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A Study On Anti L-Fuzzy Subfield Of A Field

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of anti L-fuzzy subfield of a field.
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Introduction:

After the introduction of fuzzy sets by L.A.Zadeh[13], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups, anti-fuzzy subgroups, fuzzy fields and fuzzy linear spaces was introduced by Biswas.R[4, 5]. In this paper, we introduce the some theorems in anti L-fuzzy subfield of a field.

Preliminaries

Definition: Let X be a non-empty set and L be a complete lattice. A L-fuzzy subset A of X is a function $A : X \to L$.

Definition: The union of two fuzzy subsets A and B of a set X is defined by $(A \cup B)(x) = A(x) \vee B(x)$, for all x in X.

Definition: Let (F, +, ·) be a field. A L-fuzzy subset A of F is said to be an anti L-fuzzy subfield(ALFSF) of F if the following conditions are satisfied:

(i) $A(x+y) \leq A(x) \vee A(y)$, for all x and y in F,
(ii) $A(x) \leq A(x)$, for all x in F,
(iii) $A(xy) \leq A(x) \vee A(y)$, for all x and y in F,
(iv) $A(x^{-1}) \leq A(x)$, for all x in F−{0}, where 0 is the additive identity element of F.

Properties of anti l-fuzzy subfields

Theorem: If A is an anti L-fuzzy subfield of a field (F, +, ·), then $A(x) = A(x)$, for all x in F and $A(x^{-1}) = A(x)$, for all x in F−{0} and $A(x) \geq A(0)$, for all x in F and $A(x) \geq A(1)$, for all x in F, where 0 and 1 are identity elements in F.

Proof: For x in F and 0, 1 are identity elements in F. Now, $A(x) = A((x)) \leq A(x) \leq A(x)$. Therefore, $A(x) = A(x)$, for all x in F. And, $A(x) = A((x^{-1})^{-1}) \leq A(x^{-1}) \leq A(x)$. Therefore, $A(x^{-1}) = A(x)$, for all x in F−{0}. And, $A(0) = A(xx) \leq A(x) \vee A(x) = A(x)$. Therefore, $A(0) \leq A(x)$, for all x in F. And, $A(1) = A(xx^{-1}) \leq A(x) \vee A(x^{-1}) = A(x)$. Therefore, $A(1) \leq A(x)$, for all x in F.

Theorem: If A is an anti L-fuzzy subfield of a field (F, +, ·), then (i) $A(xy) = A(0)$ gives $A(x) = A(y)$, for all x and y in F, (ii) $A(xy^{-1}) = A(1)$ gives $A(x) = A(y)$, for all x and $y \neq 0$ in F, where 0 and 1 are identity elements in F.

Proof: Let x and y in F and 0, 1 are identity elements in F. (i) Now, $A(x) = A(xy+y) \leq A(xy) \vee A(y) = A(0) \vee A(y) = A(y) = A(x(xy)) \leq A(xy) \vee A(x) = A(0) \vee A(x) = A(x)$. Therefore, $A(x) = A(y)$, for all x and y in F. (ii) Now, $A(x) = A(xy^{-1}y) \leq A(xy^{-1}) \vee A(y) = A(1) \vee A(y) = A(y) = A((xy^{-1})^{-1}x) \leq A(xy^{-1}) \vee A(x) = A(1) \vee A(x) = A(x)$. Therefore, $A(x) = A(y)$, for all x and $y \neq 0$ in F.

Theorem: A is an anti L-fuzzy subfield of a field (F, +, ·) if and only if $A(x\ y) \leq A(x) \vee A(y)$, for all x and y in F and $A(xy^{-1}) \leq A(x) \vee A(y)$, for all x and $y \neq 0$ in F.

Proof: Let A be an anti L-fuzzy subfield of a field (F, +, ·) and x,y in F. Then, $A(xy) \leq A(x) \vee A(y) \leq A(x) \vee A(y)$. Therefore, $A(xy) \leq A(x) \vee A(y)$, for all x and y in F. And, $A(xy^{-1}) \leq A(x) \vee A(y^{-1}) \leq A(x) \vee A(y)$. Therefore, $A(xy^{-1}) \leq A(x) \vee A(y)$, for all x and $y \neq 0$ in F. Conversely, if $A(xy) \leq A(x) \vee A(y)$, for all x and y in F and $A(xy^{-1}) \leq A(x) \vee A(y)$, for all x and $y \neq 0$ in F, replace y by x, then $A(x) \geq A(0)$, for all x in F and $A(x) \geq A(1)$, for all x in F. Then, $A(x) = A(0x) \leq A(0) \vee A(x) = A(x)$. Therefore, $A (x) \leq A(x)$, for all x in F. And, $A(x^{-1}) = A(1.x^{-1}) \leq A(1) \vee A(x) = A(x)$. Therefore, $A (x^{-1}) \leq A(x)$, for all x in F−{0}. Then, $A(x+y) = A(x(y)) \leq A(x) \vee A(y) \leq A(x) \vee A(y)$. Therefore, $A(x+y) \leq A(x) \vee A(y)$, for all x and y in F. Then, $A(xy) = A(x(y^{-1})^{-1}) \leq A(x) \vee A(y^{-1}) \leq A(x) \vee A(y)$. Therefore, $A(xy) \leq A(x) \vee A(y)$, for all x and y in F. Hence A is an anti L-fuzzy subfield of F.

