Research Paper Mathematics

Commutativity Theorems on Near-Rings With (Σ,Σ)Derivation

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ABSTRACT

Let N be a 3-prime right near ring and D be a (σ,σ)-derivation such that $\sigma D = D\sigma$, where σ is an automorphism. In this paper we investigate commutativity of right near ring with help of (σ,σ) -derivation on Jordan ideal in 3-prime near ring.

KEYWORDS near-ring, commutativity, prime near-ring, Jordan Ideal

INTRODUCTION

Throughout this paper N will be a zero-symmetric right near-ring. The right near ring N is said to be 3-prime if, $xNy = 0$ for $x, y \in N$ implies $x = 0$ or $y = 0$. The symbol $Z(N)$ will denote the multiplicative center of N. A near-ring N is called zero-symmetric if $x0 = 0$, for all $x \in N$ (recall that right distributivity yields $0x = 0$). An additive mapping $d: N \to N$ is said to be derivation if $d(xy) = xd(y) + d(x)y$ for all $x, y \in N$, or equivalently, $d(xy) = xd(y) + d(x)y$ for all $x, y \in N$. we will write for all $x, y \in N$, $[x, y] = xy - yx$ and $x \circ y = xy + yx$ for the Lie product and Jordan product, respectively. The ring N is said to be 2-torsion free if $2x = 0$ implies $x = 0$ for all $x \in N$.

Many results in literature shows that how the structure of a ring is connected with the additive mappings defined on that ring. In this direction, many authors have studied the structure of prime and semi prime rings admitting suitably constrained additive mappings, as automorphisms, derivations, skew derivations and generalized derivations acting on appropriate subsets of the rings.

Motivated by the concept of Jordan ideals on right near rings, which is given by lachen Oukhitite et. al., in the present paper we study the structure of 3-prime near rings in which (σ,σ)-derivation satisfy certain identities involving the Jordan Ideal. The following lemma will be used in our theorems

Lemma 1(Golbasi L., 2004) Let D be a (σ,τ)-derivation of a near-ring N. Then $D(xy) = D(x)\sigma(y) + \tau(x)D(y)$ for all $x \in N$.

Lemma 2 (Golbasi L., 2004) Let D be a (σ,τ)-derivation on a near-ring N and $a \in N$. Then for all $x, y \in N$,
$\tau(a)(\tau(x)D(y) + D(x)\sigma(y))$
$= \tau(a)\tau(x)D(y) + \tau(a)D(x)\sigma(y)$.

Lemma 3 (Golbasi L., 2004) Let D be a nonzero (σ,τ)-derivation on a near-ring N and $a \in N$.
(i) If $D(N)a = 0$ then $a = 0$.
(ii) If $\tau(a)D(N) = 0$ then $a = 0$.

Lemma 4 (Golbasi L., 2004) Let N be a 2-torsion-free near-ring and let D be a (σ,τ)-derivation such that $\sigma D = D\sigma$ and $\tau D = D\tau$. If $D^2 = 0$ then $D = 0$.

Lemma 5 (Oukhtite L., Mamouni A., 2014) Let N be a 3-prime near-ring and J a nonzero Jordan ideal of N. If $Jx = 0$, then $x = 0$.

Lemma 6(Oukhtite L., Mamouni A., 2014) Let N be a 3-prime near ring and J a Jordan ideal of N. If $j^2 = 0$ for all $j \in J$, then $J = 0$.

Main Results:

Theorem 1 Let N be a 2-torsion free 3-prime near-ring and J a nonzero Jordan ideal of N. If N admits a nonzero (σ,σ) derivation d such that $d([n, j]) = 0$ for all $n \in N, j \in J$, then N is a commutative ring.

Proof: Assume that
$$d[n, j] = 0 \quad \forall n \in N, j \in J$$
Replacing n by nj we get,
$$\sigma[n, j]d(j) = 0$$
$$\Rightarrow [n, j]d(j) = 0 \text{ (as } \sigma \text{ is an automorphism)}$$
$$njd(j) = jnd(j)$$
Replacing n by nt, we get,

$$[n, j]Nd(j) = 0$$

$\Rightarrow J \subseteq Z(N)$ or $d(j) = 0$ for all $j \in J$

If $d(j) = 0$ for all $j \in J$ $\exists j_0 \in J$ such that $d(j_0) = 0$. Again

$d[n, j_0] = 0 \Rightarrow d(n)\sigma(j_0) = \sigma(j_0)d(n)$

Replacing n by $nd(m)$, we get,

$$d(nd(m))\sigma(j_0) = \sigma(j_0)d(nd(m))$$

$$\Rightarrow \sigma(n)d^2(m)\sigma(j_0) + d(n)\sigma(d(m))\sigma(j_0)$$

$$= \sigma(j_0)\sigma(n)d^2(m) + \sigma(j_0)d(n)\sigma(d(m))$$

Using $\sigma d = d\sigma$, we get,

$$\sigma(n)d^2(m)\sigma(j_0) = \sigma(j_0)\sigma(n)d^2(m)$$

Replacing n by nt,

$$\sigma(n)\sigma(t)d^2(m)\sigma(j_0) = \sigma(j_0)\sigma(n)\sigma(t)d^2(m)$$

$$\Rightarrow \sigma(nj_0 - j_0n)\sigma(N)d^2(m) = 0$$

$$\Rightarrow \sigma[n, j_0]Nd^2(m) = 0$$

$$\Rightarrow j_0 \in Z(N) \text{ or } d^2(m) = 0$$

$$\Rightarrow [n, j_0]Nd^2(m) = 0$$

If $d^2(m) = 0 \Rightarrow d = 0$ but $d \neq 0$

N is commutative ring .

Theorem 2 Let N be a 2-torsion free 3-prime near-ring and J a nonzero Jordan ideal of N. Then N admits no nonzero $(\tau, 1)$ derivation d such that

(i) $d(noj) = noj$ for all $n \in N, j \in J$

(ii) $d(noj) = -(noj)$ for all $n \in N, j \in J$.

Proof: i) Assume that N admits a non zero $(1, \tau)$ derivation d such that $d(noj) = noj$

Replacing n by nj, we get,

$$\tau(noj)d(j) = 0 \forall n \in N, j \in J$$

$$\Rightarrow njd(j) = -jnd(j)$$

Replacing n by nt we get ,

$$ntjd(j) = (-j)ntd(j)$$

$$\Rightarrow [n, j]Nd(-j) = 0 \forall j \in J, n \in N$$

$$\Rightarrow d(j) = 0 \text{ or } j \in Z(N) \forall j \in J$$

If $d(j) = 0$ $\exists j_0 \in J$ such that $d(j_0) = 0$

Again

$$\tau(j_0)d(n)j_0 = nj_0 + j_0n$$

Replacing n by nj_0

$$\tau(j_0)d(n)j_0 = j_0nj_0 \forall n \in N$$

Replacing n by nj_0t, we get

$$\tau(j_0)d(nj_0t)j_0 = j_0nj_0tj_0$$

$$\Rightarrow \tau(j_0)\tau(n)\tau(j_0)d(t)j_0 = 0$$

$$j_0nj_0Nj_0 = 0$$

$$\Rightarrow j_0 = 0$$

$\Rightarrow N$ is commutative ring. Hence

$$d(jn) = jn$$

Replacing n by nm we get ,

$$\tau(jn)d(m) + d(jn)m = jnm$$

$$\Rightarrow jnd(m) = 0 \forall m, n \in N$$

$$\Rightarrow jNd(m) = 0 \forall n \in N, j \in J$$

$\Rightarrow d = 0$ a contradiction

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