 | Special Issue Oct- 2015 • ISSN No 2277 - 8179

MODELING OF ORTHOTROPIC MATERIAL BASED COMPOSITE BEAM

Engineering

KEYWORDS: -finite element method; composite material; orthotropic beam; Timoshenko beam theory

Lucky Bagha1	M.E. Student Mechanical Engineering Department, University Institute of Engineering and Technology Panjab University, Chandigarh
Shankar Sehgal	Assistant Professor Mechanical Engineering Department, University Institute of Engineering and Technology Panjab University, Chandigarh
Amit Thakur	Assistant Professor Mechanical Engineering Department, University Institute of Engineering and Technology Panjab University, Chandigarh

ABSTRACT A finite element model is developed to study the dynamic behavior of composite beam. Orthotropic materials are widely use in astronautic and aerospace industries. Due to their high strength to weight ratio and high stiffness, composite are used in structural applications. In this study the free vibration responses of a composite beam using Timoshenko beam theory.

1. INTRODUCTION

Orthotropic materials have different properties in different directions. Their properties depend upon direction in x, y and z .axis. Composite materials are the orthotropic materials and possess generally high strength to weight ratio and high stiffness than other structural materials. Composite materials have many layers of a composite matrix and fibers. Each layer with different fiber alignment under different sequence and may have identical or non identical material properties. Dynamic characteristics of composite material are important to analyze because composite material is widely used in industrial applications as well as research works. Applications of composite materials include aircraft, automobile parts, turbine blades and choppers.

A composite is made by numbers of layers of stacking in the z direction. Each layer may have different fiber orientation and thin in size. Response from the laminate is influenced by fiber orientation, material properties and arrangements. The governing equations for evaluating the structural dynamic behaviour for a composite beam or plate are similar. Figure 1 shows the geometric details of a composite beam, whose finite element (FE) model can be developed in MATLAB and later can be used to evaluate its structural dynamic properties such as natural frequencies, mode-shapes and frequency response functions. The composite beam is made up of many layers of orthotropic materials and the principal axis of a layer may be oriented at an arbitrary angle with respect to the x- axis.

In this paper, different methods of developing the FE model of composite materials have been presented. This is followed by the development of FE model of a composite beam using first order shear deformation theory. These concepts can be implemented using MATLAB to develop the FE model of the composite beam under consideration [1]

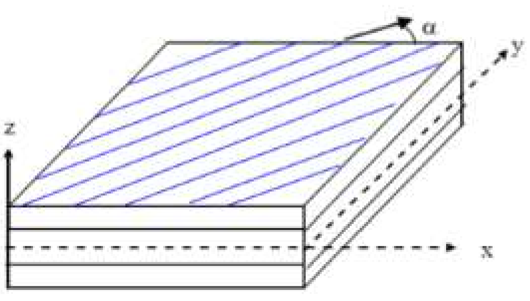

Figure 1. Geometric details of composite bea2.

2. LITERATURE REVIEW

Chandrasekhara and Bangera [2] used higher order deformation theory with Poisson's effect in order to develop the FE model for smart Timoshenko beam. Friedman and Kosmatka [3] used Hamilton's theory to develop two node Timoshenko beam element. The Timoshenko beam stiffness was exact equal to resulting matrix. Elshaferi [4] developed mass and stiffness matrix for isotropic and orthotropic beam using first order shear deformation theory. Abadi and Daneshmer [5] considered two different theories i.e. Euler Bernoulli and Timoshenko beam theories for governing equations, boundary conditions and initial conditions of micro composite laminated beam.

Vertappen and Pearse [6] used iterative solution method for modeling of composite material with patterns fiber constrained layer damping. Miranda [7] developed the FE model by using high order shear deformation theory for static and dynamic analysis of laminar beams. Behesti-Aval and Lezgy-Nazargah [8] developed the FE model for composite beam with piezoelectric layer using sinus model.

Review of the existing research paper shows that the fiber matrix composite consisting of multiple layers of material, called laminate, may be different because of the arrangement of layers. Each layer is thin and may have a different fiber orientation.

