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Collage in Partial Languages and in Partial Array Languages

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ABSTRACT

An operation called collage on one-dimensional (respectively two-dimensional) language was introduced in [2]. It is obtained by piling up, one on top of the other, words of a given recognizable language on a previously empty one-dimensional (respectively two-dimensional) array. In this paper we extend this operation to partial languages and partial array languages. We show that if P is a partial language then Collage(P) is recognizable using marked local language and if P is a partial array language then Collage(P) is recognizable using tiling system.

KEYWORDS Words, Partial words, Arrays, Partial arrays, catenation, collage, Recognizability.

1. Introduction

Syntatic methods play an important role in picture generation and description on account of their structure handling ability. Berstel and Boasson introduced the partial words in the content of gene comparison [1].

In [2] collage was introduced on one dimensional and two dimensional recognizable languages. The resulting language is the set of words seen from above, a position of the array is labeled by the topmost letter. In the one dimensional case, the language is always recognizable. Giammarresi and Restivo [3] have provided a definition of recognizability of two dimensional array languages in terms of local array languages.

Consider a collection of strips of wall papers of different textures in such a way that it forms a recognizable collection. Starting from an empty frame we can paste these strips one at a time, in any arbitrary way, with the possible overlapping but without rotations. At each position the visible pixel is that belonging to the last pasted strip. If we start with a recognizable collection of strips reduced to one column (one row) then all possible collages form again a recognizable collection.

We extend the **collage operation to partial languages and partial array languages**. Also this paper deals with the notion of recognizable collection of partial pictures, a partial picture being a matrix whose entries are taken from a finite alphabet $\Sigma \cup \{\Diamond\}$ where $\Diamond$ is the do not know symbol

2. Preliminaries and Notations

In the first two sections we recall some basic and important notations concerning words and partial words, collage on words, arrays and partial arrays. In the third section we provide some motivation from molecular biology.

2.1. Words and partial words

Definition 1.

A word of length n over the finite alpha bet Σ can be defined by a total function, $u : \{1, 2, ..., n\} \to \Sigma$ and is usually represented as $u = a1\ a2\ ...$ an with $ai = u(i)$. The length of u is denoted by $|u|$. The word of length '0' is called the empty word and is denoted by ε.

Definition 2.

Let W be a collection of strings or words, let n be an integer and let P be a finite sequence of pairs $(x, u) \in N \times W$ called stack. the Pile_p^n defined by an array of n words in Σ^* is defined by induction on the length of the sequence P.

If P has length 0, then Pile_p^n is an array of n empty words. Otherwise let P' be the sequence P deprived of its last pair (x, u). Let ℓ be the length of u. For all $1 \le i \le n$,

$$\text{Pile}_p^n[i] = \begin{cases} \text{Pile}_{p'}^n[i]\, u[i - x + 1] & \text{if } x \le i \le x + \ell - 1 \\ \text{Pile}_{p'}^n[i] & \text{otherwise} \end{cases}$$

The height of the pile is the maximum of the lengths of its entries.

Definition 3.

Given a pile $p = \text{Pile}_P^n$, its associated collage is the word w of length n, denoted by Collage_p^n, where for i = 1, 2, ..., n the following holds:

$$w[i] = \begin{cases} \square & \text{if } p[i] = \varepsilon \\ a & \text{if } p[i] = ua, u \in \Sigma^*, a \in \Sigma \end{cases}$$

In other words, the collage is obtained by selecting the topmost letter of each column and by taking □ when the column is empty

If the resulting word w does not contain the symbol □, i.e., if $w \in \Sigma^*$, it is said to be completely covered by W. The operation of collage is extended to subsets by setting for all $W \subseteq \Sigma^*$

$$\text{Collage}(W) = \{w \in (\Sigma \cup \Box)^* / w = \text{Collage}_p^n, \; n \in N, \; P \in (N \times W)^*\}$$

Definition 4.
A partial word u of length n over Σ is a partial function u : N → Σ. For 1 ≤ i ≤ n, if u(i) is defined, then we say that i belongs to the domain of u (denoted by i ∈ D(u)); Otherwise, we say that i belongs to the set of holes of u (denoted by i ∈ H(u)). A word over Σ is a partial word over Σ with an empty set of holes.

