Volume : 4 | Issue : 10 | Special Issue Oct- 2015 • ISSN No 2277 - 8179

Mathematical Models on Epidemiology

Mathematics

KEYWORDS: Epidemic Disease, Mathematical Modelling, Compartmental Model, Epidemiology.

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ABSTRACT

In the field of epidemiological, this work is describing the role of mathematical modelling and their future aspects in the field of epidemiology. Here, we are using the ordinary differential equations for different infectious diseases. Models have discussed various levels of diseases as susceptible – exposed – infective – recovered etc. with the concept of basic reproductive rate and equilibrium points and investigated their stability by using Rough's Stability criterion.

1. Introduction

In the history of epidemic diseases "Bubonic Plague" (Black Death) caused by the bacterium Yersinia pestis, devastated population in Europe from the 14th-16th centuries, killing between one-half to one-third of the entire population (Langer 1970) and according to WHO reports 1000-3000 cases of Bubonic Plague every year [1]. The dynamic models for infectious disease are mostly based on their compartment structure. The compartment structures for well – recognized SIR (Susceptible – Infective – Recovered) are firstly given by Kermack and Mc Kendrick in 1927 and are developed by many other bio-mathematicians in 1932 [2], [3]. Modified SIR model is providing for analysis of influenza infection with bacterial disease (Pneumonia). This model describe the proportion of the population to quarantine after being infected with influenza, and how better treatments of bacterial infections all would play into reducing the total number of deaths [4]. Dynamic of compartment model for epidemic is an important method of analysing the effect of infectious disease. It is based on the specific property of population growth, to define spread rule of infection disease, and the related factors etc. To construct mathematical models reflecting the dynamic properties of infection disease numerically, to analyse the dynamical behaviour and to do some simulations. The research result is helpful to predict the growth of infection disease, to determine the key factors of the spread of infection disease and to seek the optimum strategies of preventing and controlling the spread of infection diseases [5]. A various types of mathematical models have been used in mathematical epidemiology. By modifying the basic SIR model, we have obtained proposed infectious diseases compartmental ODE – models for further development in the future epidemiology.

2. Basic forms of Compartmental Models for various type of disease:

Definition of parameters of the model:
S = Susceptible Class;
I = Infective Class;
R = Recovered Class;
E = Exposed Class.

Parameters	Meaning	Dimension
μ	Recovery rate	Per year
β	Transmission rate	Per year
α	Exposed rate	Per year
δ	Recovery rate	Per year
t	Time	Per year

(I) SI Model

In this model, when the infective cannot be recovered from infection. It is given by following diagram:

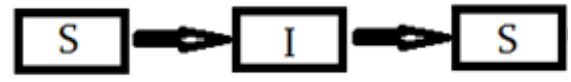
The governing equations are as follows:

$dS/dt = -\beta SI$ (i)

$dI/dt = \beta SI$ (ii)

(II) SIS Model

In this model, when the infective are recovered with no immunity. It is given by following diagram:



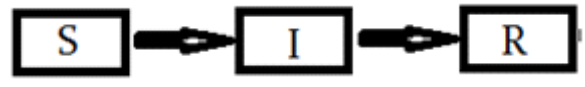
The governing equations are as follows:

$dS/dt = -\beta SI + \mu I$ (i)

$dI/dt = \beta SI - \mu I$ (ii)

(III) SIR Model

In this model, when the infective gain permanent immunity to the disease after recovered from infection. It is given by following diagram:



The governing equations are as follows:

$dS/dt = -\beta SI$ (i)

$dI/dt = \beta SI - \mu I$ (ii)

$dR/dt = \mu I$ (iii)

(IV) SIRS Model

In this model, when the recovered individuals have temporary immunity after they recovered from infection. It is given by following diagram:

The governing equations are as follows:

$dS/dt = -\boldsymbol{\beta} SI + \delta R$ (i)

$dI/dt = \beta SI - \mu I$ (ii)

$dR/dt = \mu I - \delta R$ (iii)

(V) SIRI Model

In this model, the infective cannot obtain permanent immunity to the disease after recovered from infection. It is given by following diagram:

The governing equations are as follows:

$dS/dt = -\beta SI$ (i)
$dI/dt = \beta SI - \mu I + R$ (ii)
$dR/dt = \mu I - \delta R$ (iii)

(VI) Model SEIRS

This model is providing the condition of re-susceptible of the mathematical dynamic models for the disease Influenza.

