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ANALYSIS OF PLASMA LAYER VISCOSITY IN TWO LAYERED BLOOD FLOW IN STENOTIC ARTERY

Mathematics

KEYWORDS:

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ABSTRACT

This paper deals with a model for the influences of plasma layer on blood flow through an artery with a non-dimensional shaped constriction. The model consists of a peripheral plasma layer free from red cells and a core region represented by a casson fluid. It is assumed that the fluids of both the regions (core and peripheral) having different viscosities. The present analysis demonstrates that the existence of the plasma layer may obviously change the characteristics of flow such as velocity profile, rate of flow, pressure drop etc. The analytic expression for the thickness of the peripheral layer and core viscosity has been obtained in terms of (flow rate, pressure drop and wall shear stress). The plasma layer thickness varies along the length of the tube of stenosis and attains its minimum value at the throat of stenosis. The effect of the variable plasma layer thickness on the flow characteristics have been shown graphically for different parameter values to enable a better understanding of the problem. It has been shown that the resistance to flow and the wall shear stress decreases as the peripheral viscosity decreases; however, it increases as the height of the stenosis increases. These analyses have special physiological significance in blood circulation system.

Introduction

The study of blood flow through mammalian circularly system has been the subject of scientific research for about a couple of centuries. Like most of the problem of nature and life sciences, it is complex one due to the complicated structure of blood, the circulatory system and their constituent material. The experimental studies and the theoretical treatments of blood flow phenomena are very useful for the diagnosis of a number of cardiovascular disease and development of pathological patterns in human or animal physiology and the other clinical purposes and practical applications. A large number of theoretical and experimental efforts have been made in literature to explain the blood flow behavior when it flows through the vessels of circulatory system of living beings. To understand the flow pattern in stenosed arteries, Young 1968 have analyzed the flow of blood through an arterial stenosis.

It is well known that at various location in the arterial system, atherosclerosis generally occur near the branching in the arteries where there tends to be flow irregularities. A build up of fatty deposits occurs in the blood vessel in such regions making it to lose its elasticity. Such hardening of the arteries and the narrowing of the lumen can cause severe thermos and even high blood pressure and blockage of the vessels. Determination of such regions becomes very critical and can be very beneficial if done at an earlier stage. Atherosclerosis is the leading cause of death in many countries.

Many researchers studied the non- Newtonian behavior and pulsatile flow of blood through stenosed arteries. Bugliarello and Sevilla (1970) have shown experimentally that blood flowing through narrow blood vessels, there a peripheral layer of plasma and a core region of suspension of all the erythrocytes. Thus, for a realistic description of the blood flow, it is appropriate to treat blood as a two fluid model with the suspension of all the erythrocytes in the core region as non-Newtonian fluid and plasma in the peripheral region as a Newtonian fluid. Shukla et. al. (1980) have taken two layered models and analyzed the influence of peripheral plasma viscosity on flow characteristics. Haldar et. al. (1996) worked on two-layered model of blood flow through stenosed arteries and obtained that the plasma layer thickness varies along the length of the tube of stenosis and attains its minimum value at the throat of the stenosis.

Ponalagusamy (2007) worked on blood flow through an artery with stenosis and obtained the analytical expression for pheripheral layer thickness, core viscosity, slip velocity, wall shear stress and resistance to flow. Misra et. al. (2008) designed a mathematical model of blood flow through an arterial segment with time dependent stenosis an observed the variation of velocity profile, the volumetric flow rate, the radius of the core region, the wall shear stress and the resistance to the blood flow. In the same year Srivastava worked on theoretical model for blood flow in small vessels and obtained the volumetric flow rate and pressure gradient. In the same field Kumar et. al. (2009) designed a model for the Newtonian and non-Newtonian blood flow in tapered tubes and observed that the semi empirical models provide much better estimates of pressure drop, pressure gradient and flux.

The present model is based on the investigation of red cell concentration and peripheral layer on blood flow characteristics due to the presence of mild stenosis. In the above mentioned models the thickness of the peripheral layer has been treated as constant. But actually it cannot be constant; It should be change along the length of the stenosis In the present investigation a two layered model is considered, in which the peripheral plasma layer behaves as Newtonian fluid and the core is represented by a casson fluid with the supposition that the two fluids are immiscible. The effect of variable thickness of the peripheral layer on the flow characteristics of the problem is investigated for different combinations of the parameter values. The purpose of this work is, therefore, to investigate the flow of blood in small vessels involving a two fluid model. The mathematical model considers a two layered model of blood consisting of a core region of suspension of all the erythrocytes (small spherical non flexible particles) assumed to be a particle fluid suspension (i.e. a suspension of red cells in plasma) and a peripheral layer of plasma.

