Research Paper Engineering

Analysis of Carnot Cycle, Based on Entropy Generation Principle, with Variable Heat Source and Heat Sink Temperature

Neeraj Yadav | Department of Mechanical Engineering, Institute of Engineering and Technology -DAVV, Indore, India

Dr.Govind Maheshwari | Department of Mechanical Engineering, Institute of Engineering and Technology -DAVV, Indore, India

ABSTRACT

Entropy generation is often used for thermodynamic analysis of any thermal system to maximize the work output of the system. In this paper minimum entropy generation principle is discussed and applying on Carnot cycle to optimize it. The analysis of Carnot cycle is based on output power, entropy generation rate and heat work conversion efficiency. Carnot cycle is working between two heat exchangers with variable temperature, relationship is derived for above define parameter to evaluate the performance of the cycle. In the presence of various inlet temperature streams and prescribed condition is applying on the cycle and result show that the minimum entropy generation rate is not always corresponds to maximum work done and highest work heat conversion efficiency for prescribed conditions.

KEYWORDS Carnot Cycle, Minimum Entropy Generation Rate, Variable Heat Source and Heat Sink Temperature

INTRODUCTION

Energy consumption increases day by day, energy resources depleted continuously and it will obsolete soon. The utilization of energy in efficient way is the toughest task for an engineer. Efficient system design reduces the energy loss, improving system efficiency, reducing fuel prices and global warming. The second law of thermodynamics provides a scientific basis for analyzing how effectively a system utilizes available energy. Entropy generation is one of the most important concepts in optimizing heat transfer and heat–work conversion. Both heat transfer and heat–work conversion are irreversible and non-equilibrium processes from the thermodynamic viewpoint except for the ideal reversible physical processes which could not be achieved in practice [1-2]. In the heat-work conversion optimization, the entropy generation minimization is usually used as the optimization criterion to thermodynamic cycles because the entropy generation represents the ability loss of doing work [3]. The minimization of entropy generation is equivalent to the maximization of power output [4]. To minimize the irreversibility of any proposed design, the engineer must use the relations between temperature differences and heat transfer rates, and between pressure differences and mass flow rates [5]. Heat exchanger is a device which is used to exchange heat between temperature gradient. Improving their effectiveness has often been regarded as the key issue in energy conservation [6]. Any heat transfer process is characterized by two kinds of losses, heat transfer through a finite temperature difference and pressure drop in heat transfer fluids. Entropy generation combines the two processes in one property that can be evaluated; the entropy generation number is the basic parameter in describing heat exchanger performance [7].W. A. Khan el al. determine an optimal tube bank by minimizing the dimensionless entropy generation rate for different design variables including D, L, cross-sectional area of tube bank, W H, and Q. In each case, the optimum approach velocity/Reynolds number is determined corresponding to the minimum entropy generation rate [8]. Wang et al. analyzed an endoreversible Carnot cycle and found that in the presence of various inlet temperatures of streams, larger entransy loss rate still leads to larger output power, while smaller entropy generation rate does not [9]. In this paper, the preconditions for the application of these optimization methods are discussed with Carnot cycle.

GENERATION OF ENTROPY IN THERMODYNAMIC CYCLE

A thermodynamics system consists of finite capacity heat source and heat sink with work producing device as shown in figure 1. Heat source temperature is decrease from T_{hi} to T_{ho} and this hot stream is dumped into environment at temperature T_o, the total heat supplied by hot source into the system is Q_{sh} and heat rejected or dumped out of the system from hot side is Q_{eh}. Heat sink temperature is increases from T_{li} to T_{lo} and this stream is dumped into environment at temperature T_o, the total heat gain by sink is Q_{lh} and heat dumped out of the system from cold side is Q_{el}.

