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AN ITERATIVE METHOD FOR FRACTIONAL TRANSPORTATION PROBLEM

Mathematics

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ABSTRACT

In the present paper an iterative method is carried out to obtain the solution of the Fractional Transportation Problem. In the present method first a basic feasible solution is found by an existing method and thereafter tested for optimality. An another approach of finding basic feasible solution provides better optimum solution in a smaller number of iterations has also presented.

1. INTRODUCTION

The problem of optimization of a function which is characterized by one or several ratios of given functions is called Fractional Functional Programming, henceforth to be termed as FFP.

A number of real life optimization problems take into account the minimizing or maximizing of a ratio of physical and/or economic functions subject to certain constraints. These situations lead of fractional programming problems. The ratios commonly dealt in FFP are expected cost/time, profit/capital, return/risk, performance/cost etc.

Following may be an illustration of the fractional transportation problem.

A situation where an individual, a group or a community is forced to maintain good ratios between some very important crucial parameters concerned with the transportation of availability to requirement. This may be the situation for an enterprise board confronted with the optimization of total return/total investment on production, total profit/total time etc. All these and other areas of application of fractional programming have been presented by Schaible (1977, 1981).

2. MATHEMATICAL FORMULATION OF FRACTIONAL TRANSPORTATION PROBLEM

The mathematical formulation of the fractional transportation problem is as follows;

$$\text{Minimize } P^o = \frac{N(x)}{D(x)} = \left\{\frac{\sum_i \sum_j c_{ij} x_{ij}}{\sum_i \sum_j d_{ij} x_{ij}}\right\} \quad \ldots\ldots 1$$

subject to the conditions

$$\sum_j x_{ij} = a_i \qquad i \varepsilon I \qquad \ldots\ldots 2$$

$$\sum_i x_{ij} = b_j \qquad i \varepsilon I \qquad \ldots\ldots 3$$

$$x_{ij} > 0 \qquad \forall (i,j) \varepsilon I \times J \qquad \ldots\ldots 4$$

where $I = \{1, 2, ..., m\}$ is the index set of origins.

$J = \{1, 2, ..., n)$ is the index set of destinations.

x_{ij} = the amount transported from the i^{th} origin to the j^{th} destination.

c_{ij} = the cost per unit transported from the i^{th} origin to the j^{th} destination.

d_{ij} = the profit per unit transported from the i^{th} origin to the j^{th} destination.

and it is assumed that $D(x) > 0$ for every feasible solution X.

3. METHODOLOGY AND DESCRIPTION OF THE ALGORITHM

The problem is to find $x_{ij} \geq 0$ which minimize (1) subject to (2), (3) and (4). Firstly, find an initial basic feasible solution by any existing method. Thereafter check whether this feasible solution is optimum with respect to other solutions.

Consider the problem (RP^o) given by:

$$RP^o = \text{Min}\{N(x) - q.D(x)\}$$
$$= \text{Min}\{N(x) - q.D(x)\}$$
$$= \text{Min}\{SSc_{ij}x_{ij} - q.SSd_{ij}x_{ij}\}$$
$$= \text{Min}\{SSc'_{ij} \times x_{ij}\}$$

subject to (2), (3), (4)

where $c'_{ij} = (cij - q.d_{ij}) \qquad \forall (i,j) e I \times J$

Let $F(q) = \text{Min}\{SSc'_{ij}x_{ij}\}$

The problem P^o is solved by Dinkelbach's approach in the following way.

Find the optimal solution of (RP^o) at $q = q^1 = 0$. Compute $F(q^1) = 0$, then the corresponding solution x'(say) is the optimal solution for P^o. If not then compute $q^2 = N(x') / D(x')$.

For $r \geq 2$ compute $F(q^r)$

where $q^r = N(x^{r-1}) / D(x^{r-1})$; x^{r-1} being the optimal solution at $q = q^{r-1}$.

If $F(q^r) = 0$, then x^r is the optimal solution of p^o, otherwise repeat general step for r = r+1.

Problem (RP^o) is different from the classical transportation problem in that its objective function includes the variable cost.

4. Numerical Illustration

A numerical example will clarify the above statements.

Table – 1

				a_i
	5 / 4	3 / 1	6 / 3	4 / 4	30
	3 / 3	4 / 3	7 / 2	8 / 2	15
	9 / 5	6 / 5	5 / 2	8 / 4	15
b_j	10	25	18	7	

In the above table the value of c_{ij}'s are given in the top left corner and those of d_{ij}'s in the low right corner. The values of a_i's and b_j's are also given for i = 1,2,3 and j = 1,2,3,4.

A basic feasible solution of RP^o by applying minima method with respect to variable cost only is given in the table 2. Corresponding value of dual variable are also given in table 2.

Table – 2

					a_i
	5 4	3 **(25)** 1	6 3	4 **(5)** 4	30
	3 **(10)** 3	4 3	7 **(3)** 2	8 **(2)** 2	15
	9 5	6 5	5 **(15)** 2	8 4	15
b_j	10	25	18	7	

According to algorithm, table 2 gives the optimum solution of RP^o at $q = q^1$.

