STATISTICAL ANALYSIS OF AN ATM: A QUEUING-SCIENCE PERSPECTIVE

Mathematics

KEYWORDS: Queuing system, ATM, Single server, Multiple Server.

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ABSTRACT

ATM which stands for "Automated Teller Machine" are an easy and convenient way to access your bank account anytime and from anywhere. Standing in long queues can cause irritation, displeasure. People are forced to leave without any service or to use another bank's ATM. This causes loss in customer. The systematic study of queuing system willguide the management of Bank in taking certain decisions that may improve the service quality and reduce losing customers. The data for this study was collected from an ATM in New Delhi. Data was collected by observation by recording number of customers arriving, arrival time, service time during 9A.M. to 7 P.M. for a week. The main purpose of the paper is to check whether it is beneficial for the bank to increase number of servers.

INTRODUCTION:

Queuing theory is the mathematical study of waiting lines or queue. The theory enables mathematical analysis and simulation of several related processes, including arriving at the queue, waiting in the queue and being served by the server(s) at the front of the queue. The theory permits the derivation and calculation of several performance measures including the average waiting time in the queue or the system, the expected number waiting or receiving service and the probability of encountering the system in certain states, such as empty, full, having an available server or having to wait a certain time to be served.

In everyday life, many situations arise where customers require some service, and where the service facility has limited capacity to supply this service. If a customer cannot immediately receive service, he/she may leave or wait.In the latter case, he/she joins a queue. Queues form because resources are limited. In fact it makes economic sense to have queues.

In real life, integral to queuing situations is the idea of uncertainty in, for example, inter arrival times and service times. This means that probability and statistics are needed to analyze queuing situations.
In terms of the analysis of queuing situations the types of questions in which we are interested are typically concerned with measures of system performance and might include:

- How long does a customer expect to wait in the queue before they are served, and how long will they have to wait before the service is complete?
- What is the average length of the queue?
- What is the probability that the queue will exceed a certain length?
- What is the probability of a customer having to wait longer than a given time interval before they are served?
- What is the expected utilization of the server and the expected time period during which he will be fully occupied (remember servers cost us money so we need to keep them busy). In fact if we can assign costs to factors such as customer waiting time and server idle time then we can investigate how to design a system at minimum total cost.

These are questions that need to be answered so that management can evaluate alternatives in an attempt to control/improve the situation. Some of the problems that are often investigated in practice are:
- Is it worthwhile to invest effort in reducing the service time?
- How many servers should be employed?
- Should priorities for certain types of customers be introduced?
- Is the waiting area for customers adequate?

This paper uses the queuing theory to study the optimization of a bank ATM having single server. The main aim of this paper is to study whether it is advisable to have second server installation.

Customers arrive in this ATM randomly. The data is collected for a week from Monday to Sunday in the time period of 9:00 A.M. to 7:00 P.M. We used M/M/1 model and M/M/s model (s being 2) to observe and analyze the situation.

ATM QUEUING MODEL

1. (M/M/1/FCFS/∞/∞):

Based on the observation on the current situation of ATM facility M/M/1 queuing model is employed. M/M/1 queuing model means that the arrival and service time are exponentially distributed

ASSUMPTIONS MADE ON THE SYSTEM

1. Poisson Arrival – Birth (Exponential Inter- Arrival time)
2. Poisson Departure –Death (Exponential Service time)
3. Single server
4. There is an infinite population from where customers can originate.
5. There is an infinite capacity.
6. The customers are served on First Come First Served Basis.

Let P_n = Probability of having n customers in the system during the steady state.
λ= Average Arrival Rate
μ= Average Service Rate
Equating the rate of inflow and rate of outflow from each state.
Moving from state n=0 to n=1 at the rate of λ and from n=1 to n=0 at the rate of μ.

$$P_1 = \left(\frac{\lambda}{\mu}\right) P_0 = \rho P_0$$

Where ρ is utilization factor.\

$$P_n = \rho^n P_0$$

The system at any time will be in either of states 0,1,2,...,n,... with probability $P_0, P_1, P_2, \ldots, P_n, \ldots$
Therefore,

$$\sum_{n=0}^{1} P_n = 1$$

$$\Rightarrow \sum_{n=0}^{1} \rho^n P_0 = 1$$

$$\Rightarrow P_0 \sum_{n=0}^{1} \rho^n = 1$$

$$\Rightarrow P_0\left(\frac{1}{1-\rho}\right) = 1$$

$$\Rightarrow P_0 = 1-\rho$$

$$\Rightarrow P_n = \rho^n(1-\rho)$$

(1) L_q= Expected number of customers in queue

$$= \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\rho^2}{1-\rho}$$

(2) L_s= Expected number of customers in system (waiting + being served)

$$= \frac{\lambda}{\mu-\lambda} = \frac{\rho}{1-\rho}$$

(3) W_q= Expected time spent by a customer in queue

$$= \frac{\lambda}{\lambda(\mu-\lambda)} = \frac{L_q}{\lambda}$$

(4) W_s= Expected time spent by a customer in system (waiting time + service time)

$$= \frac{1}{\mu-\lambda} = \frac{L_s}{\lambda}$$

(5) Probability that a queue is non-empty = P(atleast 1 customer in queue) =
P(atleast 2 or more customers in system) = $P_2+P_3+\cdots$
$=1-(P_0+P_1)= 1- P_0- \rho P_0 = 1-(1+\rho)(1-\rho)=1-(1-\rho^2)= \rho^2$