The governing equations of the model SEIRS are as follows:

$dS/dt = -\beta SI + \delta R$ (i)
$dE/dt = \beta SI - \alpha E$ (ii)
$dI/dt = \alpha E - \delta I$ (iii)
$dR/dt = \delta I - \delta R$ (iv)

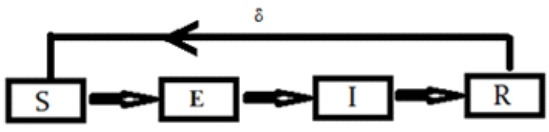


Figure: 3 SEIRS

3. Numerical Solution:

Model 1: General Solution of simultaneous differential equation of model SI is given by:

$S(t) = \frac{N(N+I_0)}{N+e^{(N+I_0)\beta t}}$ and $I(t) = \frac{(N+I_0)e^{(N+I_0)\beta t}}{N+e^{(N+I_0)\beta t}}$; Where $S + I = N + I_0$.

Here, we can obtain the numbers of susceptibility and infected person.

Model 2: General Solution of simultaneous differential equation of model SIS is given by:

$$S(t) = \frac{Ne^t - \mu}{e^t - \beta} \text{ and } I(t) = \frac{\mu - N\beta}{e^t + \beta}$$

The general solution provides the numbers of re-suspected person.

Model 3: General Solution of simultaneous differential equation of model SIR by Picard's method is given by:

$$S_n = S_0 + \int_{t_0}^{t} \{-\beta S_{n-1} I_{n-1}\} dt$$

$$I_n = I_0 + \int_{t_0}^{t} \{\beta S_{n-1} I_{n-1} - \mu I_{n-1}\} dt$$

$$R_n = R_0 + \int_{t_0}^{t} \{\mu I_{n-1}\} dt$$

Picard's method provides a unique solution with their existence.

Model 4: General Solution of simultaneous differential equation of model SIRS can solve by Jacobi method:

$$J(t) = \begin{pmatrix} -\beta & 0 & \delta \\ \beta & -\mu & 0 \\ 0 & \mu & -\delta \end{pmatrix}$$ and their eigen values are 0, $-\beta$ and $-(\delta+\mu)$.

The general solution provides the re-suspected cycle.

Model 5: General Solution of simultaneous differential equation of model SIRI is given by Jacobi method:

$$J(t) = \begin{pmatrix} -\beta & 0 & 0 \\ \beta & -\mu & \delta \\ 0 & \mu & -\delta \end{pmatrix}.$$

The general solution provides the re-infected cycle.

Model 6: General Solution of simultaneous differential equation of model SEIRS is given by Jacobi method:

$$J = \begin{pmatrix} -\beta I & 0 & -\beta S & \delta \\ \beta I & -\alpha & \beta S & 0 \\ 0 & \alpha & -\delta & 0 \\ 0 & 0 & \delta & -\delta \end{pmatrix}$$ and their eigen values are $\lambda(\delta+\lambda)$

$$[\lambda = \frac{-(\alpha+\delta) \pm \sqrt{[(\alpha+\delta)^2 - 4(\alpha\beta - \beta\rho\alpha)]}}{2}] = 0$$

With the help of Routh's stability method result is obtained and appears the condition of re-susceptible of the mathematical dynamic models for the disease Influenza.

4. Conclusion:

A wide range of mathematical models have been used in biological mathematical modelling on epidemiology. By modifying the basic SIR model, we have obtained useful result insight on the effect of a bacterial or viral infection during epidemic, endemic or pandemic. This modified SIR model can possibly allow decision makers to make a better decision on the preventing and controlling of the spread of infectious diseases and to determine the appropriate limitations for deadly complications with the help of MATLAB.

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