Let $q = q^1 = 0$

$F(q) = \text{Min}\{\Sigma\Sigma c'_{ij}x_{ij}\}$

$= N(x^1) = \text{Min}\{\Sigma\Sigma c_{ij}x_{ij}\}$ since $c'_{ij} = (c_{ij} - q.d_{ij})$

$= cij - 0.d_{ij} = c_{ij}$

hence

$F(q^1) = 3\times25 + 4\times5 + 3\times10 + 7\times3 + 8\times2 + 5\times15 = 237 \neq 0$

and

$D(x^1) = 25\times1 + 5\times4 + 10\times3 + 3\times2 + 2\times2 + 15\times2 = 115$

then

$q^2 = 237/115$ 2.1

Now, by using the following relation, table 3 will be as

$c'_{ij} = c_{ij} - q^2 d_{ij}$

Table – 3

					a_i
	–3.4 **(10)**	0.9	–0.3 **(13)**	–4.4 **(7)**	30
	–3.3	–2.3 **(10)**	2.8 **(5)**	3.8	15
	–1.5	–4.5 **(15)**	0.8	–0.4	15
b_j	10	25	18	7	

Now in the same manner,

$N(x^2) = 321$

$D(x^2) = 222$

then

$F(q^2) = (N(x^2) - \{N(x^1)/D(x^1)\}.D(x^2)$

$= 321 - \{237/115\}.222$ 0

Hence, optimum solution is not reached.

Then for next iteration

$q^3 = 321 / 222$ 1.5

Again the reduce matrix will be as in table 4.

Table – 4

					a_i
	–1.0	1.5 **(5)**	1.5 **(18)**	–2.0 **(7)**	30
	–1.5 **(10)**	–0.5 **(5)**	4.0	5.0	15
	1.5	–1.5 **(15)**	2.0	2.0	15
b_j	10	25	18	7	

$N(x^3) = 291$

$D(x^3) = 207$

then

$F(q^3) = N(x^3) - \{N(x^2)/D(x^2)\}.D(x^3)$

$= 291 - \{321 / 222\}.207 \neq 0$

Hence, optimum solution is not reached.

Then for next iteration

$q^4 = 291 / 207$ 1.4

Again the reduce matrix will be as in table 5.

Table – 5

					a_i
	–0.6	1.6 **(5)**	1.8 **(18)**	1.6 **(7)**	30
	–1.2 **(10)**	–0.2 **(5)**	4.2	5.2	15
	2.0	–1.0 **(15)**	2.2	2.4	15
b_j	10	25	18	7	

Now,

$N(x^4) = 291$

$D(x^4) = 207$

then

$F(q^4) = N(x^4) - \{N(x^3) / D(x^3)\}.D(x^4)$

$= 291 - \{291/207\}.207 = 0$

Hence, optimum solution for RP^o is reached at $q = q^4$, thus x^4 being the optimal solution

i.e. $Z_{opt.} = 1.4$

5. Remark

A technique for initial basic feasible solution giving a better solution is explained as below.

Step 1 : Firstly calculate $p_{ij} = c_{ij} / d_{ij}$ for each cell.

Step 2 : The Vogel's approximation method is applied taking

$E = \{p_{ij}\}$ as a cost matrix

This IBFS gives better solution as compared to the solution obtained from Dinkelback's approach.

Consider the problem given in table 1. The reduced cost matrix $E = \{p_{ij}\}$ will be as given in table 6.

Table – 6

		a_i			
	1.3	3.0	2.0	1.0	30
	1.0	1.3	3.5	4.0	15
	1.8	1.2	2.5	2.0	15
b_j	10	25	18	7	

Now using the Vogel's approximation method, an IBFS is obtained as given table 7.

Table – 7

					u_i
	1.3 **(5)**	3.0	2.0 **(18)**	1.0 **(7)**	0
	1.0 **(5)**	1.3 **(10)**	3.5	4.0	–0.3
	1.8	1.2 **(15)**	2.5	2.0	–0.4
v_j	1.3	1.6	2.0	1.0	

Hence the N(x) = 306 and D(x) = 222

Therefore $Z_{opt.} = 1.378$

This solution is better than obtained by the previous one. The number of iterations is also small.

6. Conclusion

The numerical example sited by Arora and Ahuja (2000) on the basis of Dinkelbach (1987) approach does not seem appropriate. They have not used the algorithm in the way as it was provided by Dinkelbach (1987).

A modified form of the same algorithm has presented here as a new approach to solve a Fractional transportation problem where no philosophical deviation of theory is being allowed. The alteration in the present algorithm from Dinkelbach (1987) algorithm is :

In Dinkelbach (1987) algorithm better solution comes after taking the ratio N(x) / D(x) in the last step of each iteration while, in the present algorithm is started after evaluating the ratio c_{ij}/d_{ij}.

The prime objective of workers in developing the algorithm for FPP is to convert the objective function into a linear form which is being considered in the present technique. The same theory has a great importance in optimization problems when the problems are formulated through the goal programming technique.

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