Research Paper Technology

Bevel Gear Optimization Using Heuristic Ant Colony Optimization Approach

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ABSTRACT

In this paper, ant colony algorithm has been used for the classical bevel gear optimization problem by satisfying design, manufacturing and operational constraints in a non-lubricated condition. Instead of applying the conventional gear design approach, the proposed ant colony algorithm employs a novel method inspired from ant's behavior to identify the best suitable parameters which can reduce the size, thickness, weight and increase the efficiency of the gear pair without affecting the performance in less computational time. Besides, the gear had also been modeled and analyzed in Ansys software to validate the developed algorithm. The proposed algorithm had been simulated for various instances and obtained competitive results in a non-lubricated environment.

KEYWORDS | Ant colony algorithm; Bevel Gear; Optimization; Non-lubrication;

1. Introduction

The problem addressed in this paper arises in the context of the classical bevel gear design optimization problem. Bevel gears are used in machine tools and transmission systems to transmit power between the angular/intersecting shafts. So these gear pair subject to more stresses. Due to standardization and manufacturing constraints, standard sized gears are used for the machine tools and lead to larger size. On the other hand, due to automation and digital revolution, a need arises to reduce the weight and size of the power transmission systems. So in this research, a set of bevel gears have to been designed for the user defined conditions and the parameters had been optimized with several instances subject to both design and operational constraints. The objective is to minimize the size and weight of the gear with increase in the efficiency.

In general, bevel gear design involves lot of design and manufacturing constraints. Some of these constraints will conflict and makes the bevel gear design problem into a NP-hard in the strong sense. Due to the complexity of the design, it is unrealistic to solve even a medium-sized problem by using traditional design techniques. The traditional techniques and the mathematical models can design the bevel gear by satisfying the stress constraints and by considering the design standardization aspects. But the optimization of the bevel gears involves lot of rework and more time-consuming. Some of the optimization algorithms, such as branch and bound schemes, integer programming, etc also might not be suitable for the problem with conflicting constraints. Therefore, meta-heuristics such as taboo search, genetic algorithm, ant's colony algorithm and simulated annealing have been studied much in recent years. However, each meta-heuristics have its own pros and cons. Therefore, most of the researchers have tried to develop hybrid and heuristic algorithms expecting to achieve improved effectiveness and efficiency. In this paper, a heuristic algorithm for bevel gear design had been proposed. Due to its exploration and information learning abilities, ACO is expected to provide appropriate design parameters by satisfying the constraints, which can thereby be enhanced to an optimal and feasible solution iteratively. In order to reduce the complexity of the problem and to convert the non-linear problem into a linear problem, heuristic penalty fitness function had been used.

The remainder of this paper is organized as follows. In the next section, the various research works carried out in the field and the gaps have been discussed, followed by bevel gear design problem formulation in the section 3. In Section 4, the proposed ACO framework is introduced and analyzed. Finally, computational results and conclusion are provided in the section 5 and 6 respectively.

2. Literature Survey

Townsend (1992) investigated the variations in the sliding velocities with respect to the line of action. As the result of the research, it has been found that the sliding velocity at the pitch line became zero and increases as it travels away from the pitch line on either direction. Blok's contact temperature theory was used to identify the variations in the temperature distribution and heat generation due to contact pressure and sliding velocity. Thus it has been paved a way for the design of non-lubricated gears. The demand for the gears and encroachments of digital machines piloted the bevel gear design using computer programs. Madhusudan and Vijayasimha (1987) programmed the gear procedure and simulated the same using computer with specific conditions and assumptions. Lin *et al.,* (2009) also developed a programme for gear box design which automated the gear design for the given conditions. Huang *et al.,* (2005) formulated a interactive physical programming and thereby optimized the three-stage spur gear unit used for speed reduction. Abersek *et al.,* (1995) created an expert system using computer programming for designing and manufacturing of a gearbox unit under some specified conditions. Tudose (2010) made an attempt to automate the helical gear designing for speed reducer application. Thompson *et al.,* (2000) considered multiple objectives to optimize the two-stage and three-stage gear reduction units. Savsani *et al.,* (2010) attempted to reduce the weight of the spur gear unit using particle swam optimization. Li *et al.,* (1996) investigated the American Gear Manufacturers Association (AGMA) procedures to reduce the weight of the gears. Renner and Ekart (2003) also used evolutionary concepts in solving computer aided problems. So ant colony algorithm has been used to optimize the bevel gear parameters in this research.

