Research Paper Mathematics

A Mathematical Version of Cournot's Oligopoly for N Firms

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ABSTRACT

Monopoly and Perfect Competition in general are two types of market exist in several countries of world. But after Economical reform there exist a new kind of market in India know as Oligopoly, which is neither perfect competition nor monopoly. The term Oligopoly comes from two Greek words: Oligoi meaning "few"and poleein meaning "to sell". Oligopoly market refers to a kind of market mechanism that only a few manufacturers control production and vendition of product in entire market .The perfect example of oligopoly in India is Petroleum companies, they follows Cournot's model. In this paper the original cournot model explained with the help of Mathematical expression. Second part of the paper is a mathematical model for n firms in oligopoly market. Third part of paper explained the need of new model for Indian market. The conclusions have certain academic guiding sense and practical application value. At last, some suggestions for government to control the price of petroleum products and for companies about their product and profits.

KEYWORDS Oligopoly, Duopoly, Nesh Equilibrium.

1. Introduction

The main market structures include perfect competition (many buyers and sellers), oligopoly (few sellers and many buyers), monopoly (single seller and many buyers) and monopsony (single buyer and many sellers). One of the main distinguishing factors that we have identified between the different market structures is the level of barriers to entry. Market entry barriers are relevant when considering new market entry (Niu, Dong & Chen, 2011). Market barriers can adversely affect the speed and extent of entry into a new market (Gruca & Sudharshan, 1995; Varadarajan & Peterson, 1992). The main barriers to entry include Cost advantages of incumbents, Product differentiation of incumbents, Capital requirements, Customer switching costs, Access to distribution channels, Government policy (Porter, 1980; Johnson, Scholes & Whittington, 2011). It is very important to understand country characteristics and other environmental issues before selecting the right entry mode for a new market. Another problem that might arise from not selecting the appropriate entry mode strategy is a substantial limitation of strategic options open to the firm as well as a blockage of opportunities, (Ekeledo & Sivakumar 2004; Alderson 1957). The works of such scholars raise certain questions such as: What factors affect entry mode decisions? What barriers commonly exist in oligopolistic market situations? Which entry modes can a firm use to enter an oligopolistic market situation? How sufficient is the information available to companies contemplating entry into a new market for making comparisons between entry modes and how accessible is this information?

In a market situation where a few suppliers dominate the market, also known as an oligopolistic market, a firm must respond to their rival's choices and rivals would also respond to the firm's choices. In an oligopolistic market situation, there is tension between cooperation and self interest. Thus, entry and survival might be different from a more open market. Firms in an oligopolistic market situation would obviously want to protect their market share and thus would act in ways so as to make entry into their market difficult (Karakaya & Stahl, 1989. It is therefore interesting to find out the most appropriate entry mode that a firm should use when entering an oligopolistic market situation, factors that may influence the choice for entry mode and the challenges that such a firm would face due to barriers.

Basar and Ho [l] consider a duopoly model with quadratic cost functions. They show existence and uniqueness of affine equilibrium strategies and that, in equilibrium, expected profits of firm i increase with the precision of its information and decrease with the precision of the rival's information.Clarke [2] considers an n-firm oligopoly model and shows that there is never a mutual incentive for all firms in the industry to share information unless they may cooperate on strategy once information has been shared. Harris and Lewis [6] consider a duopoly model where firms in period one decide on plant capacity before market conditions are known. In period two they choose a level of production contingent on the state of demand and their plant size. They argue that observed differences in firm size and market share may be explained by producers having access to different information at the time of their investment decisions.

