Research Paper Mathematics

Exact Solution to Unsteady Mhd Dusty Fluid Flow Over a Moving Horizontal Plate Through a Porous Space

A.Mohamed Ismail | Department of Mathematics,Sathyabama University,Chennai,India

S.Ganesh | Department of Mathematics,Sathyabama University,Chennai,India

ABSTRACT

This work is focused on the study of unsteady magnetohydrodynamic for a viscous laminar incompressible electrically conducting flow of dusty fluid between two parallel plates through a porous space with one in uniform motion and the other plate at rest with uniform suction at the stationary plate is discussed. The governing system of linear partial differential equations are transformed into ordinary differential equations using similarity transformation method. The axial velocity,transverse velocity of the dusty fluid and velocity of the dust particles were presented. Analytical expression is given for the velocity field and the effects of the various parameters on th flow field entering into the problem are discussed. . It is found that velocity distribution for clean fluid and dust fluid decreases with the increase of fluid particle interaction parameter.

KEYWORDS | Fluid flow, MHD flow, parallel plates, dust particles, similarity transformation,porous medium.

INTRODUCTION

The magnetohydrodynamic (MHD) flow of a viscous fluid through a porous medium in the presence of a magnetic field has become the basis of many scientific and engineering applications. The necessities of modern machinery have motivated the interest in fluid flow studies, which involve the interaction of several phenomena. One such study is presented, when a viscous fluid flows over a porous surface has its significance in many engineering problems such as flow of liquid in a porous bearing (Joseph and Tao L.N [1]), in the field of water in river beds, in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs, in chemical engineering for filtration and purifications process. Hassaninen I.A. Mansour M.A. [2] has investigated the magnetic flow through the porous medium between two infinite plates. Hamza E. A. [3] has studied the suction and injection effects of flow between parallel plates. Soundalgekar V and Uplekar A [4] studied the effect of heat transfer considering constant temperature. Sing C.B and Ram P.C [5] considered laminar flow of an electrically conducting fluid through a channel in the presence of transverse magnetic field under the influence of periodic pressure gradient and solved the resulting differential equation by the method of Laplace transform. Cunningham R.E and Williams R. J [6] also reported several geophysical applications of flow in porous medium, viz. porous rollers and its natural occurrence in the flow of rivers through porous banks and beds and the flow of oil through underground porous rocks. The mathematical theory of the flow of fluid through a porous medium was initiated by Darcy H [7]. For the steady flow, he assumed that viscous forces were in equilibrium with external forces due to pressure difference and body forces. In the most of the examples, the fluid flows through porous medium, have two regions. In region I, the fluid is free to flow and in region II, the fluid flows through the porous medium.The authors C.S.Bagewadi and B.J.Gireesha [8, 9] have studied two-dimensional dusty fluid flow under varying temperature and time time dependent pressure gradients. The present work deals with the study of laminar flow of an unsteady dusty fluid in porous medium through uniform pipe with sector of a circle as cross-section. Initially, the fluid and dust particles are assumed to be at rest. The motion of fluid is due to influence of time dependent pressure gradient. The analytical expressions are obtained for velocities of fluid and dust particles. For each case the skin friction at boundaries are obtained. The effects of porosity parameter, number density of the dust particles and time on the components of velocity of both fluid and dust particles are studied with the help of graphs. Bikash Chandra Ghosh and Ghosh N.C [10] obtained the solution for MHD flow of a visco-elastic fluid through porous medium. Brent Sleep E [11] discussed the modelling transient organic vapour transport in porous media with the dusty gas model.Hamdan M.H [12] has obtained the dusty gas flow through porous medium.Madhura K.R and Gireesha B.J [13] have discussed the laminar flow of a unsteady dusty fluid in porous medium through an open rectangular channel in Frenet frame field system. Recently Mohamed Ismail A, Ganesh S[14] Unsteady Stokes Flow of Dusty Fluid between Two Parallel Plates through Porous Medium. They considered the fluid being withdrawn through both walls of the channel at the same rate.

Formulation of the problem

The flow of an incompressible viscous fluid between two parallel porous plates $y = -h$ and $y = h$ is measured in the presence of a transverse magnetic field which is applied perpendicular to the walls in a parallel plate channel bounded by a loosely packed porous medium.