Theorem: Let A be a L-fuzzy subset of a field (F, +, ·). If $A(e) = A(e^{1}) = 0$, $A(xy) \leq A(x) \vee A(y)$, for all x and y in F and $A(xy^{-1}) \leq A(x) \vee A(y)$, for all x and $y \neq e$ in F, then A is an anti L-fuzzy subfield of F, where e and e^{1} are identity elements of F.

Proof: Let e and e^{1} be identity elements of F and x, y in F. Now $A(x) = A(ex) \leq A(e) \vee A(x) = 0 \vee A(x) = A(x)$. Therefore, $A(x) \leq A(x)$, for all x in F. And, $A(x^{-1}) = A(e^{1}x^{-1}) \leq A(e^{1}) \vee A(x) = 0 \vee A(x) = A(x)$.

Therefore, $A(x^{-1}) \leq A(x)$, for all x in F−{e}. And, $A(x+y) = A(x (y)) \leq A(x) \vee A(y) \leq A(x) \vee A(y)$. Therefore, $A(x+y) \leq A(x) \vee A(y)$, for all x and y in F. And, $A(xy) = A(x(y^{-1})^{-1}) \leq A(x) \vee A(y^{-1}) \leq A(x) \vee A(y)$. Therefore, $A(xy) \leq A(x) \vee A(y)$, for all x and y in F. Hence A is an anti L-fuzzy subfield of F.

Theorem: If A is an anti L-fuzzy subfield of a field (F, +, ·), then $H = \{x / x \in F: A(x) = 0 \}$ is either empty or is a subfield of F.

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then $A(xy) \leq A(x) \vee A(y) = A(x) \vee A(y) = 0 \vee 0 = 0$. Therefore, $A(xy) = 0$, for all x and y in F. We get xy in H. And, $A(xy^{-1}) \leq A(x) \vee A(y^{-1}) = A(x) \vee A(y) = 0 \vee 0 = 0$. Therefore, $A(xy^{-1}) = 0$, for all x and $y \neq e$ in F. We get xy^{-1} in H. Therefore, H is a subfield of F. Hence H is either empty or is a subfield of F.

Theorem: If A is an anti L-fuzzy subfield of a field (F, +, ·), then $H = \{x \in F: A(x) = A(e) = A(e^{1}) \}$ is either empty or is a subfield of F, where e and e^{1} are identity elements of F.

Proof: If no element satisfies this condition, then H is empty. If x and y satisfies this condition, then $A(x) = A(x) = A(e)$, for all x in F and $A(x^{-1}) = A(x) = A(e^{1})$, for all x in F−{e}. Therefore, $A(x) = A(e)$, for all x in F and $A(x^{-1}) = A(e^{1})$, for all x in F−{e}. Hence x , x^{-1} in H. Now, $A(xy) \leq A(x) \vee A(y) \leq A(x) \vee A(y) = A(e) \vee A(e) = A(e)$. Therefore, $A(xy) \leq A(e)$ -------- (1).

And, $A(e) = A((xy)(xy)) \leq A(xy) \vee A((xy)) \leq A(xy) \vee A(xy) = A(xy)$. Therefore, $A(e) \leq A(xy)$ ----------(2). From (1) and (2), we get $A(e) = A(xy)$, for all x and y in F. Now, $A(xy^{-1}) \leq A(x) \vee A(y^{-1}) \leq A(x) \vee A(y) = A(e^1) \vee A(e^1) = A(e^1)$. Therefore, $A(xy^{-1}) \leq A(e^1)$ ---------- (3). And, $A(e^1) = A((xy^{-1})(xy^{-1})^{-1}) \leq A(xy^{-1}) \vee A((xy^{-1})^{-1}) \leq A(xy^{-1}) \vee A(xy^{-1}) = A(xy^{-1})$.

Therefore, $A(e^1) \leq A(xy^{-1})$ ---------- (4). From (3) and (4), we get $A(e^1) = A(xy^{-1})$, for all x and $y \neq e$ in F. Hence $A(e) = A(xy)$, $A(e^1) = A(xy^{-1})$. We get xy, xy^{-1} in H. Hence H is either empty or is a subfield of F.

Theorem: Let A be an anti L-fuzzy subfield of a field $(F, +, \cdot)$. Then (i) if $A(xy) = 0$, then $A(x) = A(y)$, for x and y in F.

(ii) if $A(xy^{-1}) = 0$, then $A(x) = A(y)$, for all x and $y \neq e$ in F, where e and e^1 are identity elements of F.