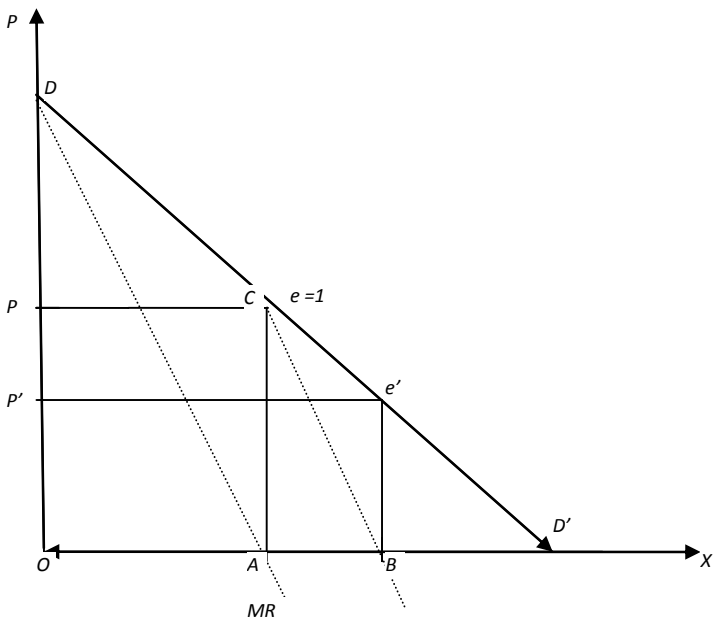
Gal-Or [5] considers an oligopoly model with two stages. At the first firms observe a private signal and decide whether to reveal it to other firms and how partial this revelation will be. At the second, they choose the level of output. They shows that no information sharing is the unique Nash equilibrium of the game both when private signals are completely uncorrelated and when they are perfectly correlated.

In our model Cournot competition with a homogenous product is a particular case. Our findings for this case are consistent with those of the authors who use the Normal model. The demand structure (with no uncertainty) we consider is a symmetric version of a duopoly model proposed by Dixit [4] the duality and welfare properties of which are analyzed in Singh and Vives [13].

2. Cournot's Model

The Model may present many ways but in the original version, it makes the assumption that the two firms have identical product and cost. Cournot in his model takes two firms owning a spring of mineral water, which is produced at zero cost. Here we will present briefly the same version and then we will genelise it to n firm by using mathematical equations.

Cournot assumed that there are two firms each owning a mineral well and operating with zero cost. In today's word which is not possible. we are explaining only his Ideas. The firm sells their out put in a market with a straight line demand curve. The firm assumes that its competitor will not change its out put and decides its own out put to maximize profit.



Assume that firm X_1 is first to start producing and selling mineral water. It will produce quantity *A* at price *P* where profits are maximum and because *MR=MC=0* firm has maximum profit at this point. Now firm X_2 assume that X_1 will keep its output fixed (At *OA*) and hence considers that its own demand curve is CD'. Clearly firm X_2 will produce half the quantity AD' because at this level AB of output is revenue and profit is at a maximum. Firm X_2 produce half of the market which has not been supplied by X_1. Output of $X_2 = \frac{1}{2}.\frac{1}{2} = \frac{1}{4}$ of total market.

For Next Period , firm X_1 assume that X_2 will retain his quantity constant in the next period. So he will produce one half of the market which is not supplied by X_2, i.e $\frac{1}{2}(1-\frac{1}{4}) = \frac{3}{8}$ of total market. Similarly form X_2 assume the same and produce ½(1-3/8)=5/16 of total market.

For Third period , In third period firm X_1 will continue to assume that X_2 will not change its quantity and thus will produce one half of the remainder of the market i.e $\frac{1}{2}(1-\frac{5}{16})=\frac{11}{32}$ and so on we will reached at equilibrium in which each firm produce 1/3 of the total market. To gather they cover the two thirds of the total market. Each firm maximizes its profit in each period but the industry profit are not maximized. That is , the firms would have higher joint profits if the recognized their interdependence , after their failure in forecasting the correct reaction of their rival. Recognition of their interdependence would lead them to act as 'a monopolist', producing one half of the total market output, selling it at the profit- maximizing price *P*, and sharing the market equally, that is, each producing one-quarter of the total market(instead of one-third).

1. The production of firm X_1 in successive periods is ;

Period 1 : $\frac{1}{2}$

Period 2 : $\frac{1}{2}(1-\frac{1}{4})=\frac{3}{8}=\frac{1}{2}-\frac{1}{8}$

Period 3 : $\frac{1}{2}(1-\frac{5}{16})=\frac{11}{32}=\frac{1}{2}-\frac{1}{8}-\frac{1}{32}$

Period 4 : $\frac{1}{2}(1-\frac{42}{128})=\frac{43}{128}=\frac{1}{2}-\frac{1}{8}-\frac{1}{32}-\frac{1}{128}$

We observe that the out put of X_1 decline gradually. We rewrite this expression as follows