Analysis

Consider the steady, laminar and one dimensional, axially, fully developed flow of blood through an artery provided with the mild constriction. Here the flow of blood (incompressible fluid) is represented by a two layered model (a core of red blood cell suspension) surrounded by a peripheral layer of plasma. Also suppose here that the stenosis is symmetrical (Shukla, el. at. 1980 and Young 1968). The radius of the vessel is given by:

$$R(x)=\begin{cases}1-\eta\left\{b^{n-1}(x-a)-(x-a)^{n}\right\} & for \quad a\le x\le a+b\\ 1 & for \quad \text{otherwise}\end{cases} \tag{1}$$

Here R is the radius of the constricted artery, R_0 is the radius of the normal artery, b is the length of the stenosis, δ is the height of the stenosis and a represents its location then,

$$x = a + \frac{b}{a^{1/(n-1)}} \tag{2}$$

$$\eta = \frac{\delta n^{n/(n-1)}}{R_0 b^n (n-1)} \tag{3}$$

The basic equation governing the flow in the arterial system is given by the following equation:

$$0 = -\frac{dp}{dz} + \frac{1}{r}\frac{\partial}{\partial r}\left[\mu r \frac{\partial v}{\partial r}\right] \tag{4}$$

where v is the axial velocity, p is the fluid pressure and μ is the viscosity function, solving equation (4) with boundary condition $\frac{\partial v}{\partial r} = 0$ at $r = 0$ and $v = 0$ at $r = R$

$$v = \left(-\frac{1}{2}\frac{dp}{dz}\right)\int_r^R \frac{r}{\mu} dr \tag{5}$$

Here we consider the blood flow in two layers, a peripheral plasma layer near the wall free from red cells and other is a core layer which is represented by a Casson fluid. It is also supposed that the fluids in both the regions are immiscible. The consistency function μ may be written as:

$$\begin{aligned} \mu &= \mu_c \quad for \quad 0 \le r \le R_1 \\ \mu &= \mu_p \quad for \quad R_1 \le r \le R \end{aligned} \tag{6}$$

μ_p and μ_c are the viscosities of the two fluid, v_p and v_c are the axial velocity in the two regions, then the equations which governs the fluid motion in the plasma and core layer are respectively:

$$\frac{dv_p}{dr} = -\frac{pr}{2\mu_p} \qquad for\ R \ge r \ge R_1 \tag{7}$$

$$\frac{dv_c}{dr} = -\frac{p}{2\mu_c}\left(r^2 - r_0^2\right) \qquad for\ R \ge r \ge R_1 \tag{8}$$

Here r_0 is the plug region, R_1 is the radius of the interface separating of the two medium. The boundary conditions for equations (7) and (8) are the no slip conditions;

$$v_p = 0 \qquad at \quad r = R \tag{9a}$$

$$v_p = v_c \qquad at \quad r = R_1 \tag{9b}$$

Solving equation (7) and (8) by using the boundary condition(9a,b) then the velocity in peripheral layer is;

$$v_p = \frac{p}{4\mu_p}\left[R^2 - r^2\right] \tag{10}$$

The velocity in core layer is defined as;

$$v_c = \frac{p}{2\mu_c}\left[\frac{4}{3}\sqrt{r_0}\left(r^{\frac{3}{2}} - R_1^{\frac{3}{2}}\right) - \frac{1}{2}\left(r^2 - R_1^2\right) - r_0\left(r - R_1\right) + \frac{p}{4\mu_p}\left(R^2 - R_1^2\right)\right] \tag{11}$$

Let U be the velocity of the plug region then the expression for U is obtained from equation (11) with $r = r_0$ as:

$$U = \frac{p}{2\mu_c}\left[\frac{1}{2}R_1^2 + r_0 R_1 - \frac{4}{3}\sqrt{r_0}R_1^{\frac{3}{2}} - \frac{1}{6}r_0^2 + \frac{p}{4\mu_p}\left(R^2 - R_1^2\right)\right] \tag{12}$$

