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MHD EFFECT ON UNSTEADY VISCOUS INCOMPERSSIBLE FLUID FLOW THROUGH POROUS MEDIUM WITH HALL CURRENT

Mathematics

KEYWORDS: MHD, Hall Current, Porous Medium, Radiation, Chemical reaction, Unsteady.

VINEET KUMAR SHARMA — Deptt. of Mathematics, Rawal Institute of Engineering and Technology, Faridabad (INDIA)

NISHA K.R. — Deptt. of Mathematics, Rawal Institute of Engineering and Technology, Faridabad (INDIA)

ABSTRACT

The present paper investigates the MHD effects on unsteady, viscous, incompressible, electrically conducting fluid flow through porous medium along a semi- infinite vertical plate in the presence of radiation and chemical reaction with hall current. The equations of motion are transformed into non-dimensional equations and solved with the help of perturbation technique. The results have been discussed through table and graphs. Expressions for the velocity distribution, temperature distribution, concentration distribution, Shear stress, Nusselt number and Sherwood number are obtained.

Introduction

MHD flow of viscous incompressible fluid plays important role in different areas of applied science & engineering such as oil exploration industry, plasma physics, blood flow, complex pumping station, chemical industry and so on.

Chamkha [1] investigated unsteady MHD convective heat and mass transfer past a semi- infinite vertical permeable moving plate with heat absorption. Chauhan and Kumar[2] discussed the effects of slip conditions on forced convection and entropy generation in a circular channel occupied by a highly porous medium: Darcy extended Brinkman-Forchheimer model. Prakash and Ogulu [3] investigated unsteady two dimensional flow of a radiating and chemically reacting MHD fluid with time-dependent suction. Singh and Gupta [4] considered MHD free convective flow of viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer. Bhargava et. al. [5] investigated finite element solutions for non-Newtonian pulsatile flow in a non-Darcian porous medium conduit. Reddy and Reddy [6] studied Unsteady MHD convective heat and mass transfer past a semi-infinite vertical porous plate with variable viscosity and thermal conductivity. Krishnambal and Anuradha [7] discussed effect of radiation on the flow of a viscoelastic fluid and heat transfer in a porous medium over a stretching sheet. Satya et. Al [8] considered Effects of Hall current and radiation absorption on MHD micropolar fluid in a rotating system. Jen and Yan [9] dicussed developing fluid flow and heat transfer in a channel partially filled with porous medium. Venkateswarlu and Satya [10] dicussed chemical reaction and radiation absorption effects on the flow and heat transfer of a nanofluid in a rotating system. Vyas and Ranjan [11] discussed the dissipative MHD boundary layer flow in a porous medium over a sheet stretching nonlinearly in the presence of radiation. Mishra, et. al [12] investigated unsteady viscous fluid flow with porous medium in the presence of radiation and chemical reaction, Chinese Journal of Physics VOL. 52, NO. 1-I February

In the present study we investigated the MHD effect with chemical reaction and thermal radiation on a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate in the presence of Hall current.

Problem Formulation

We considers the problem of MHD fluid flow with hall current on unsteady, laminar, boundary layer flow of a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate in the presence of uniform porous medium with chemical reaction and thermal radiation. A uniform magnetic field of strength B0 is applied normal to the flow of fluid. A time-dependent suction is assumed and governing equations are:

$$\frac{\partial v'}{\partial y'} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta(T-T_\infty) + g\beta'(C-C_\infty) - \frac{\nu}{K'}u' - \frac{\sigma}{\rho(1+m^2)}B_0^2 u' \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\left(\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y'}\right) = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y'^2} + \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\frac{\partial C}{\partial t'} + v'\frac{\partial C}{\partial y'} = D\frac{\partial^2 C}{\partial y'^2} - k'_r(C-C_\infty) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

By using Rosseland approximination for the radiation, we take

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y'} \qquad \ldots\ldots\ldots\ldots\ldots (5)$$

Where T and C are dimensional temperature and concentration, u and v are velocity in the direction of x and y, x and y are cartesian coordinates, g is acceleration due to gravity, ρ is density of the fluid, σ is electrical conductivity, β and   is coefficient of volume expansion due to temperature and concentration, C_p is specific heat at constant pressure, k - Thermal conductivity, t - Time, B_0 - magnetic induction, D is Molar diffusivity

T^4 can be expanded in a Taylor series about the free stream temperature T∞ so that after rejecting higher order terms we have

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \qquad \ldots\ldots\ldots\ldots\ldots (6)$$