The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Equations of momentum are:

$$\frac{\partial u}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial x}=\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0^2 u}{\rho}-\frac{K^* N(u-u_p)}{\rho}-\frac{\mu u}{K\rho} \qquad (2)$$

$$\frac{\partial v}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial y}=\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad (3)$$

where u and v are the velocity components along the x- and y-directions at time t and u_p is the velocity of the dust particles respectively, n denotes the kinematic viscosity, r denotes the density of the fluid, s is the electrical conductivity of the fluid and B_0 is the magnitude of the uniform magnetic field acting along the y-axis. The induced magnetic field is assumed to be negligible. We impose the boundary conditions

$$u(x,h)=U, \qquad u(x,-h)=0$$
$$v(x,h)=0 \qquad v(x,-h)=-v_0 \qquad (4)$$

The Motion of dust particles is governed by Newton's second law

$$m_p\frac{\partial u_p}{\partial t}=K^*(u-u_p) \qquad (5)$$

General Solutions to the problem

Let us take the solutions of the equations (1)-(3) respectively as

$$u = u(x,y)e^{-nt}$$
$$v = v(x,y)e^{-nt}$$
$$p = p(x,y)e^{-nt} \quad (6)$$
$$u_p = u_p(x,y)e^{-nt}$$

Using (5) in (4)

$$u_p = \frac{K^*}{K^* - m_p}u \quad (7)$$

Let h = y/h be the dimensionless distance and

n =μ / r be the kinematic viscosity and the equations (1), (2) and (3) become

$$\frac{\partial u}{\partial x} + \frac{1}{h}\frac{\partial u}{\partial \eta} = 0 \quad (8)$$

$$-nu = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{h^2}\frac{\partial^2 u}{\partial \eta^2}\right) - \frac{\sigma B_0^2 u}{\rho} - \frac{K^* N(1-c^2)u}{\rho} - \frac{\mu u}{K\rho} \quad (9)$$

$$-nv = -\frac{1}{\rho h}\frac{\partial p}{\partial \eta} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{1}{h^2}\frac{\partial^2 v}{\partial \eta^2}\right) \quad (10)$$

where $C^2 = \frac{K^*}{K^* - m_p}$

The boundary conditions are converted into

$$u(x,1) = U, \quad u(x,-1) = 0$$
$$v(x,1) = 0 \quad v(x,-1) = -v_0 \quad (11)$$

The stream function $\psi(x,y)$ is defined as

$$\psi(x,\eta) = (h\ (0) - v_0 x)f(\eta) \quad (12)$$

and $u(x,y) = \frac{\partial \psi}{\partial y}$ $v(x,y) = -\frac{\partial \psi}{\partial x}$

From equations (2), (3), (9) and (10), we have

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = (hu(0) - v_0 x)\left[-nf' - \frac{\nu}{h^2}f'''(\eta) + \frac{\sigma B_0^2}{\rho}f' + \frac{K^* N(1-c^2)}{\rho}f' + \frac{\mu}{K\rho}f'\right] \quad (13)$$

$$-\frac{1}{\rho h}\frac{\partial p}{\partial \eta} = -n v_0 f(\eta) - \frac{\nu v_0}{h^2}f''(\eta) \quad (14)$$

Partially differentiating the equations

(13) & (14) with respect to 'h' & 'x' respectively

$$\frac{\partial^2 p}{\partial \eta \partial x} = (hu(0) - v_0 x)\frac{\partial}{\partial \eta}\left(-nf' - \frac{\nu}{h^2}f'''(\eta) + \frac{\sigma B_0^2}{\rho}f' + \frac{K^* N(1-c^2)}{\rho}f' + \frac{\mu}{K\rho}f'\right) \quad (15)$$

$$\frac{\partial^2 p}{\partial x \partial \eta} = 0 \quad (16)$$

From equations (15) & (16), we get

$$\frac{d}{d\eta}\left(-nf' - \frac{\nu}{h^2}f'''(\eta) + \frac{\sigma B_0^2}{\rho}f' + \frac{K^* N(1-c^2)}{\rho}f' + \frac{\mu}{K\rho}f'\right) = 0 \quad (17)$$