3. Problem definition and notation

In the bevel gear design problem, the basic dimensions of the gear pair have to be identified such a way that the stress values should be within the allowable limit. Major parameters such as module, material, number of teeth's, etc should follow a predefined standard. In bevel gear design problem, each gear parameter can posses its own constraints and inter dependent on the other parameter, which cannot be interrupted separately. So optimizing the bevel gear design parameters is the most complicated problem and thus the design became strongly NP hard problem. A feasible bevel gear design is to build a permutation of sets of gear parameters by satisfying

the constraint and achieving the objectives. The objective of this research is to find feasible bevel gear parameters that minimize the weight and size of the gear by increasing the efficiency of the gear. The objective function *O(x)* used in this research can mathematically be defined as given in the Equation 1.

$$O(x) = (\max.\ \eta,\ \min.\ \S,\ \min\ \hat{W}) \quad (1)$$

Whereas, *max.* η, *min.* $\S$ *and min.* $\hat{W}$ denotes maximization of efficiency, minimization of size and weight respectively by satisfying the constraints set *C(x)* given in the Equation 2.

$$\text{Subject to } C(x) = \{D(c),\ M(c),\ Op(c),\ nl(c)\} \quad (2)$$

Whereas, D(c), M(c), Op(c) and nl(c) denotes the design, manufacturing, operational and non-lubrication constraints to be satisfied by the gear pair respectively.

The set of parameters like materials, speed ratio, speed of rotation, torque required, power to be transmitted, number of teeth, module, twisting moment, gear thickness, etc have to be considered for the optimal design of bevel gear pair. For the experimentation purpose, to reduce the computational time, the parameter set considered in this research had been converged to power, module and thickness. Remaining parameter had been frozen to a standard values based on the literatures. Let *P(x) = {Po, Md, Th}* denote the permutation of power, module and thickness respectively and the probability of the parameters are *P(x) = {(Po=8, …..32) kw, (Md = 1, …, 16) mm, (Th=5,…., 50)mm}.* Hence, a feasible and optimal sequence of the design is defined by the Equation 3.

$$P(optimal) = \{Po(i),\ Md(j),\ Th(k)\}. \quad (3)$$

These three parameters have been considered as the input to the bevel gear design problem and the remaining parameter used to manufacture the bevel gear pair will be calculated.

3.1 Objective Functions

The formula for calculating the efficiency is given in the Equation 4.

$$\eta = 100 - P_L \quad (4)$$

where, 'P_L' = Power loss function which can be calculated by the Equation 5.

$$P_L = \frac{50f}{\cos\Phi} \times \frac{(H_a^2 + H_r^2)}{(H_a + H_r)} \quad (5)$$

'H_a' = Specific sliding velocity at start of approach action

'H_r' = Specific sliding velocity at end of recess action

'f' = Coefficient of friction

'Φ' = Pressure angle in degrees

'H_a' and 'H_r' are calculated by the Equations 6 & 7 respectively.

$$H_r = \frac{(i+1)}{i} \times \sqrt{\left(\left[\frac{r_o}{r}\right]^2 - \cos^2\Phi\right)} - \sin\Phi \quad (6)$$

$$H_a = (i+1) \times \sqrt{\left(\left[\frac{R_o}{R}\right]^2 - \cos^2\Phi\right)} - \sin\Phi \quad (7)$$

Whereas, 'R' & 'R_o' = Pitch and Outside circle radius of gear in mm.

'r' & 'r_o' = Pitch and Outside circle radius of pinion in mm

R_0 = R + one addendum

The outside radius of the bevel gear is calculated using the Equation 8 and 9.