Definition 5.
If u is a partial word of length n over Σ, then the companion of u (denoted by u◊) is the total function u◊ : N → Σ ∪ {◊} defined by

$$u_\Diamond(i) = \begin{cases} u(i) & \text{if } i \in D(u) \\ \Diamond & \text{otherwise, where } \Diamond \notin \Sigma \end{cases}$$

The symbol '◊' is viewed as a 'do not know' symbol and not as a 'do not care' symbol as in pattern matching.

A partial language L over the alphabet Σ is any subset of Σ_p^*

where Σp = Σ ∪ {Δ}.

2.2. Arrays and partial arrays
Definition 6.
Let Σ be a finite alphabet. A rectangular array over Σ of size (m, n) is a rectangular arrangement of elements of Σ having m rows and n columns where m, n ≥ 0.

The set of all arrays over Σ (includ ing an empty array Λ) is denoted by Σ** and Σ++ = Σ** – {Λ}. Let Σ* and Σ* denotes the horizontal sequences and vertical sequences of letters from Σ respectively. We use θ for row catenation Φ for column catenation.

$$\text{If } X = \begin{matrix} a_{m1} & \cdots & a_{mn} \\ \vdots & \ddots & \vdots \\ a_{11} & \cdots & a_{1n} \end{matrix} \text{ and } Y = \begin{matrix} b_{m'1} & \cdots & b_{m'n'} \\ \vdots & \ddots & \vdots \\ b_{11} & \cdots & b_{1n'} \end{matrix} \text{ then } X \theta Y \text{ is defined only when at least one}$$

$$\text{of them is } \Lambda \text{ or } n = n' \text{ and is given by } X\theta Y = \begin{matrix} a_{m1} & \cdots & a_{mn} \\ \vdots & \ddots & \vdots \\ a_{11} & \cdots & a_{1n} \\ b_{m'1} & \cdots & b_{m'n'} \\ \vdots & \ddots & \vdots \\ b_{11} & \cdots & b_{1n'} \end{matrix}.$$

X Φ Y is defined by only when at least one of them is Λ or m = m′ and is given by

$$X\Phi Y = \begin{matrix} a_{m1} & \cdots & a_{mn} & b_{m'1} & \cdots & b_{m'n'} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{11} & \cdots & a_{1n} & b_{11} & \cdots & b_{1n'} \end{matrix}$$

Definition7.

A partial array A of size (m, n) over Σ is a partial function where Z+ is the set of all positive integers. For 1 ≤ i ≤ m, 1 ≤ j ≤ n if A(i, j) is defined then (i, j) ∈ D(A), otherwise (i, j) ∈ H(A). An array over Σ is a partial array over Σ with an empty set of holes.

If A is a partial array of size (m, n) over Σ, then the companion of A (denoted by A◊ is the total function defined by

$$A_\Diamond(i, j) = \begin{cases} A(i, j) & \text{if } (i, j) \in D(A) \\ \Diamond & \text{otherwise where } \Diamond \notin \Sigma \end{cases}$$

The set of all partial array over Σ is denoted by

Σ_p^{**} where $\Sigma_p = \Sigma \cup \{\Diamond\}$. $\Sigma_p^{++} = \Sigma_p^{**} - \{\Lambda\}$, Λ empty array with no symbols.

The set of all partial arrays over Σ of size (k, r), k £ m, r £ n is denoted by $\Sigma^{p(k,r)}$

2.3. Motivation
The genetic information in almost all organisms is carried by molecules of DNA. A DNA molecule is a quite long but finite string of nucleotides of 4 possible types (A, T, C and G). In nature {A, T, C, G} the four bases occur in pairs [A ↔ U, C ↔ G]. A protein is a string of aminoacids of 20 possible types. Proteins are synthesized from genes.

Copying operations on single strands do not work without errors and this result in mismatches. Partial words were introduced in the context of gene (or protein) comparison by Berstel and Boasson.

Partial words play an important role in DNA sequencing. The partial DNA arrays are used to study difference in gene expressions between wild type and the signaling mutants.