Integrating equation (17), we have

$$f'''(\eta) + \left(\frac{nh^2}{\nu} - \frac{\sigma B_0^2 h^2}{\rho\nu} - \frac{K^* N(1-c^2)h^2}{\rho\nu} - \frac{\mu}{K\rho}\right)f'(\eta) - = C_1$$

$$f'''(\eta) - (\alpha^2 h^2 + M^2)f'(\eta) = C_1 \quad (18)$$

where M is the Hartmann number

$$M = \sqrt{\frac{\sigma B_0^2 h^2}{\rho\nu}} = B_0 h\sqrt{\frac{\sigma}{\mu}} \quad \& \quad \alpha^2 = \frac{-n}{\nu} + \frac{K^* N(1-c^2)}{\rho\nu} + \frac{\mu}{K\rho}$$

Equation (18) reduces to the form

$$(D^3 - (\alpha^2 h^2 + M^2)D)f(\eta) = C_1 \quad (19)$$

with the boundary conditions

$$f(1) = 0, \quad f(-1) = -1$$
$$f'(1) = 0, \quad f'(-1) = 0 \quad (20)$$

Hence the solution of (17) subject to the boundary condition (20) is

$$f(\eta) = \frac{\left((1-\eta)\sqrt{\alpha^2 h^2 + M^2}\cosh\sqrt{\alpha^2 h^2 + M^2} + \sinh\left(\sqrt{\alpha^2 h^2 + M^2}\,\eta\right) - \sinh\sqrt{\alpha^2 h^2 + M^2}\right)}{2\left(\sinh\sqrt{\alpha^2 h^2 + M^2} - \sqrt{\alpha^2 h^2 + M^2}\cosh\sqrt{\alpha^2 h^2 + M^2}\right)} \quad (21)$$

Substituting the value of f(h) in the stream function

Hence the Axial Velocity

$$u = u(x,y)e^{-nt}$$
$$= \frac{1}{h}\frac{\partial \psi}{\partial \eta}e^{-nt}$$
$$= \frac{1}{h}(h\ (0) - v_0 x)f'(\eta)e^{-nt}$$
$$= \left(u(0) - \frac{v_0 x}{h}\right)e^{-nt}\frac{\left(\cosh\left(\left(\sqrt{\alpha^2 h^2 + M^2}\right)y/h\right) - \sqrt{\alpha^2 h^2 + M^2}\cosh\sqrt{\alpha^2 h^2 + M^2}\right)}{2(\sinh\sqrt{\alpha^2 h^2 + M^2} - \sqrt{\alpha^2 h^2 + M^2}\cosh\sqrt{\alpha^2 h^2 + M^2})}$$

Transverse Velocity

$$v = v(x,y)e^{-nt}$$
$$= -\frac{\partial \psi}{\partial x}e^{-nt}$$
$$= v_0 e^{-nt}\frac{\left((1-(y/h))\sqrt{\alpha^2 h^2 + M^2}\cosh\sqrt{\alpha^2 h^2 + M^2} + \sinh\left(\left(\sqrt{\alpha^2 h^2 + M^2}\right)y/h\right) - \sinh\sqrt{\alpha^2 h^2 + M^2}\right)}{2(\sinh\sqrt{\alpha^2 h^2 + M^2} - \sqrt{\alpha^2 h^2 + M^2}\cosh\sqrt{\alpha^2 h^2 + M^2})}$$

Results and discussions

Analytical solutions of this problem is obtained and the outcome are illustrated graphically in Figs. 1–9 to show the velocity of the fluid based on the parameters Hartmann number(M), no. of dust particles(N) and density(r). Figs. 1-9 present the effect of axial velocity of fluid, velocity of fluid and dust particles, transverse velocity of fluid This paper analyses the performance of dusty fluid in accordance with various parameters.