The minimization function

$$r_o = r + m = \frac{d_1}{2} + m \quad (8)$$

$$R_o = R + m = \frac{d_2}{2} + m \quad (9)$$

for weight, total weight of the gear should be as minimum as possible. The formula for calculating the weight of the gear is given in the Equation 10.

$$\hat{W} = \{[\frac{\pi}{4} \times d_1^2 \times b \times \rho] + [\frac{\pi}{4} \times d_2^2 \times b \times \rho]\} \quad (10)$$

Where, 'd_1' & 'd_2' = Pitch circle diameter of pinion and gear in mm

'b' & 'ρ' = Thickness and Density of the material in kg/mm^3

The minimization function for size is to reduce the cone distance between the gears and thereby the size of the gears can be reduced. The formula for calculating the cone distance is given in Equation 11.

$$\S = \frac{(d_1 + d_2)}{2} = \frac{m}{2}(Z_1 + Z_2) \quad (11)$$

Where, 'Z_1', 'Z_2' = Number of teeth in pinion and gear respectively.

Thus the obtained solution can be the optimal solution and that solution needs to be checked for the constraint satisfaction. Because, obtained solution should be a feasible solution, so that the bevel gear pair can be manufactured. The basic function of the bevel gear is to transmit the power between the intersecting shafts, so the gear should withstand the stress generated in the gear teeth.

3.2 Bending and Compressive Stress

The major design stresses considered in this research are bending stress and compressive stress. The condition for the safe limit of the bevel gear pair under bending stress is given in the Equation 12.

$$\sigma_b \le [\sigma_b]_{al} \quad (12)$$

Whereas, '$[\sigma_b]_{al}$' = Allowable bending stress in N/mm^2.

'σ_b' = Induced bending stress in N/mm^2 and is given in Equation 13.

$$\sigma_b = \frac{(i+1)}{(a\ m\ b\ y)} \times [M_t] \quad (13)$$

Where, 'i' = Gear ratio / Speed ratio

'a' = Center distance between gear and pinion

'y' = Form factor

'$[M_t]$' = Design twisting moment in Nmm, and is given in Equation 14.

$$[Mt] = M_t \times k \times k_d \quad (14)$$

'M_t' = Normal twisting moment transmitted by the pinion in Nmm

'K' & 'k_d' are the Load Concentration factor and Dynamic load factor

In general, during transmission of power between the gears, compressive stresses will be created at the bottom teeth surface of gears. The safe limit condition for bevel gear pair under compressive stress is given in the Equation 15.

$$\sigma_c \le [\sigma_c]_{al} \quad (15)$$

Where, '$[\sigma_c]_{al}$' = Allowable crushing stress in N/mm^2.

'σ_c' = Induced crushing stress in N/mm^2 and is given in Equation 16.

$$\sigma_c = 0.74\left(\frac{i+1}{a}\right) \times \sqrt{\left[\left(\frac{i+1}{ib}\right) \times E \times [M_t]\right]} \quad (16)$$

Where, 'E' = Young's Modulus of the gear material in N/mm^2

Once the selected gear parameters are satisfied the bending and compressive stress values, then the parameters need to be checked for the vibration and the surface temperature. i.e. the heat generated under the non-lubricated condition should be within the allowable limit for the noiseless operation and thereby the life of the gear pair could be higher.

3.3 Non-lubricated Condition

The optimization of bevel gear pair considered in this research operates in a non-lubricated environment. Thereby more heat will be generated on the gear surface within less time. The heat generated will increase the surface temperature of the gear pair and that surface temperature should be within the allowable limits. Thereby the gear life can be improved. The maximum contact temperature can be obtained by Equation 17.

$$\theta_{B\,max} = \theta_M + \theta_{fl\,max} \quad (17)$$

Whereas, 'θ_M' is tooth temperature,

'$\theta_{fl\,max}$' is maximum flash temperature along the line-of-action, calculated using Blok's equation given in the Equation 18.

$$\theta_{fl} = 31.62\ K\ \mu_m\ (X_r\ W_n / \sqrt{(b_H)})\ x\ \{(|V_{r1} - V_{r2}|) / [(B_{M1}\sqrt{(V_{r1})}) + (B_{M2}\sqrt{(V_{r2})})]\} \quad (18)$$