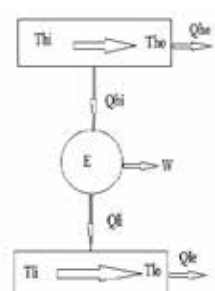


Figure 1:Thermodynamic cycle

The entropy balance is [1,4]

$$dS = S_g + dS_f \quad (1)$$

Where dSis the entropy change rate, dS_f is the entropy flow rate, and Sg is the entropy generation rate. As the system is steady, dS equals zero. The entropy generation rate could be expressed as

$$S_g = -dS_f \quad (2)$$

The entropy flow rate from heat source (S_{fh}) and sink (S_{fl}) are govern by following equation [1,6]

$$S_{fh} = \int_{T_{hi}}^{T_o} C_h \frac{dT_h}{T_h}$$

$$S_{fh} = C_h \ln \frac{T_o}{T_{hi}} \quad (3)$$

$$S_{fl} = \int_{T_{li}}^{T_o} C_l \frac{dT_l}{T_l}$$

$$S_{fl} = C_l \ln \frac{T_o}{T_{li}} \quad (4)$$

Entropy flow rate into the environment [1,4]

$$S_{fe} = \frac{Q_{eh} + Q_{el}}{T_o}$$

$$S_{fe} = \frac{C_h \left[T_{ho} - T_o\right] + C_l \left[T_{lo} - T_o\right]}{T_o} \quad (5)$$

The total entropy generation rate is calculated by adding equation (3), (4) and (5)

$$S_g = S_{fh} + S_{fl} + S_{fe} \quad (6)$$

$$S_g = \left[\left(C_h \ln \frac{T_o}{T_{hi}}\right) + \left(C_l \ln \frac{T_o}{T_{li}}\right)\right] + \left[\frac{C_h \left[T_{ho} - T_o\right] + C_l \left[T_{lo} - T_o\right]}{T_o}\right] \quad (7)$$

Work done by the system is given by the following relation

$$W = \left[Q_{sh} - Q_{eh}\right] + \left[Q_{lh} - Q_{el}\right] \quad (8)$$

Where

$$Q_{sh} = C_h \left[T_{hi} - T_o\right]$$

$$Q_{eh} = C_h \left[T_{ho} - T_o\right]$$

$$Q_{lh} = C_l \left[T_{li} - T_o\right]$$

$$Q_{el} = C_l \left[T_{lo} - T_o\right]$$

Rearranging Equation (8)

$$W = \left(Q_{sh} + Q_{lh}\right) - \left(Q_{eh} + Q_{el}\right) \quad (9)$$

The last term of equation (9) may be written as

$$W = Q_{sh} + Q_{lh} - T_o S_{fe} \quad (10)$$

The equation (6) may be written as [1]

$$S_g = S_{fh} + S_{fl} + \left[\frac{Q_{sh} + Q_{lh} - W}{T_0}\right] \quad (11)$$

CARNOT CYCLE WORKING BETWEEN VARIABLE HEAT SOURCE AND HEAT SINK TEMPERATURE

Carnot cycle composed of four processes the working fluid compressed isothermally and reversible adiabatically during processes 1-2 and 2-3 respectively. After compression working fluid expands isothermally and reversible adiabatically during process 3-4 and 4-1 respectively. During isothermal processes 1-2 and 3-4, cycle exchanging heat with variable heat sink and heat source temperature respectively.

Heat transfer to the cycle from hot side heat exchanger is [9]

$$Q_{hi} = \left[1 - \exp\left(\frac{-UA}{C_h}\right)\right] C_h \left(T_{hin} - T_h\right) \quad (12)$$

$$\varepsilon_h = \left[1 - \exp\left(\frac{-UA}{C_h}\right)\right]$$

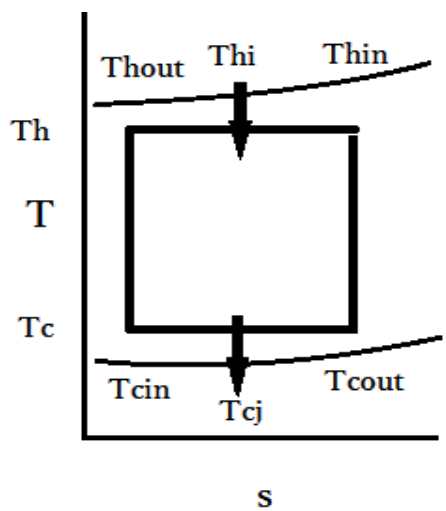


Figure 2: Carnot cycle working with variable heat source and heat sink temperature

Heat transfer to the cycle from cold side heat exchanger is

$$Q_{li} = \left[1 - \exp\left(\frac{-UA}{C_l}\right)\right] C_l \left(T_c - T_{cin}\right) \quad (13)$$

$$\varepsilon = \left[1 - \exp\left(\frac{-UA}{C_c}\right)\right]$$

UA is the thermal conductance of heat exchanger and C_h and C_l are specific heat of hot and cold working fluid respectively. $T_h$is highest temperature of Carnot cycle and T_{hi} is the inlet temperature of source side heat exchanger, T_c is lowest temperature of Carnot cycle and T_{ci} is the inlet temperature of sink side heat exchanger.