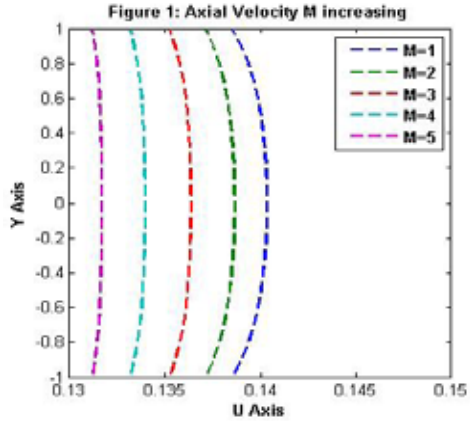


Figure 1: Axial Velocity M Increasing

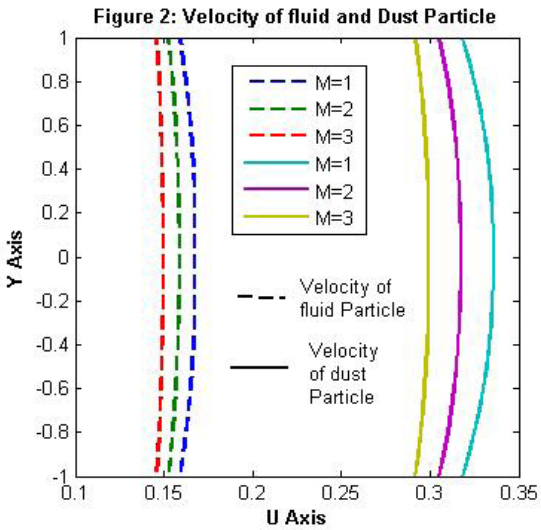


Figure 2: Velocity of the fluid and dust particles M Increasing

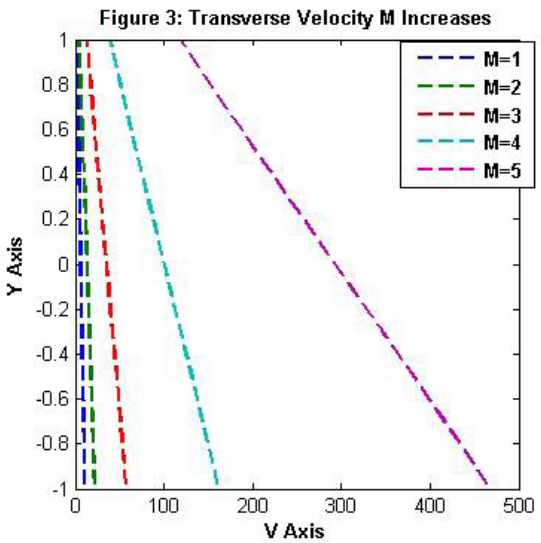


Figure 3: Transverse Velocity M Increasing

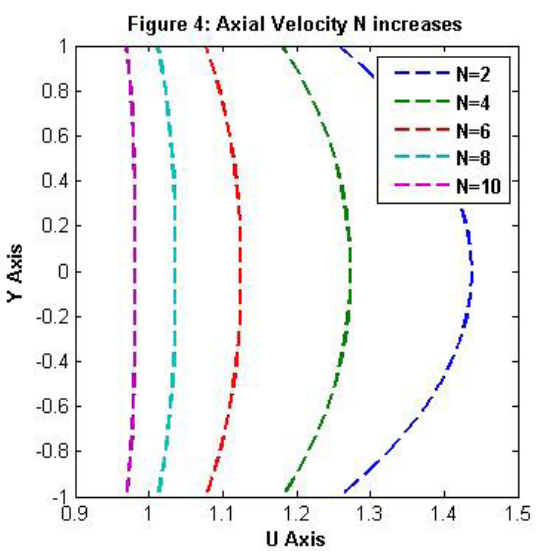


Figure 4: Axial Velocity N Increasing

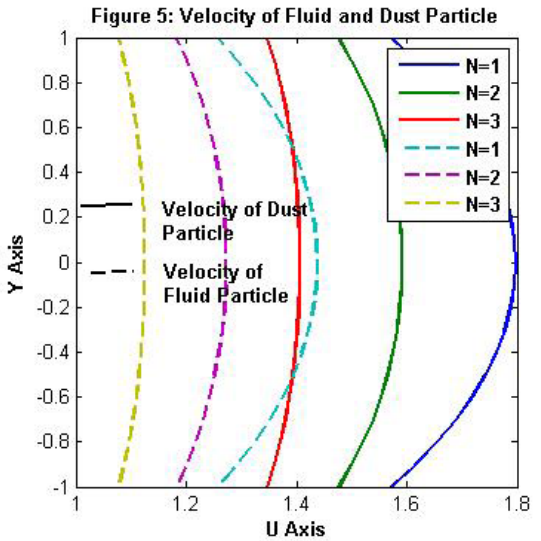


Figure 5: Velocity of the fluid and dust particles N Increasing

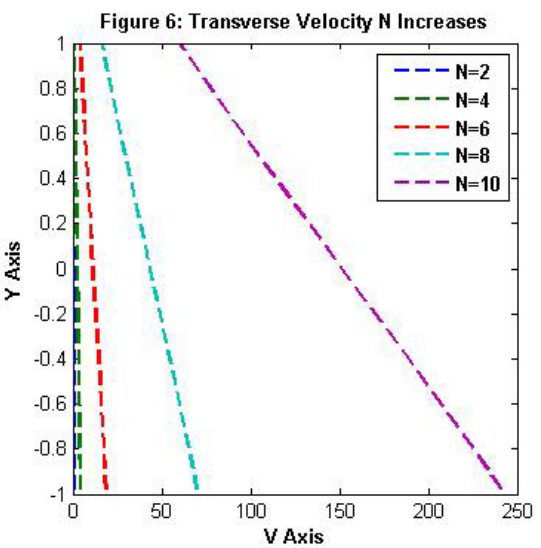


Figure 6: Transverse Velocity N Increasing

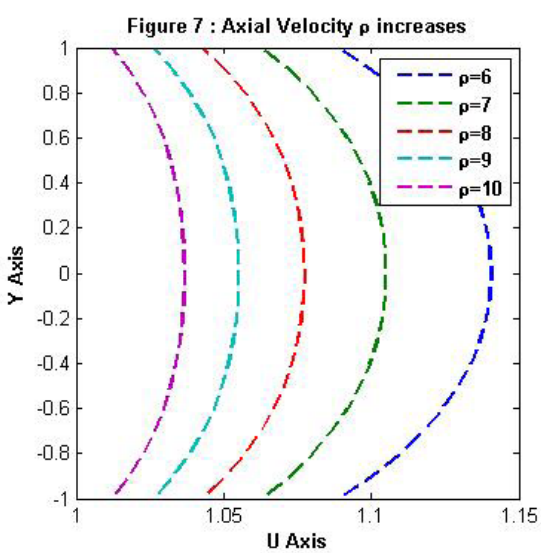


Figure 7: Axial Velocity r Increasing

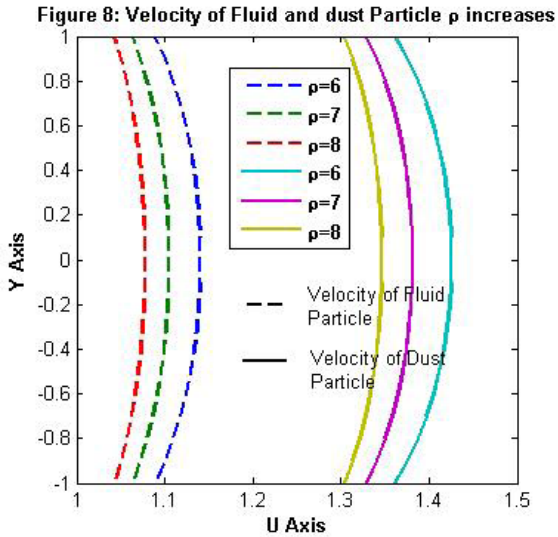


Figure 8: Velocity of the fluid and dust particles r Increasing

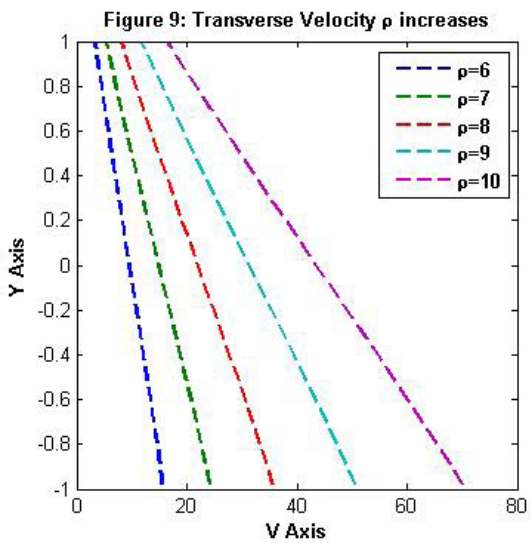


Figure 6: Transverse Velocity r Increasing

Figs. 1,4 and 7 shows that the Axial velocity of the fluid decreases when M (Hartmann no), N(no of dust particles), r(density) increases. Fig.2,5 and 7 shows that velocity of fluid and velocity of dust particles. It shows that velocity distribution for clean fluid and dust fluid decreases with the increase of fluid particle interaction parameter. It also shows that velocity of dust particle is higher than velocity of fluid particle.. Figs. 3, 6 and 9 shows that the variation of transverse velocity with respect to the variations of the parameter M,N and r. Figure 3,6 and 9 shows that Transverse velocity of fluid increases as M, N and r increases Figs. 3,6 &9 shows that the magnitude of the lower plate is higher than the magnitude of the upper plate.

