Research Paper Physics

Determining Multi-Junction Solar Cell Parameters Using Lambert-W Function

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ABSTRACT

In this communication, an analytical method to determine the parameters of multi-junction solar cells at a uniform temperature and irradiation using Lambert W-function is proposed. The parameter extraction is applied to two experimental tri-junction solar cells represented by the conventional single diode model containing five parameters at two different concentration ratios. The calculated values are compared with the experimental results showing the accuracy of the proposed approach. The effects of the parasitic resistances on the I-V characteristics are also determined which agrees with the published results. This technique could be useful to understand the behaviour of multi-junction solar cells under real conditions.

KEYWORDS | Multi-junction solar cell, parameter estimation, Lambert W-function

INTRODUCTION

In recent years, intensive research has been carried out on multi-junction solar cells (MJSC) as they are one of the most promising photovoltaic devices reaching an efficiency of 41.6% (Nishioka et al. 2006 & 2010; King et al. 2009) with theoretical efficiencies exceeding 60% (Law et al. 2010). Such cells have been investigated both at high and low concentrations respectively for terrestrial and space applications (Cotal et al. 2000; Bett et al. 2000). Several articles are available in literature for modeling and simulation of its I-V characteristics through analytical methods. Amongst these, the distributed model suggested for a tri-junction solar cell (Emelyanov et al. 2010) was validated against the published data at room temperature for concentration levels up to 5 Suns. Conventional lump single-diode model for a triple-junction cell that describes the effect of irradiance, temperature and spectrum including the subcells' parameters, is reported in (Dominguez et al. 2010) where its model coefficients are extracted. Two diodes equivalent circuit models was calibrated for InGaP/InGaAs/Ge cell data at room temperature under 1 Sun (Reinhardt et al. 2000). On the other hand, the measured I-V curve of each subcell was used in the estimation of subcells' parameters thereby the cell's performance were examined at various temperatures that agrees with the measured characteristics (Nishioka et al. 2004). Both the single- and two-diodes equivalent circuit models were analyzed and compared against published experimental data (Segev et al. 2012) with special emphasis on the temperature and concentration effects. Another method of parameter extraction proposed by Or and Appelbaum for MJSC was the minimization of the difference between the measured I-V and the theoretical I-V characteristics.

To the best of our knowledge, even though Lambert W-function has been extensively used in the study of solar cells (Jain & Kapoor, 2004), its application in case of multi-junction solar cells hasn't been reported. In this paper, a close form analytical solution of an MJSC is obtained using Lambert W-function. Here, we consider the lumped single diode equivalent circuit model of solar cells in which each subcell has a separate I-V relationship. Accordingly, an explicit solution of the terminal voltage of the cell is obtained by solving the transcendental I-V relation. Various parameters are then evaluated and compared with the experimental values. The analysis is limited to three-junction cells only as they represent the present status of photovoltaics for which there are published experimental data with which we can compare.

THEORETICAL FORMULATION

The I-V equation for the subcells of a triple-junction solar cell shown in Fig. 1is given by

$$I=I_{ph,i}-(V_i+IR_{s,i})/R_{sh,i}I_{0,i}(\exp((V_i+IR_{s,i})/n_iV_{th}) - 1) \quad (1)$$

where i represents the subcell number (1=top, 2=medium and 3=bottom). I_{ph}, I_0 and I are the photo-current due to generation of excess carriers by the absorption of sunlight, the diode reverse saturation and the load current respectively; V_{th} is thermal voltage (=k_BT/q, q is the electronic charge, k_B is Boltzmann's constant and T is the temperature), V is the voltage, n is the diode ideality factor; R_s and R_{sh} are the series and the shunt resistances, respectively. In Eq. (1) the reverse saturation current is modeled by a single term, and represents recombination in both the depletion and the quasi-neutral regions (Segev et al. 2012).

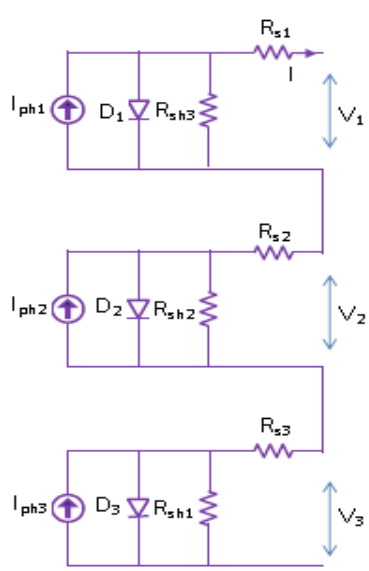


Fig. 1. Single-diode equivalent circuit cell model of a tri-junction solar cell

The terminal voltage is given by the sum of the voltages of three series-connected subcells

$$V = V_1+V_2+V_3 = \Sigma V_i \quad (2)$$

where V_i is the analytical solution of Eq. (1). Solving Eq. (1) using Lambert W-function, we have the explicit expression for V_i as

$$V_i = - IR_{s,i} - (I-I_{ph,i}-I_{0,i})R_{sh,i} - n_iV_{th}\,LambertW(x_i) \quad (3)$$

where

$$x_i = (R_{sh,i}I_{0,i}/n_iV_{th})\exp(R_{sh,i}(-I+I_{ph,i}+I_{0,i})/n_iV_{th}) \quad (4)$$