Whereas, 'K' is the Hertzian distribution of frictional heat = 0.8;

'μ_m' is the Mean coefficient of friction; X_r is the Load sharing factor;

'W_n' is the Normal unit load;

'B_{M1}' & 'B_{M2}' is the Thermal contact coefficients of the pinion and wheel;

'V_{r1}' & 'V_{r2}' is the Rolling tangential velocities in m/s of the pinion and wheel;

The design model described above can find suitable parameters for the bevel gear pair by considering major constraints and the surface temperature for the given input data. Thus the problem becomes a large combinatorial optimization NP hard problem and cannot be efficiently solved using traditional optimization methods. So in this research Ant Colony Algorithm (ACO) have been used for optimizing the bevel gear problem by considering all the above mentioned objectives and constraints.

4. Ant Colony Optimization

ACO, one of the meta-heuristics dedicated to discontinuous multi-objective optimization problems, it is inspired by the foraging behavior of real ants. Real ants are naturally having inbuilt capability of finding the shortest path to the food source from the nest without any visual cue. Instead, the ants can communicate between them about the path through the deposition of the chemical substance called pheromone. Pheromone is a chemical substance which can be generated by the ants, which is used to attract the ants. The pheromone also has evaporation properties based on the atmospheric conditions. The evaporation property helps the ant to eliminate the longer path and to select the shortest path i.e. shorter paths having higher pheromone densities compared to the longer path. Hence, the ants following the shorter paths would be higher. These behaviors can be modeled mathematically and can be successfully applied to combinatorial optimization problem with constraints. ACO has been successfully applied to the problems such as the traveling salesman problem (Dorigo and Gambardella, 1997), vehicle routing problem [21], single machine scheduling problems (Bauer et al in 1999, Den Besten et al in 2000, Gagne et al in 2002), flow shop problems (Stutzle in 1998, Tkindt et al in 2002), Job shop scheduling problem (Colorni et al, 1994), group shop scheduling problem (Blum et al, 2004), etc. The ACO procedure can be characterized as the following steps. Formulation of the ACO network, initializing the parameters, Ants trail, Global optimization, Penalization, Design re-optimization.

4.1 Formulation of ACO Network

In general, the size of the ACO network will vary from application to application. In this research, each of the design parameters considered as a single design control criteria and occupied in separate layers with multiple nodes having probability values as weightage. The constraints are treated as a bottleneck and having higher penalty values which makes the pheromone to evaporate at proportional rate. So the first layer of the network having 10 nodes for ants entry, second layer is for power with 24 nodes, third layer is for module with 160 nodes, fourth layer is for thickness with 45 nodes and fifth layer is for fitness function and constraint calculations with a node. As the nodes are arranged in sequence, each node should be connected to each other nodes in the adjacent layer.

4.2 Initialization of the Network

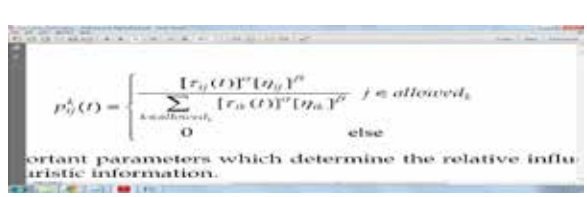
In order to increase the search space, the intensity of all the network linkages have been set to unit value. So that, the initial ants can select the path at random and thereby search space can be increased. The nodes are capable of holding digital values and allocated with all the probability of the design parameter values.