CONCLUSIONS

Analytical solutions are obtained for the Exact solution for the unsteady MHD dusty fluid flow over a moving horizontal plate through a porous space. The Similarity transformation method is used to solve the problem and the results are evaluated analytically and displayed graphically. In the light of the present investigation, following conclusions can be drawn:

Velocity distribution for clean fluid and dust fluid decreases with the increase of fluid particle interaction parameter.

The velocity of the fluid and dust particle decreases as density(r),number of dust particles(N) and Hartmann no(M) increases.

Transverse velocity of fluid increases as density(r),number of dust particles(N) and Hartmann no(M) increases.

REFERENCES

[1]Joseph, D.D., and Tao, L. N., (1966), "Lubrication of porous bearing Stokes solution". J. Appl. Mech., Trans. ASME, Series E, 88, 753–760. | [2]Hassaninen, A., and Mansour, M. A., (1990), "Unsteady Magnetic flow through a Porous Medium between two infinite parallel" Astrophysics and Space Science Springer Netherlands, Vol.163, No.2,pp.241-246. | [3] Hamza, E. A., (1999), "Suction and injection effects on a similar flow between parallel plates", J Phys 32, 656. | [4]Soundalgekar, V., and Uplekar, A., (1986), "Steady MHD coutte flow with heat transfer considering varying temperature", IEEE Trans Plasma Sci, Vol.14, p.579. | [5]Sing, C.B., and