The energy equation after substitution of equations (5) and (6) can now be written as

$$\left(\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y'}\right) = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y'^2} + \frac{16\sigma^* T_\infty^3}{3\rho C_p k^*}\frac{\partial^2 T}{\partial y'^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

From equation (1) we can see that the suction is a function of time only so we assume it in the form,

$$v' = -U_0\left(1+\varepsilon A e^{n't'}\right) \quad \text{.....................} (8)$$

$\varepsilon A << 1$, the minus sign indicates that the suction is towards the plane. It is now convenient to introduce the following dimensionless parameters:

$$u=\frac{u'}{U_0},\quad y=\frac{U_0}{\nu}y',\quad t=\frac{U_0^2}{\nu}t',\quad n=\frac{\nu n'}{U_0^2},$$

$$k_r=\frac{k_r'\nu}{U_0^2},\ \theta=\frac{T-T_\infty}{T_w-T_\infty}\quad \phi=\frac{C-C_\infty}{C_w-C_\infty}$$

$$P_r=\frac{\rho\nu C_p}{k}\ \text{(Prandtl Number)}$$

$$S_c=\frac{\nu}{D}\ \text{(SchmidtNumber)},$$

$$G_r=\frac{\nu g\beta\left(T_w-T_\infty\right)}{U_0^3}\ \text{(Grashof Number)}$$

$$G_m=\frac{\nu g\beta'\left(C_w-C_\infty\right)}{U_0^3}\ \text{(Modified Grashof Number)}$$

$$N_R=\frac{16\rho' T_\infty^3}{3k'k}\ \text{(Radiation Parameter)}$$

$$K=\frac{K'U_0^2}{\nu^2}\ \text{(Porosity Parameter)}$$

$$M=\frac{\sigma B_0^2\nu^2}{\rho U_0^2}\ \text{(Hartman Number)}$$

$$M_1=\frac{M}{1+m^2}$$

Where
U0 is Mean velocity, m is Hall current Parameter, A is Suction Parameter, n is constant exponential index, w is conditions at the wall, is Free stream conditions primes denote dimensional quantities

On substitution these into equation (2), (4) and (7), our governing equations become:

$$\frac{\partial u}{\partial t}-\left(1+\varepsilon A e^{nt}\right)\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+G_r\theta+G_r\phi-\left(M_1+\frac{1}{K}\right)u \quad \text{.....................} (9)$$

$$\frac{\partial \theta}{\partial t}-\left(1+\varepsilon A e^{nt}\right)\frac{\partial \theta}{\partial y}=\left(\frac{1+N_R}{P_r}\right)\frac{\partial^2\theta}{\partial y^2} \quad \text{.....................} (10)$$

$$\frac{\partial \phi}{\partial t}-\left(1+\varepsilon A e^{nt}\right)\frac{\partial \phi}{\partial y}=\frac{1}{S_c}\frac{\partial^2\theta}{\partial y^2}-k_r\phi \quad \text{.....................} (11)$$

With the boundary conditions

$$\left.\begin{array}{ll} u=1,\theta=1+\varepsilon e^{nt},\phi=1+\varepsilon e^{nt} & \text{at}\ \ y=0 \\ u\to U_0,\theta\to 0,\phi\to 0 & \text{as}\ \ y\to\infty \end{array}\right] \quad \text{.....................} (12)$$

Analyses

Since <<<1 so let us assume u, **θ**, f as

$$\left.\begin{array}{l} u(y,t)=u_0(y)+\varepsilon e^{nt}u_1(y)+0(\varepsilon^2) \\ \theta(y,t)=\theta_0(y)+\varepsilon e^{nt}\theta_1(y)+0(\varepsilon^2) \\ \phi(y,t)=\phi_0(y)+\varepsilon e^{nt}\phi_1(y)+0(\varepsilon^2) \end{array}\right] \quad \text{.....................} (13)$$

We now substitute eq. (13) into eqs. (10) - (12) and equating the non-harmonic terms and

Harmonic terms neglecting higher order terms in ε, to obtain:

$$u_0''+u_0'-\left(M_1+\frac{1}{K}\right)u_0=-G_r\theta_0-G_m\phi_0 \quad \text{.....................} (14)$$

$$u_1''+u_1'-\left(M_1+\frac{1}{K}+n\right)u_1=-Au_0'-G_r\theta_1-G_m\phi_1 \quad \text{.....................} (15)$$

$$\theta_0''+h\theta_0'=0 \quad \text{.....................} (16)$$

$$\theta_1''+h\theta_1'-nh\theta_1=-Ah\theta_0' \quad \text{.....................} (17)$$

$$\phi_0''+S_c\phi_0'-k_rS_c\phi_0=0 \quad \text{.....................} (18)$$

$$\phi_1''+S_c\phi_1'-\left(k_r+n\right)S_c\phi_1=-AS_c\phi_0' \quad \text{.....................} (19)$$