3. Collage in Partial Words
In this section we extend the notion of collage of one-dimensional words introduced in [2] to partial languages. The collage of partial words consists of piling up partial words belonging to a given collection one on top of the other, above a horizontal line filled with a blank symbol □.The result is the partial word or a word seen from above, the top symbol at each position obscuring all symbols under it. If the top symbol is the do not know symbol ◊, then the symbol belonging to Σ under ◊ is considered as a topmost symbol. If there is no symbol belonging to Σ under ◊, then ◊ is considered as a top most symbol.

Example 1.
Consider n = 12, and assume the partial words a ◊ a, b ◊ a ◊ c, c ◊ a, a ◊ b b a c ◊ c belong to the subset W and are placed respectively at the positions 2, 4, 10 and 1 in that order. The resulting word is at the top of the Table 1.

Table 1.

a	a	b	b	a	c	◊	c	□	c	◊	a
a	◊	b	b	a	c	◊	c				
									c	◊	a
			b	◊	a	◊	c				
	a	◊	a								
1	2	3	4	5	6	7	8	9	10	11	12

Table 2. The pile resulting from Example 1

			b4							
	$\Diamond_4$	b_4	b_3	a_4	c_4	$\Diamond_4$	C_4			
a_4	a_1	$\Diamond_1$	a_1	$\Diamond_2$	a_2	$\Diamond_2$	c_2	c_2	$\Diamond_2$	a_2

Definition 8.
Let W be a collection of partial words, let n be an integer and let P be a finite sequence of pairs (x, u) ∈ N × W called a stack. The pile Pile_P^n defined by these data is a partial array of n partial words in (Σ ∪ {◊})* which is defined by induction on the length of the sequence P as follows.

If P has length 0, then Pile_p^n is a array of n empty words. Otherwise let P′ be the sequence P deprived of its last pair (x, u). Let ℓ be the length of u. For all 1 ≤ i ≤ n, we define

$$\text{Pile}_p^N[i] = \begin{cases} \text{Pile}_p^n[i]\ u[i-x+1], & \text{if } x \le i \le x+\ell-1 \\ \text{Pile}_p^n[i], & \text{otherwise} \end{cases}$$

The height of the pile is the maximum of the length of its entries.

In the above example, the pile is a partial array with 12 elements and its height is 3 (a4, a1 ◊4, ◊1 b4, a1 b2 b4, ◊2 a4, a2 c4, ◊2 ◊4, c2 c4, ε, c3, ◊3, a3).

Definition 9.
Given a pile P = Pile_p^n, its associated collage is the word W or a partial word W of length n, denoted by Collage_p^n, where for i = 1, ..., n, the following holds:

$$W(i) = \begin{cases} \square, & \text{if } P[i] = \varepsilon, \\ a \in \Sigma \text{ if } P[i] = ua,\ u \in (\Sigma \cup \{\Diamond\})^*,\ a \in \Sigma, \\ \Diamond, & \text{if } P[i] = u\Diamond,\ u \in \{\{\Diamond\}\}^*, \\ a \in \Sigma \text{ if } P[i] = u\Diamond,\ u \in (\Sigma \cup \{\Diamond\})^*. \end{cases}$$

In other words, the collage is obtained by selecting the topmost letter of each column, if the topmost letter belongs to Σ. Otherwise the letter belonging to Σ in that column if it exists, otherwise the topmost letter is taken as the do not know symbol ◊ and by taking □ when the column is empty.

If the resulting word W does not contain the symbol □, i.e., if W ∈ (Σ ∪ {◊})* it is said to be completely covered by W. We extend the operation of collage to subsets by setting for all W ⊆ (Σ ∪ {◊})*

$$\text{Collage}(W) = \{W \in (\Sigma \cup \{\Diamond\} \cup \square)^* / W = \text{Collage}_p^n,\ n \in \mathbb{N},\ p \in (\Sigma \times \Sigma \times W)^*\}$$

Theorem 1.
If W ⊂ (Σ ∪ {◊})* is recognizable then so is Collage(W).

