Research Paper Medical Science

Effect of Aneurysm on Blood Flow through Small Arteries

Mohammed Musad Saleh | University of Aden , Oil and Mineral Faculty, Yemen

ABSTRACT

In this paper the mathematical models have been developed to study the effect of aneurysm of artery on the blood flow hydrodynamics. The artery considered to be rigid vessel and the blood flow to be non-Newtonian fluid flow of viscosity μc. It is shown that wall shear stress has an inversely proportional relation with aneurysm radius R_h and pressure drop so. Wall shear stress also has direct proportional relation with viscosity. The study also estimated the ranges of wall shear stress, pressure drop, and viscosity for small arteries of radius 0.02 and arterioles of radius 0.0025 cm. It is observed that wall shear stress have maximum value when Rh=1.4R_0, and minimum value equals yield stress at Rh=2R_0.

KEYWORDS Aneurysm, Rigid Vessel, Wall shear Stress, and non-Newtonian Fluid.

1- Introduction

The artery with small diameter μ2m considered to be rigid artery have resistance to flow, so they need a pressure difference along the length of the vessel to drive the blood flow, and the blood flow through it considers to behave like a couple stress fluid, plasma layer near the walls consisting of only the plasma with viscosity coefficient μ Newtonian fluid and core layer consisting of red cells in plasma with viscosity coefficient μ_c Non- Newtonian.(Kapur 1985). (Musad 2013) have assumed that, the non-Newtonian fluid in core layer as Casson fluid and considered that Casson fluid model holds for blood flow in tubes of diameter 0.013 cm. The flow of blood in the vessels of the circulation system can be modeled using the resistive and compliant nature of the flow. These models can be used to understand the problems in the circulation system, such as aneurysms (Irving 2007). Aneurysm is an enlarged blood vessel caused by a localized weakening and dilation. Factors that contribute to the weakening of the wall tissue include irregular wall shear stress (Kondo *et al* 1997). This weakening can lead to rupture of blood artery at aneurysm location, which can cause internal bleeding and stooped the blood supply. For instance, rupture an aneurysm in the artery supplying blood to brain can bring about strokes. Likewise in abdominal artery leads to death from internal bleeding in 75% to 90% of cases (Sheard 2008). (Shojima *et al* 2004) have reported low wall shear stress related to aneurysm growth and rupture as it promotes various mechanisms that cause arterial wall remodeling.

2- Geometry of Aneurysm Model and its Boundary Equation

The figures below present the geometry of the model and its coordinates system. The aneurysm introduced into the geometry is made to be axis symmetrical i.e. symmetrical around the axis of artery. From figure 1 it is clear that $2z_0$ is the artery aneurysm length through which the blood flows, R_0 is the radius of the artery in normal condition, *R(z)* is the radius of the aneurysm part of the artery, and *h* is the extra length on normal radius.



Figure 1: The geometry of aneurysm artery

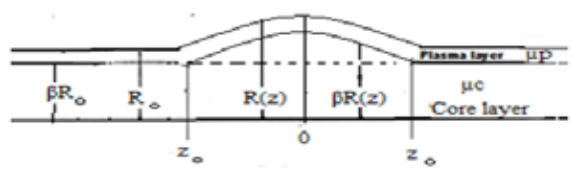


Figure 2: Coordinate system of flow in an aneurysm artery

The boundary of the aneurysm is represented in below equations

$$R(z)=\begin{cases}(1+\dfrac{h}{2R_0}(1+COS2\pi\dfrac{z}{z_0}) & -z_0<z\le z_0,\\ 1, & 2z_0<z\le -z_0\end{cases} \quad (1)$$

$$R_1(z)=\begin{cases}\beta(1+\dfrac{h}{2R_0}(1+COS2\pi\dfrac{z}{z_0}) & -z_0<z\le z_0,\\ \beta, & 2z_0<z\le -z_0\end{cases} \quad (2)$$

While the equation of velocity in aneurysm vessel can be expressed as

$$V_c=\frac{\Delta p}{4L\mu_c}[R^2(z)-R_0^2]+\frac{\Delta p}{\mu_c}[R^2(z)-(\beta R)^2(z)][\frac{\mu_c}{\mu}-1]\,0\le R_1(z)<R(z)and,0<\beta<1 \quad (3)$$

Where ΔP is pressure drop, μc is the core layer viscosity, μ is the viscosity of plasma layer, L is the length of artery, z_0 is the artery aneurysm length through which the blood flows, R_0 is the radius of the artery in normal condition, *R(z)* and $R_1(z)$ are the radii of the two layers of aneurysm part of artery, and *h* is the extra length on normal radius.