We may write the terminal voltage in terms of the Lambert W-function as

$$V = -I(R_{s,1}+R_{s,2}+R_{s,3}) - (I-I_{ph,1}-I_{0,1})R_{s,1} - (I-I_{ph,2}-I_{0,2})R_{s,2}$$

$$-(I-I_{ph,3}-I_{0,3})R_{s,3}-n_1V_{th}LambertW(x_1)-n_2V_{th}LambertW(x_2) - n_3V_{th}$$

$$LambertW(x_3) \quad (5)$$

$$V= - IR_s - (I-I_{ph,1}-I_{0,1})R_{s,1} - (I-I_{ph,2}-I_{0,2})R_{s,2} - (I-I_{ph,3}-I_{0,3})R_{s,3}-n_1V_{th}Lam$$

$$bertW(x_1) -n_2V_{th}LambertW(x_2) - n_3V_{th}\,LambertW(x_3) \quad (6)$$

Eq. (6) is the analytical close form expression of the terminal voltage for the tri-junction solar cell in terms of the current and model's parameters which is explicit in nature. Here R_s is the total series resistance ($R_s=R_{s,1}+ R_{s,2}+ R_{s,3}$) that includes the junction tunnel diodes located between the subcells. The substitution of I=0 in Eq. (6) gives the value of the open circuit voltage of the cell and the short-circuit current is obtained by replacing V=0 in Eq. (6).

The output power is

$$P = VI \quad (7)$$

More explicitly in terms of current, it is rewritten as

$$P(i) = I\Sigma (-IR_{s,i}-(I-I_{ph,i}-I_{0,i})R_{sh,i} -n_iV_{th}\,LambertW(x_i)) \quad (8)$$

At the maximum power point (MPP)

$$(\partial P/\partial I) = V + I(\partial V/\partial I) \quad (9)$$

This may be written in terms of Lambert W-function as

$$\Sigma[IR_{s,i} + (I-I_{ph,i}-I_{0,i})R_{sh,i} + n_iV_{th}\,LambertW(x_i) +$$

$$I\Sigma[R_{s,i}+R_{sh,i}+R_{sh,i}\,LambertW(x_i)/(1+LambertW(x_i))] = 0 \quad (10)$$

Solving Eq. (10), the current I_{mp} at the MPP is obtained.

The voltage at the MPP, V_{mp} is then obtained by substituting $I=I_{mp}$ in Eq. (6). Another parameter that determines the cell quality is the fill factor and is calculated by

$$FF = (I_{mp}V_{mp})/(I_{sc}V_{oc}) \quad (11)$$

RESULTS AND DISCUSSION

We apply the analytical solutions derived here to the I-V characteristics of $In_{0.49}Ga_{0.51}P$(top) / $In_{0.01}Ga_{0.99}As$ (middle)/Ge(bottom) tri-junction solar cell and extract its parameters at 25^{0}C under 1 Sun. Using the data from Sakurada et al. 2011 and Segev et al. 2012 [Table 1(a)], various parameters are extracted and compared with the experimental results; relative accuracy is also calculated [Table 1(b)]. Since the cell current is clamped by the subcell that produces the lowest current in the cell (Or and Appelbaum, 2013), I=min(I_{top}, I_{mid}, I_{bot}) where I_{top}, I_{mid}, I_{bot} are the current of the top, middle and bottom subcells, respectively, the short-circuit of the cell current is limited by that of the top subcell which is evident from the table. Similarly, using the data of another tri-junction cell [Table 2(a)] i.e., InGaP/GaAs/Ge, we estimated its parameters at 25^{0}C under 350 Suns to establish its general applicability for other concentrations also. The evaluated values are then compared with the experimental data and the relative accuracies are also determined [Table 2(b)].

Each subcell has a different I-V characteristic such that the current is limited by that of the top cell which is theoretically obtained through the solutions of Lambert W-function [Table 1(b)]. It is observed that the effect of the various shunt resistances in the top junction has a greater influence on the I-V characteristics of MJSC at 250C under 1 Sun. The influence of R_s on the I-V curve and the variation in the power generated with its terminal current are depicted. These results are in agreement with the published results of Nishioka et al. (2004).