Proof.
It suffices to prove that the set of completely covered partial words or words is recognizable. We denote this set by covered(W) then we have

Collage(W) = □*(Covered(W)□+)*Covered(W)□*.

The partial language over a given alphabet Σ ∪ {◊} is called marked local whenever it is possible to partition Σ ∪ {◊} = I ∪ H ∪ F ∪ {◊}, in such a way that the partial word belongs to the partial language if and only if its initial letter belongs to I ∪ {◊}, its final letter belongs to F ∪ {◊}, the remaining letters belong to H ∪ {◊} and the transitions between consecutive letters belong to a subset V ⊆ Σ ∪ {◊} × Σ ∪ {◊}. The recognizable language W is the image of some marked local language U in a letter to letter morphism f. Consequently we may assume W itself is marked local. But for each w ∈ Covered(W) there exists a pile of height at most 2N whose associated collage is w, where N be the number of states of an automaton recognizing W. It suffices to consider pile of partial words of length bounded by some integer h to generate all partial words in the collage of W. If we prove that the set of piles viewed as partial words over the alphabet $\ddot{A}\bigcup_{0\le i\le h}(\acute{O}\cup\{\Diamond\})^i$ is recognizable, then the collage itself is recognizable. This is achieved by showing that the language Pile(W) is marked local. The possible initial (respectively final) letters (over the alphabet δ) are of the form a1 ... ak with 0 < k ≤ h and ai ∈ I ∪ {◊} (respectively ai ∈ F ∪ {◊}) for i = 1, 2, ..., k. The allowed transitions are the pairs (A, B) where A = a1 ... ak and B = b1 ... bℓ satisfying the following condition. Let $a_{i_1}, a_{i_2}, ..., a_{i_r}$ with 1 ≤ i1 < i2 ... < ir be the sequence of the letters of the alphabet I ∪ H ∪ {◊} in A and $b_{j_1}, b_{j_2}, ..., b_{j_s}$ with 1 ≤ j1 < j2 ... < js be the sequence of the letters of the alphabet F ∪ H ∪ {◊} in B. Then r = s and the pairs (a_{i_t}, b_{j_t}) belong to V for t = 1, 2, ..., r.

We describe the above construction with the example of Table 2, without distinguishing explicitly between the three subalphabets I, F and H. Consider the transition between columns 4 and 5. Then A = $a_1b_2b_4$ and B = $\Diamond_2a_4$. For A, the subsequence of letters in I ∪ H ∪ {◊} is a_1, b_2 and b_4, for B the subsequence of letters in F ∪ H ∪ {◊} is $\Diamond_2$, a_4. Similarly consider the transition between columns 5 and 6.Then A = $\Diamond_2a_4$ and B = a_2c_4. For A the subsequence of letters in I ∪ H ∪ {◊} is $\Diamond_2$, a_4 and for B the subsequence of letters in F ∪ H ∪ {◊} is a_2, c_4.

4. Collage in Partial Arrays

Definition 10.
Let $p \subseteq \Sigma^{*\square*}$ be a collection of partial pictures called patches. Let (r, c) be a pair of integers and s be a finite sequence of triples (x, y, p) ∈ N2 × P called stack. The $\text{Collage}_S^{(r,\phi}$ is the r × c partial array of symbols in Σp ∪ {□} defined by induction on the number of elements in S as follows. If s = ϕ then $\text{Collage}_S^{(r,\phi}$ is the r × c array whose entries are all equal to the letter □. Otherwise let S′ be the sequence S deprived of its lost triple (x, y, p).

$$\text{Collage}_S^{(r,c)}(i,j) = \begin{cases} \square & \text{if } P(i,j) = \varepsilon \\ \Diamond & \text{if } P(i,j) = \Diamond \\ P(i-x+1, j-y+1) & \text{if } i \in (x, x+s(p)-1), j \in (y, y+s(p)-1) \\ \text{collage}_S^{(r,c)} & \text{if } i \notin (x, x+s(p)-1), j \notin (y, y+s(p)-1) \end{cases}$$

The resultant may the partial array or array.

Example 3.

