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DISCRETE SERVICE SYSTEM WITH BULK ARRIVALS

Mathematics

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ABSTRACT

Queueing networks are widely used for modeling and analyzing computer communication networks. In the present paper two operations, namely slotted time and non-slotted time operations are being used for analyzing discrete service time systems in digital communications, under the specified conditions. An attempt has been made to analyse the performance of different types of systems which differ in the service order of their customers. Various performance measures viz. mean queue lengths, waiting times, etc. of FCFS system are obtained using slotted time operation and generating function method. These results are further compared with those in non-slotted time operations. The result shows significant deviation under heavy traffic conditions.

INTRODUCTION

The two operations, slotted time operation and the non-slotted time operation, have played significant role in computer networking. We consider these two operations in this paper dealing with a single server queue under the given service time. The purpose of this investigation is to find out that to what extent does the two operations, slotted time and non-slotted time differ, since basically there is a very minute difference between the two and mostly the results obtained can be mistaken of one another.

Using these two operations, the properties of queueing systems with service discipline FCFS have been established by finding queue lengths and waiting times. The bifurcation of queue length under the assumption of Poisson arrival is studied with the non-slotted time system. The analysis of LCFS is also made with comparison to the slotted time system. On the whole the comparison made in this study enables us to approximate one operation on another. This analysis has been widely used in computer networking.

In 1984, David et al. gave a simple and effective approach probability function deriving bounds for bulk arrival queues by making use of the bounds for single arrival queues. Fakinos (1991) for a quite general group-arrival, group departure queue, gave a relation concerning the limiting of queue size distribution. Chaudhary (1992) presented a closed-form analysis and computationally efficient method to study the waiting time distributions of the first and random customer of an arrival group in the MX/G/1 queue. Sharma and Sharma (1996) studied the time-dependent analysis of a limited capacity queueing model with bulk arrival rate depending upon the nature of the service available in the system. Liu and Tseng (1999) discussed a cost model for the service cost, queueing cost and switching cost for bulk arrival queue.

Chaudhary and Kim (2003) presented a complete distribution for the system content of a discrete-time multi-server queue with an infinite buffer, where each customer arriving in a group requires a deterministic service time that could be greater than one slot. Chen (2006) developed a non-linear programming approach to derive the membership functions of the steady-state performance measures in bulk-arrival queueing system with varying batch sizes. Kim and Kim (2007) discussed the performance evaluation processor-sharing queue with bulk arrivals and phase-type services.

Singh (2013) consider a bulk service queue with Markovian arrival process and analyze it using the method of roots and present a simple closed-form analysis for evaluating queue-length distribution at a post-departure epoch in terms of roots of the characteristic equation associated with the MAP/R (a,b)/1 queue, where R represents the class of distributions whose Laplace–Stieltjes transforms are rational functions. Banerjee et al. (2014) consider a single-server queue with finite-buffer in a discrete-time domain where the packets are transmitted in batches (of varying size) according to minimum and maximum threshold limit, usually known as general batch service rule. Lee (2015) considers a discrete-time bulk-arrival bulk-service queueing system with variable service capacity, where the service capacity varies depending on the previous service time.

In this paper, queueing analysis of discrete service system with bulk arrivals is made. The organization of the paper is as follows. In section 2, queueing characteristics like queue size distribution and waiting time of FCFS system are considered in unslotted time. In section 3, the queue characteristic of FCFS systems are considered in slotted time. Section 4 deals with queueing characteristics of LCFS system in slotted time. Numerical results are discussed in section 5. Finally, we wind up the paper by concluding remarks in section 6.

2. FCFS SYSTEM WITH UNSLOTTED TIME OPERATIONS

With the several assumptions as discussed before, the FCFS system with unslotted time operation is the familiar $M^X/G/1$ systems. Considering the waiting time to be the time lapse between the arrival of the customer and the beginning of its service and let W(s) be the LST of waiting time cumulative distribution function. Let G(z) be the p.g.f. of batch size. Let X be the random variable denoting the batch size and let Gk denote Pr(X=k). Let Q(z) to be the probability generating function (p.g.f.) of the queue length distribution at a general time point. Then by Pollaczek-Khinchin (P-K) transform equations for $M^X/G/1$ system (cf. Takagi, 1991)

$$Q(z)=\frac{(1-\rho)(1-z)B^*[\lambda-\lambda G(z)]}{B^*[\lambda-\lambda G(z)]-z} \qquad \text{....(1)}$$

and

$$W(s)=\frac{s(1-\rho)}{s-\lambda+\lambda G[B^*(s)]} \qquad \text{....(2)}$$

where $\rho=\lambda T$ is the server utilization, and $B^*(s)=e^{-sT}$ is the LST of the service time p.d.f.