Table 1(a) Data from Sakurada et al. 2011 and Segev et al. 2012 from which various parameters of MJSC are calculated under 1 Sun.

Parameters	InGaP	InGaAs	Ge
I_{ph} (A)	6.7522×10^{-3}	7.7126×10^{-3}	10.094×10^{-3}
I_0 (A)	3.30×10^{-15}	6.00×10^{-11}	3.00×10^{-5}
n	1.97	1.75	1.96
R_s (Ω)	0.0236	0.0012	0.0008
R_{sh} (Ω)	16.0×10^6	4.5×10^6	540

Table 1 (b) Comparing the calculated values with the experimental results for $In_{0.49}Ga_{0.51}P/In_{0.01}Ga_{0.99}As$)/Ge tri-junction solar cell under 1 Sun at 25^{0}C

Parameters	Lambert W-function	Experimental Data [Sakurada et al]	Accuracy (%)
V_{oc} (V)	2.5655	2.53	1.403
I_{sc} (mA)	6.7522	6.74	0.181
V_{mp} (V)	2.2384	------	------
I_{mp} (mA)	6.5746	------	------
P_{mp} (mW)	14.716	14.475	1.665
FF	0.849	0.849	0.0

It is seen that this method requires undergoing a lot of intensive calculations to determine the parameters of the cell. However, it presents the advantage of being explicit in nature and also the close agreement between the experimental values and the theoretical results obtained validates this method.

Table 2(a) Data from Or and Appelbaum from which various parameters of MJSC are calculated under 350 Suns.

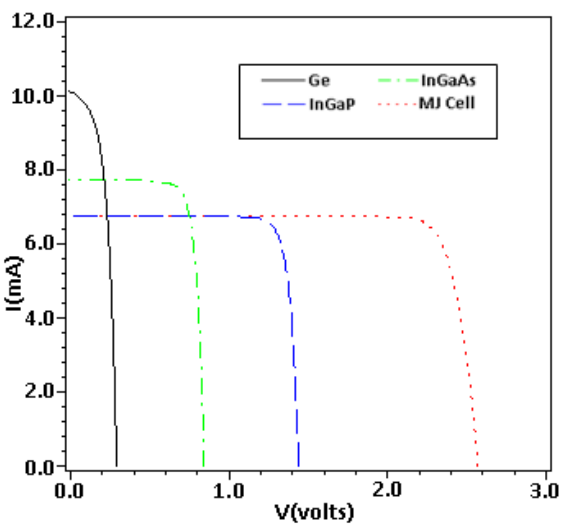
Parameters	InGaP	GaAs	Ge
I_{ph} (A)	4.292	4.792	6.101
I_0 (A)	1.101×10^{-13}	2.813×10^{-13}	8.543×10^{-7}
n	1.993	1.278	1.421
R_s (Ω)	7.195×10^{-3}	3.352×10^{-3}	2.492×10^{-4}
R_{sh} (Ω)	587	389	100

Table 2(b) Comparing the calculated values with the experimental results using the data from [14] for InGaP/GaAs/Ge tri-junction solar cell under 350 Suns at 25^{0}C

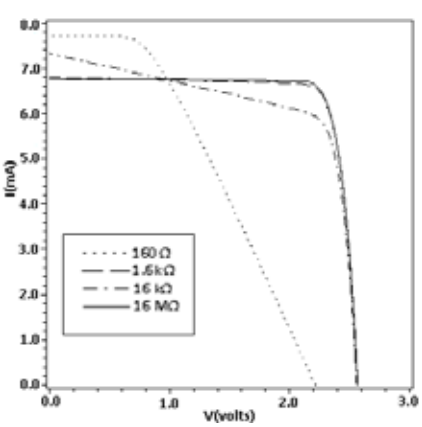
Parameters	Lambert W-function	Experimental Data [Or & Appelbaum]	Accuracy (%)
V_{oc} (V)	3.1218	3.2	2.443
I_{sc} (A)	4.2943	4.308	0.318
V_{mp} (V)	2.7711	2.845	2.597
I_{mp} (A)	4.2007	4.215	0.339
P_{mp} (W)	11.64	12.0	3.0
FF	0.868	0.87	0.23

CONCLUSIONS

A close form solution for the terminal voltage of a triple-junction solar cell in terms of the terminal current and the model's parameters using the single-diode model by the application of Lambert W-function is presented. The method optimizes the calculations and the parameter extraction. The explicit nature of the solution was put into test with the published experimental data. A theoretical analysis of the influence of parasitic resistances on the characteristics of the tri-junction cell is found to be in agreement with the results available in literature. We may say that this analytical approach is a strong theoretical one which can be applied for the investigation of MJSCs having more than three junctions for its analysis and optimal design.