Proof: Let x and y in F. Now, $A(x) = A(xy+y) \leq A(xy) \vee A(y) = 0 \vee A(y) = A(y) = A(y) = A(x+xy) \leq A(x) \vee A(xy) = A(x) \vee 0 = A(x) = A(x)$. Therefore, $A(x) = A(y)$, for all x and y in F. And, $A(x) = A(xy^{-1}y) \leq A(xy^{-1}) \vee A(y) = 0 \vee A(y) = A(y) = A(y^{-1}) = A(x^{-1}xy^{-1}) \leq A(x^{-1}) \vee A(xy^{-1}) = A(x^{-1}) \vee 0 = A(x^{-1}) = A(x)$. Therefore, $A(x) = A(y)$, for all x and $y \neq e$ in F.

Theorem: If A is an anti L-fuzzy subfield of a field $(F, +, \cdot)$, then (i) if $A(xy) = 1$, then either $A(x) = 1$ or $A(y) = 1$, for x and y in F,

(ii) if $A(xy^{-1}) = 1$, then either $A(x) = 1$ or $A(y) = 1$, for all x and $y \neq e$ in F.

Proof: Let x and y in F. By the definition $A(xy) \leq A(x) \vee A(y)$, which implies that $1 \leq A(x) \vee A(y)$. Therefore, either $A(x) = 1$ or $A(y) = 1$, for all x and y in F. And by the definition $A(xy^{-1}) \leq A(x) \vee A(y)$, which implies that $1 \leq A(x) \vee A(y)$. Therefore, either $A(x) = 1$ or $A(y) = 1$, for all x and $y \neq e$ in F.

Theorem: Let $(F, +, \cdot)$ be a field. If A is an anti L-fuzzy subfield of F, then $A(x + y) = A(x) \vee A(y)$, for all x and y in F and $A(xy) = A(x) \vee A(y)$, for all x and y in F with $A(x)$ $A(y)$.

Proof: Let x and y belongs to F. Assume that $A(x)$ $A(y)$. Now, $A(y) = A(x+x+y) \leq A(x) \vee A(x+y) \leq A(x) \vee A(x+y) = A(x+y) \leq A(x) \vee A(y) = A(y)$. Therefore, $A(x+y) = A(y) = A(x) \vee A(y)$, for all x and y in F. And, $A(y) = A(x^{-1}xy) \leq A(x^{-1}) \vee A(xy) \leq A(x) \vee A(xy) = A(xy) \leq A(x) \vee A(y) = A(y)$. Therefore, $A(xy) = A(y) = A(x) \vee A(y)$, for all x and y in F.

Theorem : If A and B are any two anti L-fuzzy subfields of a field $(F, +, \cdot)$, then their union $A \cup B$ is an anti L-fuzzy subfield of F.

Proof : Let x and y belongs to F and $A = \{x, A(x) / x \in F\}$ and $B = \{x, B(x) / x \in F\}$. Let $C = A \cup B$ and $C = \{x, C(x) / x \in F\}$, $C(x) = A(x) \vee B(x)$. (i) $C(xy) = A(xy) \vee B(xy) \leq \{A(x) \vee A(y)\} \vee \{B(x) \vee B(y)\} = \{A(x) \vee B(x)\} \vee \{A(y) \vee B(y)\} = C(x) \vee C(y)$. Therefore, $C(xy) \leq C(x) \vee C(y)$, for all x and y in F. (ii) $C(xy^{-1}) = A(xy^{-1}) \vee B(xy^{-1}) \leq \{A(x) \vee A(y)\} \vee \{B(x) \vee B(y)\} = \{A(x) \vee B(x)\} \vee \{A(y) \vee B(y)\} = C(x) \vee C(y)$. Therefore, $C(xy^{-1}) \leq C(x) \vee C(y)$, for all x and $y \neq 0$ in F.

Hence $A \cup B$ is an anti L-fuzzy subfield of a field F.

Theorem: The union of a family of anti L-fuzzy subfields of a field $(F, +, \cdot)$ is an anti L-fuzzy subfield of F.

Proof: Let $\{A_i\}_{i \in I}$ be a family of anti L-fuzzy subfields of a Field F and $A = \bigcup_{i \in I} A_i$. Then for x and y belongs to F, we Have (i) $A(xy) = \sup_{i \in I} A_i(x-y) \leq \sup_{i \in I} A_i(x) \vee A_i(y) \leq \sup_{i \in I} (A_i(x)) \vee \sup_{i \in I} (A_i(y)) = A(x) \vee A(y)$. Therefore, $A(xy) \leq A(x) \vee A(y)$, for all x and y in F. (ii) $A(xy^{-1}) = \sup_{i \in I} A_i(xy^{-1}) \leq \sup_{i \in I} A_i(x) \vee A_i(y) \leq \sup_{i \in I} \leq (A_i(x)) \vee \sup_{i \in I} (A_i(y)) = A(x) \vee A(y)$.

Therefore, $A(xy^{-1}) \leq A(x) \vee A(y)$, for all x and $y \neq 0$ in F. Hence the union of a family of anti L-fuzzy subfields of a field F is an anti L-fuzzy subfield of F.

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