The sequence of triples S = {(1, 1, p1), (2, 3, p2), (3, 2, p3), (4, 2, p4)} where

$P_1 =$

a	b
◊	c
b	a

$P_2 =$

a	c
◊	b

$P_3 =$

a	c
b	◊
a	b
◊	a

$P_4 =$

c	◊
a	a

defines the collage

1	a	b	□	□
2	◊	c	a	c
3	b	a	c	b
4	□	c	◊	□
5	□	a	a	□
6	□	◊	a	□

Example 4.
The sequence of triples

S = {(1, 1, p_1), (2, 1, p_2), (3, 2, p_3), (4, 2, p_4)}

where

P_1 = | a | b |
| ◊ | c |
| b | a |

P_2 = | a | c |
| ◊ | b |

P_3 = | a | c |
b	a
a	b
◊	a

P_4 = | c | ◊ |
| a | a |
| a | c |

defines the collage

a	b	□	□
◊	c	a	c
b	a	c	b
□	c	◊	□
□	a	a	□
□	◊	a	□

In studying the languages of partial pictures by applying the collage operation from a recognizable partial picture language requires to extend the operation to subsets of partial picture.

Definition 11.
A local system in a pair (Σ, θ) where Σ is a finite alphabet and θ a subset of $\Sigma^{2\times2}$. The language defined by the system is a set of all pictures whose 2 × 2 subpictures belong to θ.

Definition 12.
A tiling system is a quadruple (Σ, Δ, θ, h) where

Σ and Δ – finite alphabets

θ – a subset of Δ2×2

h : Δ → Σ – a projection.

The language recognized by the tiling system is the projection by h of the local language recognized by the local system (Δ, θ).

A language is tiling recognizable if it is recognized by some tiling system.

Definition 13
Given a set of partial patches $P \subseteq \Sigma_p^{**}$ its collage closure is the set

Collage(P) = {P ∈ (Σp ∪ □}*X* / p = $\text{Collage}_S^{(r,\phi}$ for some r, c and S} when the resulting partial picture or picture does not contain any □ it is said to be completely covered by P.

The collage of a set of recognizable partial patches is recognizable when the partial patches all have a unique column (unique row). Also it is recognizable when stacks associated with a collage have bounded height.

Proposition 1.
If $X \subseteq \Sigma_{P_1}^{**}$, $Y \subseteq \Sigma_{P_2}^{**}$ are recognizable languages then F(X, Y) is recognizable.

Proof.

Let $(\Sigma_1, \Delta_1, \Delta_{\theta_1}, \theta_1, h_1)$ and $(\Sigma_2, \Delta_2, \Delta_{\theta_2}, \theta_2, h_2)$ be partial tiling system recognizing X and Y. Define $\Delta_\theta = \Delta_1 \times \Delta_2$, $\theta_3 = \{t / \pi_1(t) \in \theta_1 \wedge \pi_2(t) \in \theta_2\}$, $\Delta_{\theta_3} = \Delta_{\theta_1} \times \Delta_{\theta_2}$ and $h_3(x, y) = f(h_1(x), h_2(y))$ where $\pi_1 : \Delta_1 \times \Delta_2 \to \Delta_1$, $\pi_2 : \Delta_1 \times \Delta_2 \to \Delta_2$ and $\pi_3 : \Delta_{\theta_3} \times \Delta_{\theta_3} \to \Delta_{\theta_3}$ or Δ_{θ_2} are projections. Then the system $(\Sigma_3, \Delta_3, \Delta_{\theta_3}, \theta_3, h_3)$ recognizes F(X, Y).

Proposition 2.
Let $P \subseteq \Theta_p^{*x1}$ (respectively $P \subseteq \Theta_p^{*}$) be a recognizable language of partial patches. Then the partial picture language Collage(P) ⊆ (Σp ∪ {#})*X* is again recognizable.

Proof.
A partial picture in Collage(P) is a row (column) catenation of uni-dimensional collages. Also since if $W \subseteq \Theta_p^{*}$ is recognizable then so is Collage(W) and also by the above definition Collage(P) is recognizable.

x

Theorem 2.
If P ⊆ {a ∪ {◊}}*X* is an arbitrary partial picture language then the partial picture language Collage(P) is recognizable.