4.3 Ants Trial

Initially, a colony of 10 ants have been selected and allowed to proceed through the formulated network. The ant movement should be restricted in unidirectional and allowed to pass through any nodes in a layer. Once the ant has completed its travel by passing through all the layers, the corresponding fitness function need to be calculated along with the constraint satisfaction. The ant which violates the constraints have to be penalized with the negative value, inturn the decreases the fitness function value and lead to evaporation of the pheromone in the path. On the other hand, the ant path which satisfies the constraints will add weightage and intun increases the fitness function value and lead to deposition of more pheromone on the path. The pheromone updations have been done using the Equation 19.

$$\tau_{ij}(t+1) = \Delta\tau_{ij} + (1-\rho)\ \tau_{ij}(t) \quad (19)$$

Whereas, **'ρ' denotes the evaporation rate of the pheromone and is set as 0.01 in this** research. 'τ' and 't' represent the pheromone value of the path and trails respectively. 'Δτ' represents the proportional change in the fitness function value. These values only help the forthcoming ants to take decision using ants moving rule. Ants moving rule is used for the fourth coming ants to take decision in the shortest route selection. The probability of the k_{th} ant choosing the path is given by Equation 20.



Whereas 'α' and 'β' are the parameters to determine the relative influence of the trail pheromone and the heuristic infor-

mation. Similarly 10 colonies of ants have to be allowed to identify the best path having optimal bevel gear parameters and thus iteration gets completed. Similarly 100 iterations have been allowed by applying global updation rule

4.4 Global Updating Rule

The global updating rule is used to escape from the local optimal point and to search the global optimal point in the search space. The global updating rule is given in the Equation 21.

$$\tau_{ij}(t+1) = \min^{\max}\{\Delta\ \tau_{ij(best)} + (1-\rho)\ \tau_{ij}(t)\} \quad (21)$$

whereas τ_{max} and τ_{min} are respectively the upper and lower bounds imposed on the pheromone and Δ_{best} belongs to the best tour which yields better parameters for the gears. This phase has to be performed to make the search more directed and intended to provide a larger amount of pheromone to better parameter path.

5. Computational results

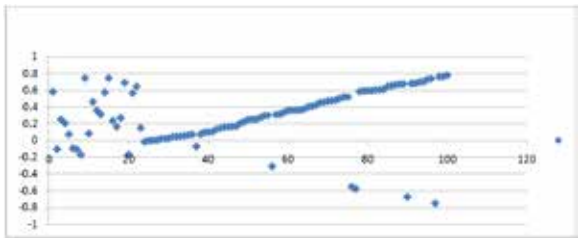
A computational study had been conducted to optimize the bevel gear design with multiple constraints, which was coded in Visual Basic programming. The process of choosing appropriate ant colony parameters is time-consuming, as it is varying from application to application. On the other hand, the execution time became reduced by proper choosing of the ant colony parameters. Fixing of higher evaporation rate dynamically for the path which violates the constraints reduces the computational time to a larger extent. ACO was tested on 100 benchmark instances of different sizes and found that the performances are satisfactory. The same data had also been used to design and analysis in ANSYS software and the obtained results are comparable with the stress values are in safe zone. The sample result obtained for a colony of ants is given in the Figure 1.

B19

	A	B	C
1	Ant	pattern	Fitness function
2	1	12-01-45	0.146418994
3	2	14-12-12	0.259666844
4	3	30-16-12	0.313938794
5	4	32-05-32	0.305430611
6	5	30-10-22	0.294656387
7	6	16-12-16	0.280012276
8	7	12-10-10	0.359666844
9	8	10-02-14	0.16193682
10	9	08-04-22	0.146418994
11	10	32-10-30	0.142857293

Figure 1: Sample Result

The convergence graph obtained for sample iteration is given in the Figure 2. Form the Figure 2, it is clear that the ACO searches in random initially and after global updation the iterations get converged towards the global optimal point. The infeasible solutions are having negative fitness values and marked below the zero x axis line. The maximum fitness function value obtained for the sample is 0.78 and obtained in 260 seconds.



6. Conclusion

In this paper, ACO have been proposed for optimizing the bevel gear design parameters by satisfying the constraints in design, manufacturing and operations. Also the design had been checked for the vibration and noise level in non-lubricated condition. To improve the performance of ACO algorithm, sensitivity analysis had been carried out to fix the design parameters. A new heuristic of penalizing and setting higher evaporation rate for the infeasible paths utilizes the history information for further search and thereby reduces the computational time. The proposed algorithm has been tested on 100 instances and the results are found satisfactory.

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