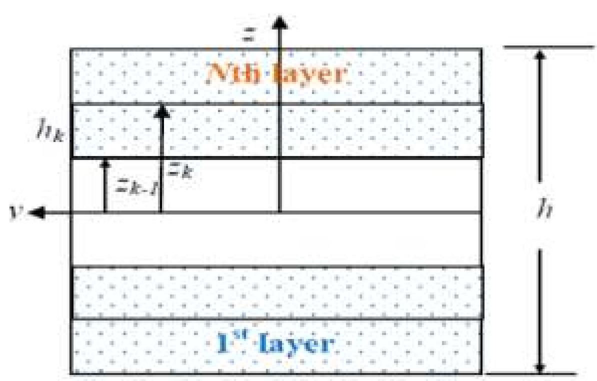

Figure 2. Cross-sectional details of composite beam

Figure 2 shows the cross-sectional details for a composite beam werein, h is the laminated beam thickness and the k_{th} layer thickness is h_k. The origin of the thickness co-ordinate, designated z, is located at the laminated geometry mid plane. The geometric mid plane may be within a particular layer or at an interface between layers. The laminate thickness is increase in the z direction from –h/2 to +h/2. The layer 1 is at the most negative location i.e. at the bottom of the beam, the next layer in as layer 2, the layer at an arbitrary location is layer k, N is the most positive layer. Z is located the interfaces of the composite beam; z_0 and z_1 bounded the 1 layer of the beam, z_1 and z_2 bounded the 2 layer, $z_{(k-1)}$ and z_k bounded the k_{th} layer and the z_{N-1} and z_N bounded the N_{th} layer[9]

3. MODELING OF COMPOSITE BEAM

Hook's law for orthotropic material is given by: {σ} = [q] {q} here [q] is called material stiffness matrix [9]. For plane stress condition

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{Bmatrix}=\begin{bmatrix}q_{11} & q_{12} & 0 & 0 & 0\\ q_{21} & q_{22} & 0 & 0 & 0\\ 0 & 0 & q_{33} & 0 & 0\\ 0 & 0 & 0 & q_{44} & 0\\ 0 & 0 & 0 & 0 & q_{55}\end{bmatrix}\begin{Bmatrix}\epsilon_{11}\\ \epsilon_{22}\\ \epsilon_{23}\\ \epsilon_{13}\\ \epsilon_{12}\end{Bmatrix}$$

The elastic constants in the principal material coordinate system are:

$$q_{11}=\frac{E_1}{1-\nu_{12}\nu_{12}},\ q_{12}=\frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}},\ q_{22}=\frac{E_2}{1-\nu_{12}\nu_{21}},$$

$$q_{44}=G_{23},\ q_{55}=G_{13},\ q_{66}=G_{12}.$$

Here E_1, E_2, G_{12}, G_{23}, G_{13} and ν_{12} are engineering parameters of the nth layer in the laminated obtained from rule of mixtures the transformed stress strain relations for each lamina can be written as:

$$\begin{Bmatrix}N_X\\ N_Y\\ N_{XY}\end{Bmatrix}=\begin{bmatrix}A_{11} & A_{12} & A_{12}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{Bmatrix}\epsilon_X^0\\ \epsilon_Y^0\\ \gamma_{XY}^0\end{Bmatrix}+\begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{Bmatrix}k_X^0\\ k_Y^0\\ k_{XY}^0\end{Bmatrix}$$

[4] Here Q_{ij} is the transformed reduced stiffness terms are given as following;

$$[Q]=[T]^{-1}[q][T]^{-1}\text{, where }[T]=\begin{bmatrix}c^2 & s^2 & -2cs\\ s^2 & c^2 & 2cs\\ cs & -cs & (c^2-s^2)\end{bmatrix}$$

with c=cos(ϕ) and s=sin(ϕ). Also ϕ is is fiber orientation angle for k_{th} lamina with respect to x-axis of beam.

The important assumption of classical lamination theory is that each point within the volume of a laminate is in a state of plane stress. Therefore, stresses can be computed if we know the strains and the curvatures of the reference surface. Given the force and moment results, we want to calculate the stresses and strains through the thickness as well as the strains and curvatures on the reference surface. We also want to do this by computing the laminate stiffness matrix. The force resultants N_x, N_y and N_{xy} can be shown to be related to the mid plane strains and curvatures k^0 at the reference surface

$$\begin{Bmatrix}M_X\\ M_Y\\ M_{XY}\end{Bmatrix}=\begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{Bmatrix}\epsilon_X^0\\ \epsilon_Y^0\\ \gamma_{XY}^0\end{Bmatrix}+\begin{bmatrix}D_{11} & D_{12} & D_{12}\\ D_{12} & D_{22} & D_{26}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{Bmatrix}k_X^0\\ k_Y^0\\ k_{XY}^0\end{Bmatrix}$$