The cycle satisfy the Carnot theorem as follows equation (14)

$$\frac{Q_{hi}}{T_h} = R\frac{Q_{li}}{T_c} \tag{14}$$

R is the irreversibility parameter.

According to law of energy conservation, the work done by the cycle is calculated as follows

$$W = Q_{hi} - Q_{li} \tag{15}$$

And the cycle is efficiency is given by the following relation

$$\eta = \frac{W}{Q_{hi}} = 1 - \frac{Q_{li}}{Q_{hi}} \tag{16}$$

Carnot cycle efficiency is also given by following relation

$$\eta = \frac{W}{Q_{hi}} = 1 - \frac{T_c}{T_h} \tag{17}$$

Rearranging equation (17)

$$W = Q_{hi}\left(1 - \frac{T_c}{T_h}\right) \tag{18}$$

$$W = \left[1 - \exp\left(\frac{-UA}{C_h}\right)\right] C_h\left(T_{hin} - T_h\right) * \left(1 - \frac{\dfrac{RT_hC_lT_{cin}}{\left(RT_hC_l - C_h(Th_{in} - T_h)\right)}}{T_h}\right) \tag{19}$$

Substituting equation (18) into equation (11), we can get the entropy generation rate as follows

$$S_g = \left[\frac{C_h\left(T_{hin} - T_o\right) + C_l\left(T_{cin} - T_o\right) - W}{T_o}\right] + \left[C_h \ln\left(\frac{T_o}{T_{hin}}\right) + C_l \ln\left(\frac{T_o}{T_{cin}}\right)\right] \tag{20}$$

$$S_g = \left[\frac{C_h\left(T_{hin} - T_o\right) + C_l\left(T_{cin} - T_o\right) - \left[1 - \exp\left(\frac{-UA}{C_h}\right)\right] C_h\left(T_{hin} - T_h\right)}{T_o}\right] * \left(1 - \frac{\dfrac{RT_hC_lT_{cin}}{\left(RT_hC_l - C_h(Th_{in} - T_h)\right)}}{T_h}\right) \Big/ T_o + \left[C_h \ln\left(\frac{T_o}{T_{hin}}\right) + C_l \ln\left(\frac{T_o}{T_{cin}}\right)\right]$$

$$N_g = \left[\frac{\left[\dfrac{C_h\left(T_{hin} - T_o\right) + C_l\left(T_{cin} - T_o\right) - \left[1 - \exp\left(\frac{-UA}{C_h}\right)\right] C_h\left(T_{hin} - T_h\right)}{T_o}\right]}{\left[1 - \dfrac{\dfrac{RT_hC_lT_{cin}}{\left(RT_hC_l - C_h(Th_{in} - T_h)\right)}}{T_h}\right]} + \left[C_h \ln\left(\frac{T_o}{T_{hin}}\right) + C_l \ln\left(\frac{T_o}{T_{cin}}\right)\right] \Big/ \right] \text{Cmin} \tag{22}$$

Considering the equations (12), (13) and (14), by using these equations we can find the value of Tc for prescribed values of T_h and R. By considering the equation (17), (19) (21) and (22), we can find the values of efficiency work done entropy generation rate and entropy generation number for prescribed operating parameter.

For a specific example [9] , Choosing C_h=3 W/K, T_{hi}=400 K, T_o=300K, Thermal conductance of heat exchangers UA=100 W/K, C_h=2 W/K, T_{ci}= 302 K. Putting prescribed values in above equations, the variation in output power, entropy generation rate with variable T_h and R=1.2 shown in figure 3 and the variation in output power, entropy generation rate with variable R and T_h=370K as shown in figure 4, when heat capacity rate of fluid remains constant.