(6) L_n= Average length of non-empty queue= $\frac{1}{1-\rho} = \frac{\mu}{\mu-\lambda}$

(7) W_n= Average time of a customer in non-empty queue = $\frac{1}{\mu-\lambda} = \frac{1}{\mu(1-\rho)} = \frac{L_n}{\mu}$

2. (M/M/s/FCFS/∞/∞):
Queuing sysytem in which queues are served by s parallel service channels where each server is independently and identically distributed exponential service time distribution and arrival process is poison.

Mean arrival rate $\lambda_n=\lambda$ for all n.
For mean service rate, if there are more than s customers in the system then all the servers will be busy with mean rate μ &
Thus, Service rate = $s\mu$
Here, s=2

Thus, Service rate $\begin{cases} \mu, & if\ one\ customer \\ 2\mu, & if\ more\ than\ one\ customer \end{cases}$

$$\rho = \frac{\lambda}{s\mu} < 1$$

$$P_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{(s)!}\left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{(s\mu-\lambda)}}$$

(1) L_q= Expected number of customers in queue

$$= \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\lambda\mu}{(s\mu-\lambda)^2}\right]P_0$$

(2) L_s= Expected number of customers in system (waiting + being served)

$$= L_q + \frac{\lambda}{\mu}$$

(3) W_q= Expected time spent by a customer in queue

$$= \frac{L_q}{\lambda}$$

(4) W_s= Expected time spent by a customer in system (waiting time + service time)

$$= \frac{L_s}{\lambda}$$

(5) L_n= Expected number of customers waiting to be served at any time 't' is= $\frac{s\mu}{s\mu-\lambda}$

(6) W_n= Average time of a customer in non-empty queue = $\frac{L_n}{s\mu}$

COST MODEL
Let us assume the total cost for the ATM with s number of servers be the sum of operating operating cost (including cost of installation, servicing, mantainence e.t.c.) and waiting cost (either indirect cost or loss of customers).

$$C=C_s+ C_w$$

Where,
C=Total cost
C_s= Cost of operating s ATM's
C_w= Waiting cost
For single server ATM

$$C=C_s+ L_s \times C_w$$

Where, L_s= Expected number of customers in system (waiting + being served)
For 's' server ATM

$$C=s\times C_s+ L_s \times C_w$$

Where,L_s= Expected number of customers in system (waiting + being served)

EXPERIMENTAL REULTS

TABLE – 1 (Customer Count)

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
136	170	210	137	154	183	122

CALCULATIONS

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
λ	λ_1=0.26667	λ_2=0.28333	λ_3=0.35	λ_4=0.28333	λ_5=0.25667	λ_6=0.305	λ_7=0.2033
μ	μ_1=0.52312	μ_2=0.56044	μ_3=0.64864	μ_4=0.51612	μ_5=0.51499	μ_6=0.58167	μ_7=0.49326

Average arrival rate =λ=0.27047 customer per minute
Average service rate =μ=0.5442 customer per minute

$$\rho = \frac{\lambda}{\mu} = 0.497 < 1$$

For Single Server ATM
(1) L_q=0.49107
(2) L_s=0.98807
(3) W_q=1.851562 minutes

(4) W_s=3.65316 minutes
(5) Probability that a queue is non-empty = 0.2479
(6) L_n=1.98807
(7) W_n=3.6532

For 2 servers ATM
Average arrival rate =λ=0.27047 customer per minute
Average service rate =μ=0.5442 customer per minute

$$\rho = \frac{\lambda}{2\mu} = 0.2485 < 1$$

P_0=0.6019
(1) L_q= 0.032711
(2) L_s=0.529711
(3) W_q=0.036357
(4) W_s=1.95848
(5) L_n=1.330676
(6) W_n=1.222598

C_s= Cost of operating s ATM's
C_w= Waiting cost

For single server ATM

$C=C_s+ L_s \times C_w$

Where, L_s= Expected number of customers in system (waiting + being served)

For 's' server ATM

$$C=s \times C_s+ L_s \times C_w$$

On calculation with Cost of operating each ATM is Rs. 20,000/- per month and the waiting cost for each server is Rs. 10,500 the total cost is Rs. 30,374.735 for single server and total cost is Rs.45,561.9655.

CONCLUSION:
This research paper has evaluated using the queuing theory for the ATMs in bank whether increasing the number of server will benefit bank in Quality of Service or not. As calculated though the cost for single server is less than cost for double server but by having the server increased length of queue is decreased and waiting time is reduced. This will help bank in retaining it's customers and it will help bank service better.

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