3- The Model Developed

We assume that the artery to be raged and the small artery under consideration for the development of model is with an axially aneurysm (fusiform). The blood flow is assumed to behave like a couple stress fluid. Peripheral layer is plasma with viscosity coefficient μ and core suspension layer is erythrocytes with viscosity coefficient μ_c.

For small artery the equations of wall shear stress and velocity are given as follows

$$\sqrt{\tau}=\sqrt{-\mu\frac{\partial v}{\partial r}}+\sqrt{\tau_y} \quad (4)$$

By taking the derivative of equation (3), we get

$$(\frac{\partial v}{\partial r})_{r=R(z)} = \frac{\Delta p}{2L\mu}[R(z) + 4(\frac{\mu_c}{\mu} - 1)R(z) - 4(\frac{\mu_c}{\mu} - 1)R_1(z)] \quad (5)$$

$$(\frac{\partial v}{\partial r})_{r=R(z)} = \frac{\Delta p}{2L\mu}[R(z) + 4(R(z)(\frac{\mu_c}{\mu} - 1) - R_1(z)(\frac{\mu_c}{\mu} - 1)] \quad (6)$$

But $R_1(z) = \beta R(z)$

$$(\frac{\partial v}{\partial r})_{r=R(z)} = \frac{\Delta P}{2L\mu}[R(z) + 4(R(z)(\frac{\mu_c}{\mu} - 1) - \beta R(z)(\frac{\mu_c}{\mu} - 1)] \quad (7)$$

Put $\beta = 0.95$ then equation (7) reduced to

$$(\frac{\partial v}{\partial r})_{r=R(z)} = \frac{\Delta p}{10L\mu}[(4 + \frac{\mu_c}{\mu})R(z)] \quad (8)$$

Then from equations 1, 4 and 8 the wall shear stress along the aneurysm artery will be as

$$\sqrt{\tau} = \sqrt{-\mu\frac{\Delta p}{10L\mu}[(4 + \frac{\mu_c}{\mu})\int R(z)dz}} + \sqrt{\tau_y} \quad (9)$$

$$\sqrt{\tau} = \sqrt{-\frac{\Delta p}{10L}[(4 + \frac{\mu_c}{\mu})\int_{-z_0}^{z_0}(1 + \frac{h}{2R_0}(z + \cos 2\pi\frac{z}{z_0}))dz + \int_{2z_0}^{-z_0}1.dz}} + \sqrt{\tau_y} \quad (10)$$

$$\sqrt{\tau} = \sqrt{-\frac{\Delta p}{10L}z_0[(4 + \frac{\mu_c}{\mu})(\frac{h}{R_0} - 1)}} + \sqrt{\tau_y} \quad (11)$$

$$\sqrt{\tau} = \sqrt{-\frac{\Delta p}{10L}z_0(4 + \frac{\mu_c}{\mu})(\frac{h}{R_0} - 1)} + \sqrt{\tau_y} \quad (12)$$

$0 < h \leq R_0$ put $h + R_0 = R_h$ then equation (12) reduce to

$$\sqrt{\tau} = \sqrt{-\frac{\Delta p}{10L}z_0(4 + \frac{\mu_c}{\mu})(\frac{R_h}{R_0} - 2)} + \sqrt{\tau_y} \quad (13)$$

Equation (13) is the equation of wall shear stress of blood flow in small artery with aneurysm. Where τ is wall shear stress, R_0 is the radius of the vessel, z_0 is the half length of aneurysm part, ΔP is pressure drop, R_h is the radius of aneurysm part, L is the total length of artery, μ, μ_c are the viscosity of plasma layer and core layer respectively and τ_y is the yield stress.