Similarly, the moment resultantsM_x, M_y and M_xy can also be show to be related to strains and curvatures at the reference surface by the following

$$\begin{Bmatrix}M_X\\ M_Y\\ M_{XY}\end{Bmatrix}=\begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{Bmatrix}\epsilon_X^0\\ \epsilon_Y^0\\ \gamma_{XY}^0\end{Bmatrix}+\begin{bmatrix}D_{11} & D_{12} & D_{12}\\ D_{12} & D_{22} & D_{26}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{Bmatrix}k_X^0\\ k_Y^0\\ k_{XY}^0\end{Bmatrix}$$

Where matrix [A],[B] and[D] are given by

$$A_{ij}=\sum_{k=1}^{N}(Q_{ij})\,(z_k - z_{k-1})$$

$$B_{ij}=\frac{1}{2}\sum_{k=1}^{N}(Q_{ij})\,(z_k^2 - z_{k-1}^2)$$

$$D_{ij}=\frac{1}{3}\sum_{k=1}^{N}(Q_{ij})\,(z_k^3 - z_{k-1}^3)$$

$$\begin{Bmatrix}N_X\\ N_Y\\ N_{XY}\\ M_X\\ M_Y\\ M_{XY}\end{Bmatrix}=\begin{bmatrix}A & B\\ B & D\end{bmatrix}\begin{Bmatrix}\epsilon_X^0\\ \epsilon_X^0\\ \gamma_{XY}^0\\ K_X^0\\ K_Y^0\\ K_{XY}^O\end{Bmatrix}$$

Strains are related with forces by the inverse of [A] matrix following the relation {ϵ}=[a]{N} where matrix [a]=[A](-1) , all the elements of the [B] matrix are identically zero in the case of a symmetric laminate.

In this paper the cantilever beam is divided into five elements each having a length L/5 , mass m and height h.

Elemental mass and stiffness matrices of orthotropic composite cantilever beam can be formed using Timoshenko beam theory [4]. Further the governing equation for the beam can be written as,

$$[M]\{\ddot{X}\}+[K]\{X\}=0$$

Where [M] is the global mass matrix, {ẍ} second order derivative of the nodal displacements with respect to time, [k] is the global stiffness matrix, {x} is the nodal displacement vector.

CONCLUSION

Dynamic characteristics can be obtained from FE modeling of a cantilever composite beam with the help of Timoshenko beam theory. Cantilever beam is divided into 5 elements and 6 nodes. Mass and stiffness element matrices can be obtained by first order shear deformation theory. The model will be used further for model updating purposes.

REFERENCE

[1]MATLAB", User's guide of MATLAB v7. New York: The Mathworks Incorporation, 2004.|[2]M. A. Elshafei, M. R. Ajala, and A. M. Riad, "K. Chandrashekhara and K. M. Bangera, 'Free Vibration of Composite Beams Using a Refined Shear Flexible Beam Element,' Computers and Structures," vol. 3, no. 11, pp. 21–33, 2014.|[3]Z. Friedman and J. B. Kosmatka, "An improved two-node timoshenko beam finite element," Computers & Structures, vol. 47, no. 3. pp. 473–481, 1993.|[4]M. A. Elshafei, "FE Modeling and Analysis of Isotropic and Orthotropic Beams Using First Order Shear Deformation Theory," Mater. Sci. Appl., vol. 04, no. 01, pp. 77–102, 2013.|[5]M. Mohammad Abadi and A. R. Daneshmehr, "An investigation of modified couple stress theory in buckling analysis of micro composite laminated Euler-Bernoulli and Timoshenko beams," Int. J. Eng. Sci., vol. 75, pp. 40–53, 2014.|[6]A. P. Verstappen and J. R. Pearse, "Modelling of patterned fibre constrained layer damping for composite materials," 2015.|[7]I. M. B. D. Miranda, "Static and Dynamic Analysis of Laminate Beams Using High Order Shear Deformation Theory," pp. 1–6.|[8]S. B. Beheshti-Aval and M. Lezgy-Nazargah, "A Finite Element Model for Composite Beams with Piezoelectric Layers using A Sinus Model," J. Mech., vol. 26, no. 03, pp. 335–344, 2010.|[9]G. Z. Voyiadjis and P. I. Kattan, Mechanics of composite materials with MATLAB, vol. 1, no. 1. 2005.