Proof:
If p belongs to P then each occurrence of a partial array q is contained in some occurrences of a partial array p satisfying r (p) ≤ r (q) and c (p) ≤ c (q). Thus Collage (P) equals Collage (Q) where Q is the set of minimal partial patches in P where minimal is meant component wise. In the unary the collage corresponds to taking logical OR on pixels. Thus f achieves the logical disjunction of the pixels. It suffices to consider the case where Q is reduced to a unique element.

Let P = {a ∪ {◊}}r×c be a singleton. Consider an element p ∈ Collage (P). Every pixel (i, j) of p can belong to a number of occurrences. Since there is only one non-blank letter, the order in which these partial patches are laid is irrelevant. The number of times a given patch is laid on a given position is also irrelevant. Position of the patches alone is relevant. Therefore we consider the set Bi,j of pairs (k, ℓ) such that the rectangle ar×c can be placed in p with its top left corner at position (i–k+1, j –ℓ+1) that is if the sub picture [i–k+1 … i–k+r] × [j–ℓ+1 … j–ℓ+c] of p is made of all a's.

Let Δ be the power set of {1, …, r} × {1, …, c}. A tiling system (Σp, Δ, Δh, θ, h) recognizing Collage (P) as follows. The primary alphabet is Σp = {a, ◊, □} and the auxiliary alphabet is Δ. Δh is set of holes. The projection h maps ϕ to □ and ◊ to ◊ and every element to a. Consider the subsets of {Δ ∪ {#} ∪ {◊}}2×2

$$\theta_1 = \left\{ \begin{array}{|c|c|}\hline x & y \\ \hline z & t \\ \hline \end{array} \quad \forall k \; \forall \ell \; (k, \ell) \in x \wedge \ell < c \Rightarrow (k, \ell+1) \in y \right\}.$$

$$\theta_2 = \left\{ \begin{array}{|c|c|}\hline x & y \\ \hline z & t \\ \hline \end{array} \quad \forall k \; \forall \ell \; (k, \ell) \in y \wedge \ell > 1 \Rightarrow (k, \ell-1) \in x \right\}.$$

$$\theta_3 = \left\{ \begin{array}{|c|c|}\hline x & y \\ \hline z & t \\ \hline \end{array} \quad \forall k \; \forall \ell \; (k, \ell) \in x \wedge k < r \Rightarrow (k+1, \ell) \in z \right\}.$$

$$\theta_4 = \left\{ \begin{array}{|c|c|}\hline x & y \\ \hline z & t \\ \hline \end{array} \quad \forall k \; \forall \ell \; (k, \ell) \in z \wedge k > 1 \Rightarrow (k-1, \ell) \in x \right\}.$$

We set $^{**}\square^{*}$ We make the condition of the form (k, ℓ) ∈ x is false wherever x = #. It is clear that all pictures in Collage (P)

are recognized by tiling system. Conversely assume a picture is recognized by the system. Then it suffices to observe that if (k, ℓ) is one element a subset of Δ or Δh which labels the pixel at position (i, j) then all pixels at position (i+α, j+β) satisfying i–k+1 ≤ i–k+r and j–ℓ+1 ≤ j–ℓ+c are labeled by a subset containing the element (k+α, ℓ+β) proving thus that the picture is the union of occurrences of the rectangle.

Discussions

Partial words are useful in the context of gene comparison. Alignment of two genes can be viewed as a construction of two partial words that are said to be compatible. DNA sequence can be viewed as a long strings in the 4 letters alphabet of nucleotides (A,C,G,T).

DNA sequencing is the process of obtaining from a DNA molecule its base sequence.

The computational task involved in DNA sequencing is called fragment assembly of DNA. There is a technique to cut random pieces of a long DNA molecule and to produce enough copies of the pieces to sequence. Suppose the input is composed of the four sequences. The only guidance to assembly apart from the target are the overlaps between fragments. Therefore assembling of the random pieces of DNA molecule is similar to applying collage opearation to partial language and partial array language.

Conclusion

In this paper we have extended the collage operation to partial languages and partial array languages and proved that if P is a partial language then Collage(P) is recognizable and if P is a partial array language then Collage(P) is recognizable.

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