Product of X_1 in equilibrium $=\frac{1}{2}-\frac{1}{8}-\frac{1}{32}-\frac{1}{128}\ldots$

$$=\frac{1}{2}-[\frac{1}{8}+\frac{1}{8}\cdot\frac{1}{4}+\frac{1}{8}\cdot\left(\frac{1}{4}\right)^2+\frac{1}{8}\cdot\left(\frac{1}{4}\right)^3\ \ldots.]$$

The expression in the brackets is a declining geometric progression with ratio $r=\frac{1}{4}$ using sum to infinity on geometric series

i.e $\frac{a}{1-r}$. we got $\frac{1}{2}-\frac{\frac{1}{8}}{1-\frac{1}{4}}=\frac{1}{2}-\frac{\frac{1}{8}}{\frac{3}{4}}=\frac{1}{2}-\frac{4}{24}=\frac{8}{24}=\frac{1}{3}$

2. The production for the firm X_2 in successive periods is

Period 2 : $\frac{1}{2}\cdot\left(\frac{1}{2}\right)=\frac{1}{4}$

Period 3 : $\frac{1}{2}(1-\frac{3}{8}) = \frac{5}{16} = \frac{1}{4}+\frac{1}{16}$

Period 4 : $\frac{1}{2}(1-\frac{11}{32}) = \frac{21}{64} = \frac{1}{4}+\frac{1}{16}+\frac{1}{64}$

Period 5 : $\frac{1}{2}(1-\frac{43}{128}) = \frac{85}{256} = \frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}$

We observe that X_2 's production increases, but at a declining rate. It may be written as

Product of X_2 in equilibrium = $\frac{1}{4}+\frac{1}{4}\cdot\left(\frac{1}{4}\right)+\frac{1}{4}\cdot\left(\frac{1}{4}\right)^2+\frac{1}{4}\cdot\left(\frac{1}{4}\right)^3 . \ . \ .$

The expression in the brackets is a declining geometric progression with ratio $r = \frac{1}{4}$ using sum to infinity on geometric series

we got $\frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

Thus the cournot solution is stable. Each firm supplies 1/3 of the market, at a common price which is lower than the monopoly price, but above the pure competitive price . It can be shown that if there are three firms in the industry, each will produce one-quarter of the market and all of them together will supply $\frac{3}{4}$ of the entire market OD' .And in general, if there are n firms in the industry each will provide $\frac{1}{n+1}$ of the market, and the industry output will be $\frac{n}{n+1}$.

3. Cournot's Oligopoly with Any N Number of Firms.

Consider the output of firm i ; x_i

Total Out put ; $X = x_1 + x_2 + x_3 + \ldots + x_n$

Out put of firm y_i (i.e opponent firm) = $X - x_i$

Constant marginal cost of Firm i ; c_i

Let inverse demand function ; $p(x)$.

Profits of firm i ;

$$\pi_i(y_i, x_i) = p(x)\, x_i - c_i x_i \qquad \ldots(1)$$

or $\pi_i(y_i, x_i) = p(x_i + y_i)\, x_i - c_i x_i$

For maximum profit

$$\frac{\partial \pi_i}{\partial x_i} = \frac{\partial p}{\partial x_i} x_i + p - c_i = 0 \qquad \ldots(2)$$

Solution defines reaction curve $x_i = g(y_i)$ which is often decreasing.

Here we consider Linear case ;

Let $p = A - BX$ *i.e* $p = A - B(x_i + y_i)$

$$\frac{\partial p}{\partial x_i} = -B$$

From (2)

$$-Bx_i + A - BX - c_i = 0 \qquad \ldots(3)$$

$$-Bx_i + A - B\,(x_i + y_i) - c_i = 0$$

$$-2Bx_i + A - B\,y_i - c_i = 0$$

$$A - B\,y_i - c_i = 2Bx_i$$

$$x_i = \frac{A - c_i}{2B} - \frac{1}{2} y_i \quad \text{as } x_i = g(y_i)$$

Now Cournot's Nesh Equilibrium;

1. Each firm maximizes profit given his/ her expectation of y_i

2. The expectation is correct.

This yields the simultaneous system of equations

$$x_i = g_i(y_i)$$

for all $i = 1, 2, 3, \ldots n$ where $x_i + y_i = X$

Now from (3)

$$-Bx_1 + (A - BX) - c_1 = 0$$

$$-Bx_2+(A-BX)-c_2=0$$

$$-Bx_3+(A-BX)-c_3=0$$

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$$-Bx_n+(A-BX)-c_n=0$$

On summations

$$-B(x_1+x_2+x_3\ldots+x_n)+n(A-BX)-n\bar{c}=0 \quad \text{where} \quad \bar{c}=\frac{c_1+c_2+c_3+\ldots c_n}{n}$$

$$-BX+n(A-BX)-n\bar{c}=0$$

which is the marginal cost in the market.