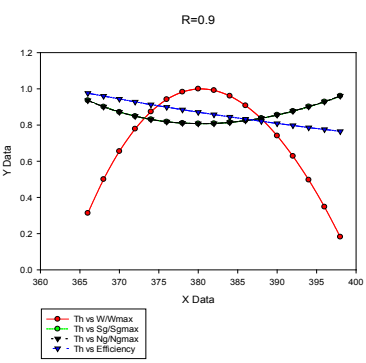


Figure 3: Variation of Work ratio (W/W_{max}), Entropy generation rate ratio (S_g/S_{gmax}), Entropy generation number ratio (N_g/N_{gmax}) and efficiency of cycle with variation T_h at R=0.9

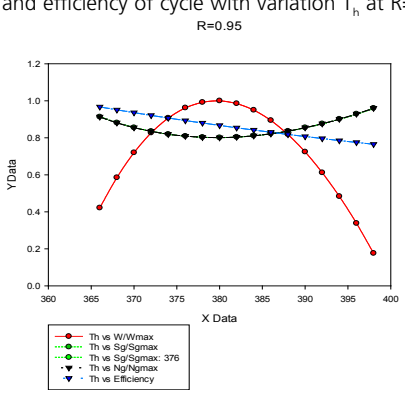


Figure 4: Variation of Work ratio (W/W_{max}), entropy generation rate ratio (S_g/S_{gmax}), Entropy generation number ratio (N_g/N_{gmax}) and efficiency of cycle with variation T_h at R=0.95

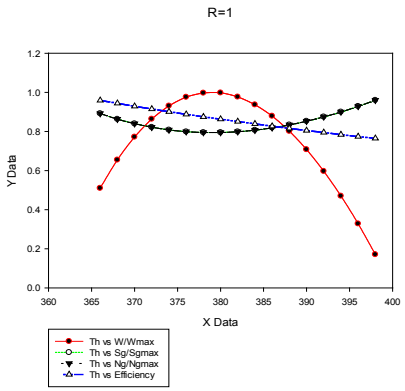


Figure5: Variation of Work ratio W/W_{max}), Entropy generation rateratio(S_g/S_{gmax}), Entropy generation number ratio (N_g/N_{gmax}) and efficiency of cyclewith variation T_h at R=1

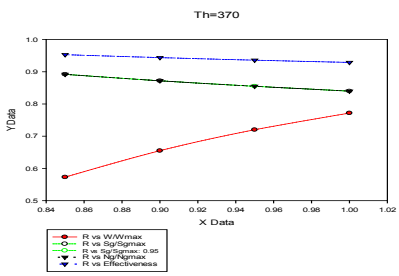


Figure 6: Variation of Work ratio (W/W_{max}), Entropy generation rate ratio (S_g/S_{gmax}), Entropy generation number ratio (N_g/N_{gmax}) and efficiency of cycle with variation of R atconstant T_h= 370 K

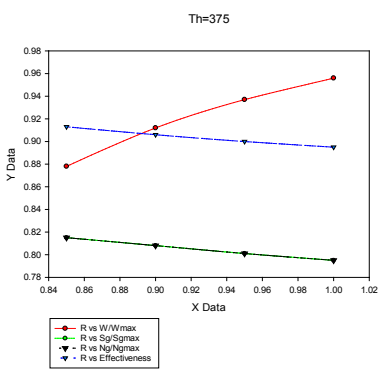


Figure 7: Variation of Work ratio (W/W_{max}), Entropy generation rate ratio (S_g/S_{gmax}), Entropy generation number ratio (N_g/N_{gmax}) and efficiency of cycle with variation of R at constant T_h= 375 K.

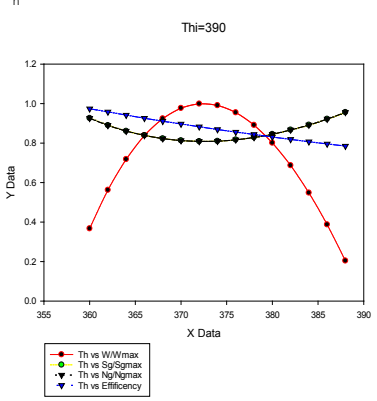


Figure 8: The variation of Work ratio (W/W_{max}), entropy generation rate ratio (S_g/S_{gmax}), entropy generation number ratio(N_g/N_{gmax}) and cycle efficiency with variation of T_h at constant T_{hi}= 390 K and R=0.9