Where $h=\dfrac{P_r}{1+N_R}$ and primes denote differentiation with respect to y. Now the boundary condition are:

$$\left.\begin{array}{ll} \left.\begin{array}{l} u_0=1,u_1=0,\theta_0=1=\theta_1, \\ \phi_0=1=\phi_1 \end{array}\right\} & \text{at}\ \ y=0 \\ \left.\begin{array}{l} u_0\to 0,u_1\to 0,\theta_0\to 0,\theta_1\to 0, \\ \phi_0\to 0,\phi_1\to 0 \end{array}\right\} & \text{as}\ \ y\to\infty \end{array}\right] \quad \text{.....................} (20)$$

Integrating (15) to (20) subject to the conditions in equations (21) we have

$$u_0 = C_3 e^{-m_1 y} - Z_1 e^{-hy} - Z_2 e^{-m_4 y} \quad \ldots\ldots\ldots\ldots (21)$$

$$u_1 = C_4 e^{-m_2 y} + Z_3 e^{-m_1 y} - Z_4 e^{-hy} - Z_5 e^{-m_4 y} - Z_6 e^{-m_3 y} - Z_7 e^{-m_5 y} \quad \ldots\ldots\ldots\ldots (22)$$

$$\theta_0 = e^{-hy} \quad \ldots\ldots\ldots\ldots (23)$$

$$\theta_1 = C_2 e^{-m_3 y} - \frac{Ahe^{-hy}}{n} \quad \ldots\ldots\ldots\ldots (24)$$

$$\phi_0 = e^{-m_4 y} \quad \ldots\ldots\ldots\ldots (25)$$

$$\phi_1 = C_1 e^{-m_5 y} + \frac{AS_c m_4 e^{-m_4 y}}{m_4^2 + S_c m_4 - (k_r + n) S_c} \quad \ldots\ldots\ldots\ldots (26)$$

Where

$$m_1 = \frac{1 + \sqrt{1 + 4\left(M_1 + \frac{1}{K}\right)}}{2},$$

$$m_2 = \frac{1 + \sqrt{1 + 4\left(M_1 + \frac{1}{K} + n\right)}}{2},$$

$$m_3 = \frac{h + \sqrt{h^2 + 4nh}}{2}, m_4 = \frac{S_c + \sqrt{S_c^2 + 4S_c k_r}}{2},$$

$$m_5 = \frac{S_c + \sqrt{S_c^2 + 4S_c (k_r + n)}}{2},$$

$$C_1 = 1 - \frac{AS_c m_4}{m_4^2 + S_c m_4 - (k_r + n) S_c},$$

$$C_2 = 1 - \frac{Ahe^{-hy}}{n}, Z_1 = \frac{G_r}{h^2 - h - \left(M_1 + \frac{1}{K}\right)},$$

$$Z_2 = \frac{G_m}{m_4^2 - m_4 - \left(M_1 + \frac{1}{K}\right)}, C_3 = 1 + Z_1 + Z_2,$$

$$Z_3 = \frac{Am_1 C_1}{m_1^2 - m_1 - \left(M_1 + \frac{1}{K} + n\right)},$$

$$Z_4 = \frac{\left(AhZ_1 + \frac{Ah}{n}\right)}{h^2 - h - \left(M_1 + \frac{1}{K} + n\right)},$$

$$Z_5 = \left[\left(Am_4 Z_2 + \frac{AS_c m_4}{m_4^2 - m_4 - (k_r + n) S_c}\right) \times \frac{1}{m_4^2 - m_4 - \left(M_1 + \frac{1}{K} + n\right)}\right]$$

$$Z_6 = \frac{C_3 m_3}{m_3^2 - m_3 - \left(M_1 + \frac{1}{K} + n\right)},$$

$$Z_7 = \frac{G_m C_1}{m_5^2 - m_5 - \left(M_1 + \frac{1}{K} + n\right)}$$

$$C_4 = -Z_3 + Z_4 + Z_5 + Z_6 + Z_7$$

Such that the velocity, temperature and concentration distributions can be expressed as