4- Boundary Conditions

1- When Rh=2R0 equation (13) become

$\tau = \tau_y$, This means that, wall shear stress equals yield stress

2- When $\mu = \mu_c$ and $\tau_y = 0$ equation (13) reduce to power low model as follow

$\tau = \frac{\Delta P}{2L} z_0 (\frac{R_h}{R_0} - 2)$, which hold good for Newtonian fluid flow

5- Results and Discussion

The results based on the numerical solution of equation (13) by using mathematical software for R_0=0.02cm, L=1cm, $z_0=R_0$, yield stress=0.3dynes/cm^2 and for aneurysm radius (R_h) range ($1.4R_0$ to $2R_0$) or 0.028 to 0.039 cm for small artery and 0.0035 to 0.005 for arteriole, show that, wall shear stress decreases with the increase of aneurysm radius R_h and pressure drop so. It is observed that wall shear stress decrease with the decrease of core layer viscosity μ_c while other parameters are constants.

The results of wall shear stress, pressure drop and viscosity of aneurysm artery of radius 0.02cm are listed in tables 1 and graph 3. Its 1shown that the range of wall shear stress is (1.12 to 0.54 dyes/cm^2) and pressure drop range is (25 to 40 dynes/cm^2) for aneurysm radius (R_h) range 0.028 to 0.039 cm, and maximum viscosity 0.04dynes-s/cm^2. And for the same range of aneurysm radius R_h and maximum pressure drop 40, the range of wall shear stress is (1.08 to 0.41 dynes/cm^2).

While in table 2 and graph 4 we listed the results of wall shear stress, pressure drop and viscosity for the arterioles of radius R_0=0.0025 cm and length L=1cm. the results predict the ranges of wall shear stress, pressure drop and viscosity are (0.41 to 0.30), (6 to 10), and (0.04 t0 0.01) respectively, for the range of aneurysm radius 0.0035 to .0050 cm.

It is clear the maximum of wall shear stress at $R_h=1.4R_0$ is suitable to maintaining the thickness of aneurysm wall. But enlarge radius of aneurysm part $R_h \geq 1.4R_0$ leads to large pressure and low wall shear stress. This change in pressure will damage the thickness of artery and become compliance artery acceptable for rupture, response to increase a pressure drop and decrease the wall shear stress.

Table 1. Estimated wall shear stress, pressure drop and viscosity of aneurysm artery for range of R_h from 0.028 to 0.039cm, $z_0 = R_0$ where R_0=0.02cm, L=1cm.

R_0	Pressure drop	viscosity	Wall shear stress
0.028	25.0	0.040	1.14
0.029	26.6	0.036	1.12
0.030	28.3	0.033	1.08
0.032	30.0	0.030	1.04
0.033	31.6	0.026	1.00
0.034	33.3	0.023	0.94
0.035	35.0	0.020	0.90
0.036	36.6	0.016	0.80
0.037	38.0	0.013	0.70
0.039	40.0	0.010	0.55

Table 2. Estimated wall shear stress, pressure drop and viscosity of aneurysm for arterioles of, R_0=0.0025cm, $z_0 = R_0$, L=1cm, and range of R_h from 0.0035 to 0.005cm

R_0	Pressure drop dynes/cm^2	Viscosity dynes-s/cm^2	Wall shear stress dynes/cm^2
0.0035	6.00	0.040	0.410
0.0036	6.44	0.036	0.404
0.0038	6.88	0.033	0.400
0.0040	7.33	0.030	0.396
0.0042	7.77	0.026	0.389
0.0043	8.22	0.023	0.382
0.0045	8.66	0.020	0.372
0.0046	9.11	0.016	0.360
0.0048	9.55	0.013	0.343
0.0050	10.0	0.010	0.300

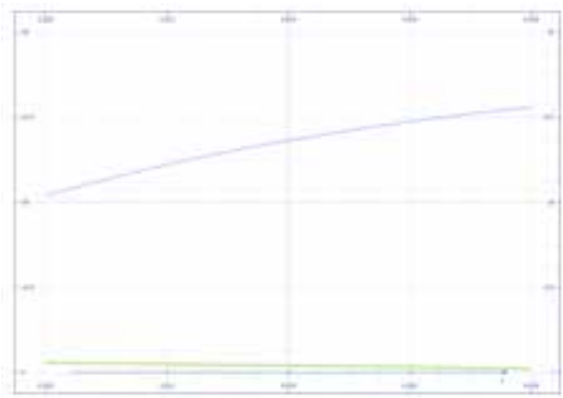


Figure 3: Graph of wall shear stress and pressure drop of blood flow in aneurysm artery of R_0=0.02cm with respect to aneurysm radius R_h.

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