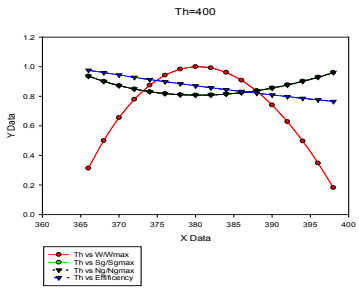


Figure 9: The variation of Work ratio (W/W_{max}), entropy generation rate ratio (S_g/S_{gmax}), entropy generation number ratio(N_g/N_{gmax}) and cycle efficiency with variation of T_h at constant T_{hi}= 400 K and R=0.9

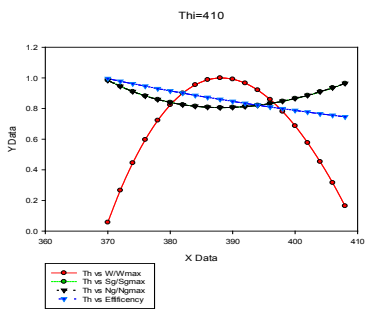


figure 10: The variation of Work ratio (W/W_{max}), entropy generation rate ratio (S_g/S_{gmax}), entropy generation number ratio(N_g/N_{gmax}) and cycle efficiency with variation of T_h at constant T_{hi}= 410 K and R=0.9

Conclusion:-

The Carnot cycle working between variable heat source and heat sink temperature, the results shows the variation of work done to maximum work done, entropy generation rate to maximum entropy generation rate, entropy generation number to maximum entropy generation number and efficiency are obtained for the variable values of T_h, R and T_{hi}. The maximum work done always corresponds to minimum entropy generation for variable value of T_h and prescribed values of operating parameter. The efficiency of the cycle for prescribed operating parameter is decreases. The maximum work done is obtained if Carnot cycle works between T_h=379 K to 382 K for T_{hi}=400 K and different values of R. The value of optimum value T_h is found by analysis of figure number 3 to figure 5. For different values of R and T_h=375 K, the work done increases and entropy generation rate decreases with increase of value of R is clearly seen from figure 6 to figure 8. The variation of T_h for different value of T_{hi} and R=0.9 shows in figure 8 to figure 10, after analyzing above figure, we found that optimum value of T_h is dependent on T_{hi}. T_{hi} increases the optimum value of T_h increases. The value entropy generation rate ratio and entropy generation number ratio is same in all operating and prescribed conditions and efficiency of the cycle is decreased for all operating and prescribed conditions.

REFERENCES

1. Cheng Xue-Tao & Liang Xin-Gang, Applicability of the minimum entropy generation method for optimizing thermodynamic cycle, Chinese physics bulletin Vol. 22 No. 1 (2013) 010508. | 2. Soteris a. Kalogirou, entropy generation minimization of imaging concentrating solar collectors | 3. Zeng D L, Ao Y, Zhang X M and Liu C 2002 Engineering Thermodynamics (Beijing: High Education Press) (in Chinese) | 4. Adrian bejan, fundamentals of exergy analysis, entropy generation minimization, and the generation of flow architecture, international journal of energy research, int. J. Energy res. 2002; 26:545}565 (doi: 10.1002/er.804) | 5. Adrian bejan, exergy analysis, entropy generation minimization, and constructal theory, mechanical engineers' handbook: energy and power, volume 4, third edition. Edited by myerkutz. | 6. Chen Qun, Wu Jing, Wang Moran, Pan Ning&GuoZengyuan, A comparison of optimization theories for energy conservation in Heat exchanger groups, hinese Science Bulletin, February 2011,vol.56 no.4-5: 449–454 | 7. Varun Singh, Vikrant Aute, ReinhardRadermacher, Usefulness of Entropy Generation Minimization through a Heat Exchanger Modeling Tool, International Refrigeration and Air Conditioning Conference. Paper 958. | 8. W. A. Khan J. R. Culham And M. M. Yovanovich, Optimal Design Of Tube Banks In Crossflow Using Entropy Generation Minimization Method, Journal Of Thermophysics And Heat Transfer Vol. 21, No. 2, April–June 2007 | 9. Wang Wen-Hua, Cheng Xue-Tao & Liang Xin-gang, Analysis of the endoreversible carnot cycle with entropy theory and entransy theory, Chinese physics bulletin Vol. 22 No. 11 (2013) 110506. | |