(a)



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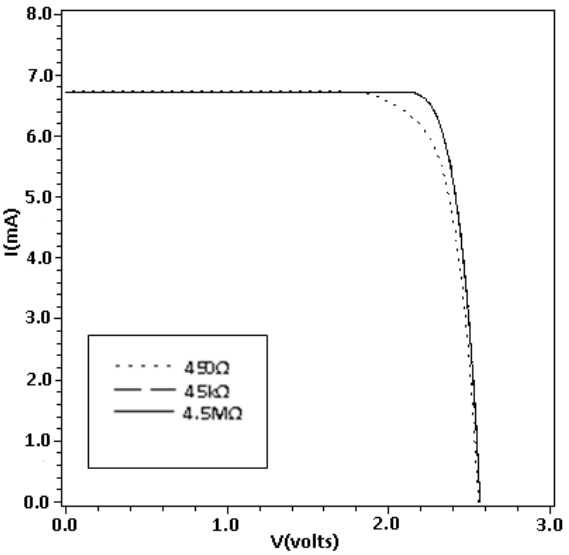

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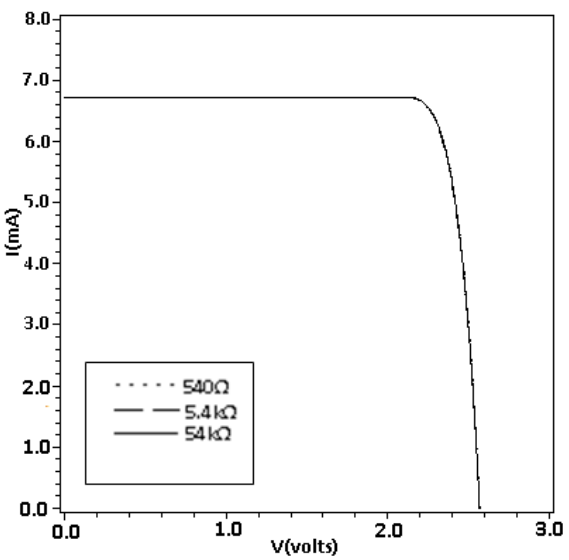

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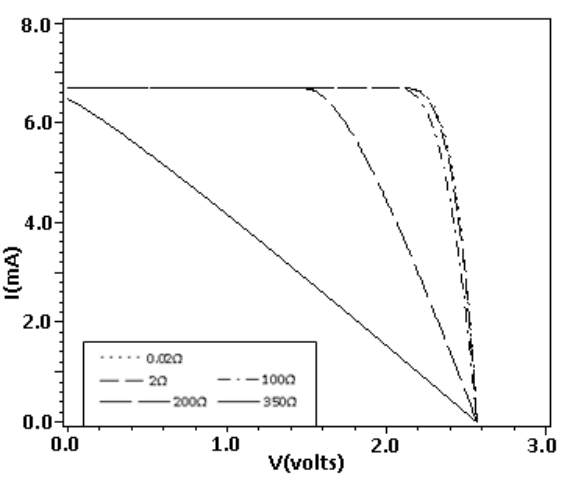

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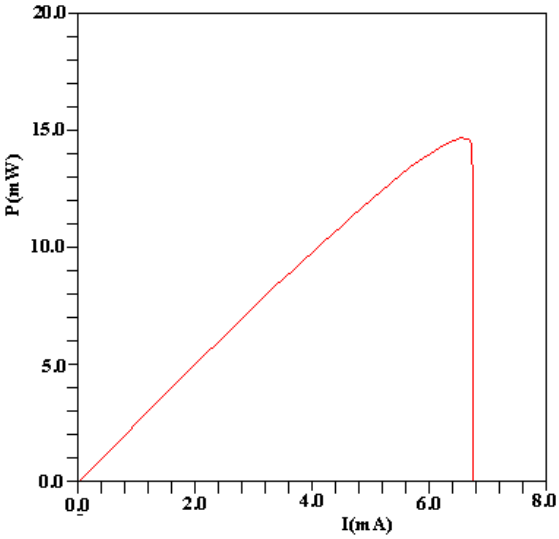

(f)

Fig. 2. Various characteristic curves of InGaP/InGaAs/Ge triple-junction solar cell at 25^{0}C under 1 Sun (a) I-V characteristics of the subcells, (b) Effect of various R_s values on the I-V curve, (c) Effect of various shunt resistance $R_{sh,1}$ in the top junction, (d) Effect of various shunt resistance $R_{sh,2}$ in the middle junction, (e) Effect of various shunt resistance $R_{sh,3}$ in the bottom junction, (f) Variation of power with current

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