Let D(s/busy) and D(s/idle) be the LST of the p.d.f. of the inter-departure time.

$\therefore \qquad D(s/busy)=B^*(s)=e^{-sT}$

Moreover with probability $1-\rho$, the server is idle, and the inter-departure time is the sum of two independent components i.e. the service time of a packet and the time taken from the last departure until the next arrival. Unconditionally we obtain D(s), the LST of the inter-departure time distribution function as

$$D(s)=\frac{(1-\rho)\lambda G(s)e^{-sT}}{[\lambda G(s)+s]}+\rho e^{-sT} \qquad \text{....(3)}$$

Busy period length 'H' in a batch arrival system is given by

$H^*(s) = B^*\{s+\lambda-\lambda G[H^*(s)]\}$ ....(4)

Now,

$$\bar{q} = Q'(1) = \frac{2\rho\left[(1-\rho)+\lambda g_2 E(B)+(\lambda g_1)^2 E(B)^2\right]}{2(1-\rho)^2} \quad ....(5)$$

$$\bar{w} = w'(0) = -\left\{\frac{\rho T+\lambda T^2 g_2}{2(1-\rho)}\right\} \quad ....(6)$$

$$\bar{d} = -D'(0) = \frac{1}{\lambda} \quad ....(7)$$

$$\bar{g} = -G'(0) = \frac{T}{1-\rho} \quad ....(8)$$

where $\bar{q}$, $\bar{w}$, $\bar{d}$ and $\bar{g}$ denote the average queue length, average waiting time, average inter departure density function and mean batch size respectively.

3. FCFS SYSTEM WITH SLOTTED TIME OPERATIONS

Let within a slot total arrivals be considered as single batch of arrivals at the slot boundaries. By a discrete time Markov chain analysis at the slotted boundaries, we can obtain $Q_D(z)$, the p.g.f. of the queue length distribution by

$$Q_D(z) = \frac{(1-\rho)(1-z)A^*(z)}{[A^*(z)-z]} \quad ....(9)$$

where $A(z) = B^*[\lambda - \lambda G(z)]$ is the p.g.f. of arrivals within one slotted time.

3.1. Queue length at post departure epoch

By modified $M^X/G/1$ queue technique, the p.g.f. of the queue length distribution as observed by a departure can be obtained as

$$\hat{Q}(z) = \frac{P_0[A^*(z)-z\tilde{A}(z)]}{[A^*(z)-z]} \quad ....(10)$$

where P_0 is the probability of an empty system seen by a departure, and A~(z) is the p.g.f. of arrivals within a time duration $e = \hat{t} + T$ from the first arrival to an empty system until its departure. By putting $\hat{Q}(1) = 1$ and using L Hospital's rule, we get

$$P_0 = (1-\rho)\rho^{-1}(1 - e^{-\rho}) \quad ....(11)$$

After substitution, we get

$$\hat{Q}(z) = \frac{1-\rho}{\rho}\frac{B^*[\lambda-\lambda G(z)][1-B^*[\lambda-\lambda G(z)]}{B^*[\lambda-\lambda z]-z} \quad ....(12)$$

3.2. Queue length observed by an arrival

In the present system the arrivals and departures occur in bulk, the arriving customers and departing customers are identically distributed. Hence

$$\hat{Q}_a(z) = \frac{1-\rho}{\rho}\frac{B^*[\lambda-\lambda G(z)][1-B^*[\lambda-\lambda G(z)]}{B^*[\lambda-\lambda z]-z} \quad ....(13)$$

3.3. Waiting Time distributions

Let)s(S^ be the LST of the p.d.f. of the system time, defined as the time lapse from the arrival of customers until the completion of their services. Then by using the queue length delay relationship, we have

$$\hat{Q}(z) = \hat{S}[\lambda - \lambda G(z)]$$

and substituting

z = 1 – s/ in equation (12), we get

$$\hat{S}(s) = \hat{Q}(1-s/\lambda) = \frac{1-\rho}{T}\frac{[1-B^*(s)]}{S-\lambda+\lambda B^*(s)} \quad ....(14)$$

We further obtain

$$\hat{W}(s) = \frac{s(1-\rho)}{s-\lambda+\lambda G[B^*(s)]}.\frac{1-G[B^*(s)]}{G[1-B^*(s)]} \quad ....(15)$$

$$\text{where } V(s) = \frac{1-G[B^*(s)]}{G[1-B^*(s)]}$$

is the LST of the residual time of the slot time observed by the bulk arrival.