$$u(y,t) = C_3 e^{-m_1 y} - Z_1 e^{-hy} - Z_2 e^{-m_4 y} + \varepsilon e^{nt} \begin{Bmatrix} C_4 e^{-m_2 y} + Z_3 e^{-m_1 y} - Z_4 e^{-hy} \\ -Z_5 e^{-m_4 y} - Z_6 e^{-m_3 y} - Z_7 e^{-m_5 y} \end{Bmatrix} \quad \ldots\ldots\ldots\ldots (27)$$

$$\theta(y,t) = e^{-hy} + \varepsilon e^{nt} \left\{ C_2 e^{-m_3 y} - \frac{Ahe^{-hy}}{n} \right\} \quad \ldots\ldots\ldots\ldots (28)$$

$$\phi(y,t) = e^{-m_4 y} + \varepsilon e^{nt} \begin{Bmatrix} C_1 e^{-m_5 y} \\ + \frac{AS_c m_4 e^{-m_4 y}}{m_4^2 + S_c m_4 - (k_r + n) S_c} \end{Bmatrix} \quad \ldots\ldots\ldots\ldots (29)$$

Shear stress, Nusselt number, Sherwood number which can be defined respectively in non-dimensions as:

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = -m_1 C_3 + hZ_1 + m_4 Z_2 + \varepsilon e^{nt} \begin{Bmatrix} -m_2 C_4 - m_1 Z_3 + \\ hZ_4 + m_4 Z_5 \\ + m_3 Z_6 + m_5 Z_7 \end{Bmatrix} \quad \ldots\ldots (30)$$

$$\frac{N_u}{Re_x} = \left.\frac{\partial \theta}{\partial y}\right|_{y=0} = -h + \varepsilon e^{nt} \left\{ -m_3 C_2 + \frac{Ah^2}{n} \right\} \quad \ldots\ldots\ldots\ldots (31)$$

$$\frac{Sh}{Re_x} = \left.\frac{\partial \phi}{\partial y}\right|_{y=0}$$

$$= -m_4 - \varepsilon e^{nt} \left\{ \begin{array}{l} m_5 C_1 \\ + \dfrac{A S_c m_4^2}{m_4^2 + S_c m_4 - (k_r + n) S_c} \end{array} \right\} \quad \ldots\ldots\ldots\ldots (32)$$

Result and Discussion:

Variation in velocity distribution u for viscous fluid in unsteady flow is shown in Tables from I & Fig - I having Graph-1 to 5 at n = 0.1, t = 0.1, $P_r = 0.71$, $N_R = 0.5$, $S_c = 0.2$, $k_r = 0.5$, e = 0.001 and A = 0.3, with different values of M, m, G_r and G^m.

Graph	M	m	G_r	G_m
I	0.2	1	2.5	3.0
II	1.5	1	2.5	3.0
III	0.2	2	2.5	3.0
IV	0.2	1	3.0	3.0
V	0.2	1	2.5	3.5

It is noticed that from table – 1 and figure -1 that all velocity graphs are increasing sharply up to y =1.1 after that velocity in each graph begins to decrease and tends to zero with the increasing in y. It is also observed from figure -I that velocity increases with the increase in G_r, and G_m. Comparing the graph I & II it is found that the velocity increases gradually till y = 2.0 after it velocity decreases gradually. Comparing the graph I & III it is noticed that the velocity increases gradually till y = 1.1 after it velocity decreases gradually.

Table No. - I

y	Graph - 1	Graph - 2	Graph - 3	Graph - 4	Graph - 5
0	0.958778	0.958862	0.957549	0.958773	0.94806
1	7.703742	8.634181	7.447087	7.806434	9.047229
2	7.103314	7.242321	6.956801	7.265254	8.447425
3	5.160208	4.998233	5.097068	5.356325	6.224809
4	3.419433	3.273663	3.387979	3.635261	4.20936

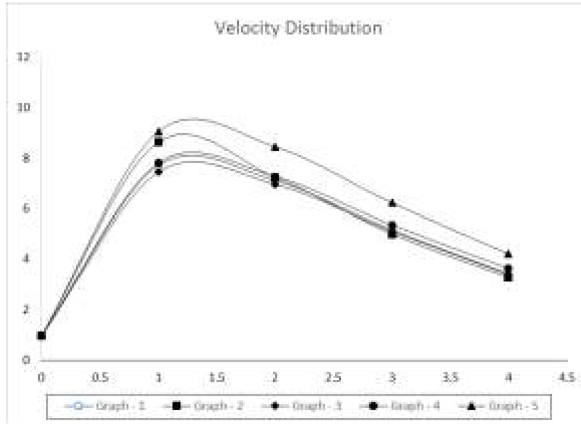

Fig.-1

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