The volumetric flow rate Q_p and Q_c in the cell free plasma layer and core region are given by;

$$Q_p = \int_{R_1}^{R} 2\pi r v_p dr \tag{13}$$

and $$Q_c = \pi r_0^2 U + \int_0^{R_1} 2\pi r v_c dr \tag{14}$$

Using the value of v_p and v_c in equation (13) and (14) then we get the total volumetric flow is;

$$Q = Q_c + Q_p$$

$$Q = \frac{\pi P}{8\mu_p}\left\{R^4 - \left(1 - \frac{\mu_p}{\mu_c}\right)R_1^4\right\} \tag{15}$$

Here $\mu = \frac{\mu_p}{\mu_c}$ then equation (15) can be written as in equation (16);

$$Q = \frac{\pi P}{8\mu_p}\left\{R^4 - R_1^4(1-\mu)\right\} \tag{16}$$

Wall shear stress and pressure drop:
Solving equation (15) and using the conditions $p = p_1$ at x=-b and $p_2 = p$ at x = b, then we get:

$$\Delta p = p_1 - p_2 = \frac{8Q_p\mu_p}{\pi}\int_{-b}^{b}\frac{dx}{\left(R^2 - R_1^2\right)^2} \tag{17}$$

If there is no stenosis, $R = R_0$, $R_1 = R_0 - \varepsilon$, being the thickness of the plasma layer, In the case of constant thickness of the plasma layer equation (17) becomes;

$$(\Delta p)_c = \frac{8Q_p\mu_p}{\pi R_0^4 \delta^2}\int_{-b}^{b}\frac{dx}{\left(\frac{2R}{R_0} - \delta\right)^2} \tag{18}$$

If there is no stenosis $(R/10=R)$ and the pressure drop across the length of the stenosis and it may be expressed as;

$$(\Delta p)_0 == \frac{16Q_p\mu_p b}{\pi R_0^4 \delta^2 (2-\delta)^2} \tag{19}$$

Dividing equation (17) and (19), then we get dimensionless pressure drop for peripheral layer is;

$$K = \frac{(\Delta p)}{(\Delta p)_0} \tag{20}$$

Using equation (16) and (17) then we get wall shear stress in equation (20);

$$\tau_1 = \frac{4\mu_p Q_p}{\pi}\cdot\frac{R}{\left(R^2 - R_1^2\right)^2} \tag{21}$$

If ε is the thickness of the plasma layer and we assumed the thickness of the plasma is to be constant then equation (20) becomes;

$$\tau_\varepsilon = \frac{4\mu_p Q_p}{\pi}\cdot\frac{R}{\varepsilon^2(2R-\varepsilon)^2} \tag{22}$$

If there is no stenosis then;

$$\tau_{\varepsilon}^{*} = \frac{4\mu_p Q_p}{\pi}\cdot\frac{R_0}{\varepsilon^2\left(2R_0-\varepsilon\right)^2} \tag{23}$$

The shear stress on the stenosis surface is given by;

$$\tau = \frac{\tau_1}{\tau_\varepsilon} = \frac{R}{R_0}\cdot\frac{\varepsilon^2\left(2R_0-\varepsilon\right)}{\left(R^2-R_1^2\right)^2} \tag{24}$$

Results and Discussion

The main purpose of this analytical expression is to quantify the influence of the resistance to flow, wall shear stress and pressure drop. The variation of the wall shear stress and the pressure drop with stenotic height are shown graphically for different values of $\alpha=R_0/r_0$ through the Fig.(a) and Fig.(b). The figures show that there are initially no appreciable changes in the solution over a small range δ/R_0. If δ/R_0 increases, beyond this range and the solutions begin to turn upwards and at higher values of these solution increases rapidly. It is noticed that the thinning of the plasma layer with increasing height of the stenosis and the wall friction and it is associated with the pressure drop. The expression for resistance to flow and wall shear stress are plotted for different value of μ, , b/b_0 through the Fig. (c) and Fig.(d). It is seen from the graph that, if the height of the stenosis is increases the resistance to flow and wall shear stress are also increases. It is also noted from the graph that the resistance to flow and wall shear stress decreases as the peripheral layer viscosity decreases.