Ram,P.C.,(1978),"Unsteady Magnetohydrodynamic Fluid flow through a channel" Journal of Scientific Research, XXVIII, No.2. | [6]Cunningham, R.E., and Williams, R. J., (1980), "Diffusion in gases and porous media". Pleanum Press, New York. | [7] Darcy, H., (1937) "The flow of fluids through porous media". Mc-Graw Hill Book Co., New York. | [8]Bagewadi, C.S., Gireesha, B.J., (2004), " A study of two dimensional steady dusty fluid flow under varying temperature, Int. Journal of Appl. Mech. And Eng., 09 647-653. | [9]Bagewadi, C.S., Gireesha, B.J., (2003), "A study of two dimensional unsteady dusty fluid flow under varying pressure gradient", Tensor, N.S., 64, 232-240. | [10]Bikash Chandra Ghosh, Ghosh, N.C.,(2001)," MHD flow of a visco-elastic fluid through porous medium", Int. J. Num. Meth. Heat and Fluid Flow, 11, No. 7, 682-698. | [11]Brent E. Sleep., (1998) "Modelling transient organic vapor transport in porous media with the dusty gas model", Adv. in Water Resources, 22, No. 3, 247-256. | [12]Hamdan, M.H., and Sawalha, K.D., (1996)," Dusty gas flow through porous medium", App. Math. Comp., 75, 59-73. | [13] Madhura, K. R., and Gireesha, B. J.,(2009)," Exact Solutions of Unsteady Dusty Fluid flow through Porous Media in an Open Rectangular Channel", Adv. Theor. Appl. Mech., Vol. 2, 2009, no. 1, 1 – 17. | [14]Mohamed Ismail A., Ganesh S., (2014),"Unsteady Stokes Flow of Dusty Fluid between TwoParallel Plates through Porous Medium ",Applied Mathematical Sciences, Vol. 8, no. 5, 241 – 249. | |