3.4. Busy duration

Two types of busy durations are being identified as virtual busy duration and actual busy duration. In virtual busy duration the first arrival is not served i.e. the server is actually not busy until the next slot boundary. Let Hv(s) be the LST of the density function of the duration from the instant of the first arrival in the empty system until the server becomes idle again. Hence,

$$\hat{H}_v(s) = \hat{V}\{s+\lambda-\lambda G[H(s)]\}.B^*\{S+\lambda-\lambda G[H(s)]\} \quad ....(16)$$

and

$$\hat{H}(s) = \hat{V}\{\lambda-\lambda G[H(s)]\}.B^*\{s+\lambda-\lambda G[H(s)]\} \quad ....(17)$$

4. LCFS SYSTEM IN SLOTTED TIME

In order to analyse the system using the slotted time operations we consider $\hat{W}_L(s)$ to be the LST of the waiting time density function. Let $_{LA}$(s) be the LST of the waiting time density function seen by first arrival to an empty system. Let $_{LB}$(s) be the LST of the waiting time density function by an arrival to the first residual slot time of the virtual busy duration and $_{LC}$(s) be the LST of the waiting time density function of arrivals to the remainder of the virtual busy duration. By unconditioning respective probabilities, we have

$$\hat{W}_L(s) = \frac{(1-\rho)[1-G\{H^*(s)\}]}{G[1-H^*(s)]} + \frac{\lambda[1-G\{H^*(s)\}]}{s+\lambda-\lambda G[H^*(s)]} \quad ....(18)$$

5. NUMERICAL RESULTS

MATLAB software has been used to develop the computer program for numerically analyzing the system performance. It is observed that the results obtained are in tandem with the analytical deductions.

Table 1: Average queue length by varying arrival and service time

l	T=2	T=4	T=6
0	0	0	0
0.2	0.043021	0.093689	0.154041
0.4	0.093689	0.226696	0.423565
0.6	0.154041	0.423565	0.939898
0.8	0.226696	0.72666	2.029245
1	0.315053	1.212678	4.644359

The effects of arrival rate (λ) on the average queue length E(Q), are shown via table and figures 1-2. Table 1 depicts that the average queue length increases on increasing the arrival rate for different service times. It is also noted that the average queue length increases on increasing the service time. It is clear from figures 1-2 that the average queue length first increases gradually, then sharply afterwards on increasing the arrival rate. Moreover, the average queue length increases on increasing either batch size or service time and this increment is more prominent for higher values of arrival rates. We see that these results tally profoundly with the results existing in the literature.

Finally, we conclude that on average queue length increases on increasing the arrival rate, service time and batch size which is as our expectation.

6. CONCLUDING REMARKS

This paper analyses queue characteristics in discrete service time with bulk arrivals. We discuss both slotted time and non-slotted time operations that are commonly used in discrete service time systems. Analytic equations have been derived for FCFS and LCFS systems. We observe that the non-slotted time operation is less complex to analyse as the equations are simpler in this case. They are further used to approximate the slotted time operation closely. These approximations find applications in various fields such as digital systems, communication networks and computer operating systems.

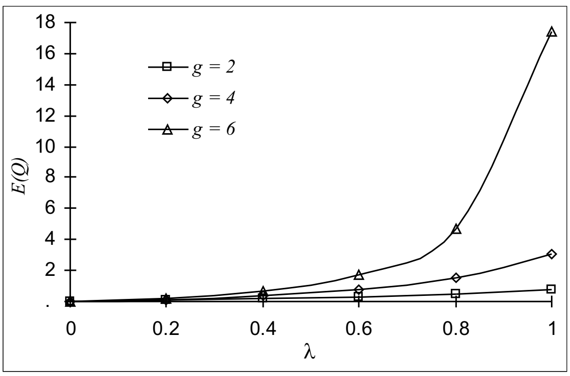

Fig. 1: Average queue length vs. l for different batch size

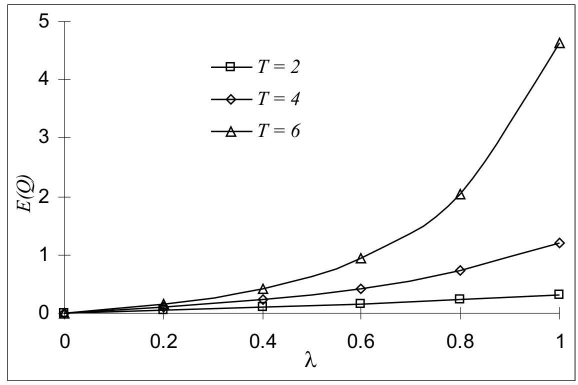

Fig. 2: Average queue length vs. λ for different time slot

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