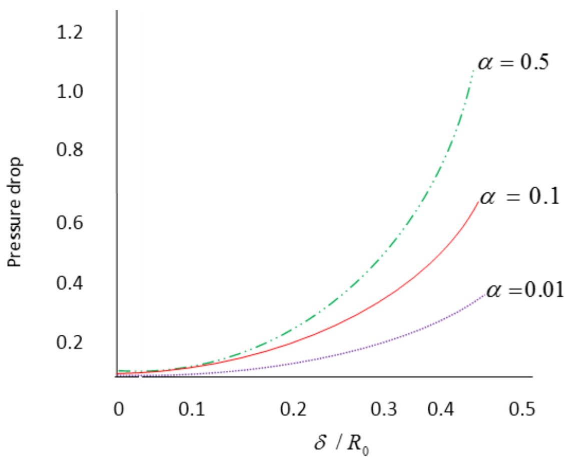

Fig.(a): Variation of the pressure drop with δ / R0 for different values of α

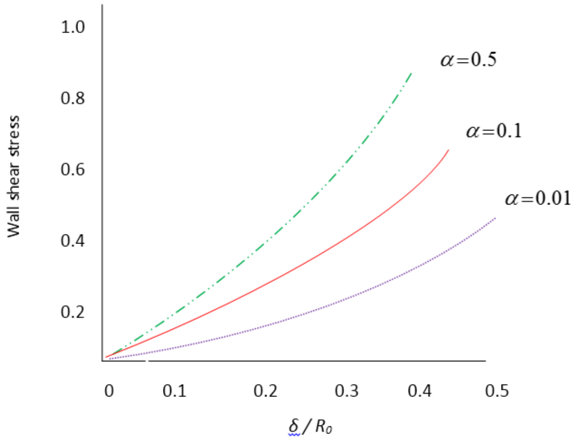

Fig.(b): Variation of the pressure drop with δ / R0 for different values of α

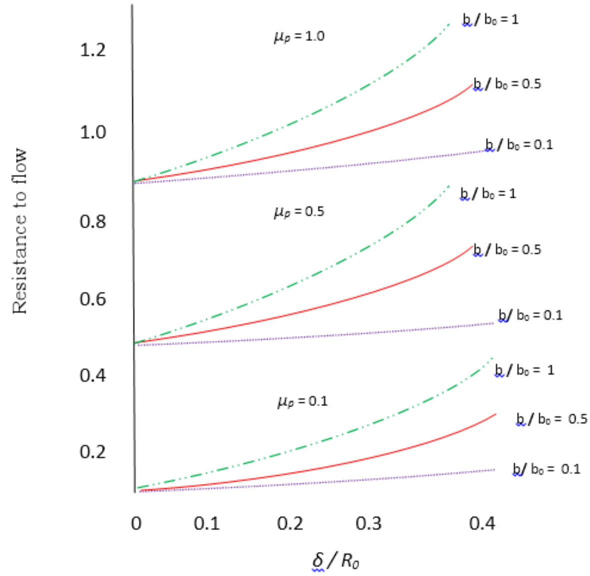

Fig.(c): Variation of the resistance to flow with δ / R0 for different values of μp and b /b0

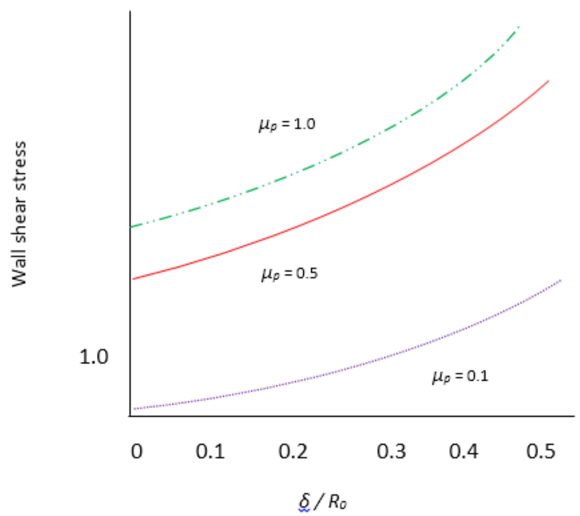

Fig.(d): Variation of the resistance to flow with δ / R0 for different values of μp and b / b0

Conclusion

A model for the effect of peripheral layers viscosity on the blood flow in the presence of mild stenosis in the lumen of the artery has been investigated in this chapter. This chapter consists of a core (red cell suspension) surrounded by a peripheral plasma layer. Both the fluids (core and peripheral layer) having different viscosities. It is concluded that the resistance to flow and wall shear stress decreases as the viscosity of the peripheral layer decreases but these characteristics increases as the size of the stenosis increases. Thus, it may be remarked that the presence of the peripheral layer helps in the functioning of the diseased arterial system.

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