Thus we can deduce the total quantity produced and the price in the market.

$$(n+1)BX=n(A-\bar{c})$$

$$X=\frac{n}{n+1}\left(\frac{A-\bar{c}}{B}\right)$$

$$p=A-BX=A-B\frac{n}{n+1}\left(\frac{A-\bar{c}}{B}\right)$$

But $$p=\frac{(n+1)A-n(A-\bar{c})}{n+1}$$

$$p=\frac{1}{n+1}A+\frac{n}{n+1}\bar{c}$$

As $n\to\infty$, $p\to 0+\bar{c}$

$p\to\bar{c}$

Production of n-Firms in Oligopoly

$$x_i^n = \frac{A - BX - c_i}{B} = \frac{A - c_i}{B} - \frac{n}{n+1}\left(\frac{A - \bar{c}}{B}\right)$$

From (3)

$$= (1 - \frac{n}{n+1})\frac{A}{B} + (\frac{n}{n+1}\bar{c} - c_i)\frac{1}{B}$$

$$= \frac{1}{n+1}\frac{A}{B} + \frac{n\bar{c} - nc_i - c_i}{(n+1)B}$$

$$= \frac{1}{n+1}\frac{A}{B} + \frac{n(\bar{c} - c_i) - c_i}{(n+1)B}$$

Assume that each firm have identical marginal cost $c_i = \bar{c} = c.$ Then

$$p = \frac{1}{n+1}A + \frac{n}{n+1}c \quad and \quad p \to c \; as \; n \to \infty$$

$$x_i^n = \frac{1}{n+1}\frac{A}{B} - \frac{c_i}{(n+1)B} == \frac{A - c}{(n+1)B} \quad and \quad x_i^n \to 0 \; as \; n \to \infty$$

from (1)

$$\pi_i^n(y_i, x_i) = px_i^n - cx_i^n = (p - c)x_i^n$$

$$= \left[\frac{1}{n+1}A + \frac{n}{n+1}c - c\right]\frac{(A - c)}{(n+1)B}$$

$$= \frac{(A - c)^2}{(n+1)^2 B}$$

$$n\pi_i^n = \frac{n(A - c)^2}{(n+1)^2 B} \to 0 \; as \; n \to \infty$$

The total profit in the industry decreases with every additional firm entering the market since all n>1

4. Analyzing the Behavior of Oligopolists

Oligopoly presents the greatest challenge to economists

- The essence of oligopoly is strategic interdependence
- Each Firm anticipates actions of its rivals when making decisions
- In order to understand and predict behavior in oligopoly markets Economists have had to modify the tools used to analyze other market structures and to develop entirely new tools as well .
- One approach{game theory}has yielded rich insights into oligopoly behavior
- Game theory deals with any situation in which the reward of any one player (called the payoff) depends on not only his or her own actions but also on those of other players in the game.
- In the case of a two-player game, the interaction is depicted using the Payoff-matrix

5. Conclusion:

The difficulty in studying oligopoly in general is that there are so many possible ways in which the firms might interact with each other. They might collude; ie. Cooperate with each other so as to maximize their joint profits, dividing up the profit among the firms. Such collusion might take the form of overt collusion or cartel. They might also act non-cooperatively, acting in their own self-interest, but taking into account the actions of the other firms. In many ways, the actions of the firms become similar to playing games, such as Monopoly or Risk.

6. Some Suggestions:

It is basic of the economics that as more firms are assumed to exist in the industry, the higher the total quantity supplied and hence the lower the price. The larger the number of firms the closer is output and price to the competitive level. Where cartel is an arrangement among several producers to obey output restrictions in order to increase their joint profits and they act like Monopolist and Simple divides the market among members of the cartel. The most famous example of this is the Organization of Petroleum Exporting Countries(OPEC) .

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