Research Paper Mathematics

Homomorphism and Anti Homomorphism in Interval Valued Q-L Fuzzy Subhemirings of a Hemiring

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ABSTRACT

In this paper, we study some of the properties of interval valued Q-L fuzzy subhemiring of a hemiring under homomorphism, antihomomorphism and prove some results on these.
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KEYWORDS

Interval valued fuzzy subset, interval valued Q-L fuzzy subhemiring, and interval valued Q-L fuzzy normal subhemiring.

INTRODUCTION:

There are many concepts of universal algebras generalizing an associative ring (R, +, .). Some of them in particular, near rings and several kinds of semirings have been proven very useful. Semirings (called also half rings) are algebras (R,+ ,.) share the same properties as a ring except that (R, +) is assumed to be a semi group rather than a commutative group. Semi rings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R,+ ,.) is said to be a semi ring (R, +) and (R ,.) are semi groups satisfying a.(b+c)=a.b+a.c and (b+c).a=b.a+c.a for all a,b and c in R. A semi ring R is said to be additively commutative if a+b=b+a for all a, b and c in R. A semi ring R may have an identity 1, defined by 1.a=a=a.1 and a zero 0, defined by 0+a=a=a+0 and a.0=0=0.a for all a in R. A semi ring R is said to be a hemi ring if it is an additively commutative with zero. Interval valued fuzzy sets were introduced independently by Zadeh [13], Grattan-Guiness [4], Jahn [6], in the seventies, in the same year. An interval valued fuzzy set (IVF) is defined by an interval-valued membership function Jun. Y.B and Kin.K [7] defined an interval valued fuzzy R-subgroups of nearrings. Solairaju. A and Nagarajan.R [10] defined the characterization of interval valued Anti fuzzy Left h-ideals over hemirings. Azriel Rosenfeld [2] defined fuzzy groups. Osman Kazanci, Sultan yamark and serife yilmaz in [8] have introduced the Notion of intuitionistic Q-fuzzification of N-subgroups (subnear rings) in a near-ring and investigated some related properties. Solairaju. A and Nagarajan.R [11], have given a new structure in the construction of Q-fuzzy groups and subgroups [12]. We introduced the concept of interval valued Q-L fuzzy subhemiring of a hemiring under homomorphism, anti homomorphism and established some results.

PRELIMINARIES:

1.1 Definition: Let X be a non-empty set and L= (L, 0 and greastest element 1.

1.2 Definition: Let X be any nonempty set. A mapping is called an interval valued L-fuzzy subset(briefly, IVLFS) of X, where D[0,1] denoted the family of all closed subintervals of [0,1] and [M](x)=[] ,for all x in X, where and are fuzzy subsets of X such that , for all x in X. Thus [M](x) is an interval (a closed subset of [0,1]) and not number from the intervak [0,1] as in the case of fuzzy subset. Note that [0] = [0,0] and [1]=[1,1].

1.3 Definition: Let X be a non-empty set and Q be a non-empty set. A Q, L--fuzzy subset A of X is function

1.2 Remark: Let be the set of all interval valued L-fuzzy subsets of X, where D means D[0,1].

1.4Definition: Let be any two interval valued L -fuzzy subsets of x. We define the following relations and operations:

i)$[M] \subseteq [N]$ if and only if $M^-(x,q) \leq N^-(x,q)$ and $M^+(x,q) \leq N^+(x,q)$,for all x in X.
ii)$[M]= [N]$ if and only if $M^-(x,q) = N^-(x,q)$ and $M^+(x,q) = N^+(x,q)$,for all x in X.
iii)$[M] \cap [N] = \{((x,q), [\{M^-(x,q) \wedge N^-(x,q)\}, \{M^+(x,q) \wedge N^+(x,q)\}]) / x \in X, q \text{ in } Q\}$.
iv)$[M] \cup [N] = \{((x,q), [\{M^-(x,q) \vee N^-(x,q)\}, \{M^+(x,q) \vee N^+(x,q)\}]) / x \in X, q \text{ in } Q\}$.
v)$[M]^c = [1] - [M] = \{((x,q)[1 - M^+(x,q), 1 - M^-(x,q)]) / x \in X, q \text{ in } Q\}$

1.5 Definition: Let (R, +, .) be a hemiring. A interval valued Q-L fuzzy subset [M] of R is said to be an interval valued Q – L fuzzy subhemiring (IVFSHR) of R if the following conditions are satisified:

(i) $[M](x+y,q) \geq ([M](x,q) \wedge [M](y,q))$
(ii)$[M](xy,q) \geq ([M](x,q) \wedge [M](y,q))$, for all x and y in R, and q in Q.

1.6 Definition: Let (R, +, .) be a hemiring. A interval valued Q-fuzzy subhemiring [A] of R is said to be an interval valued Q-fuzzy normal subhemiring (IVFNSHR) of R if [A](xy,q)=[A](yx,q) , for all x and y in R and q in Q.

1.7 Definition: Let XxQ and be any two sets. Let be any function and [A] be an interval valued Q-fuzzy subset in X, [V] be an interval valued Q-fuzzy subset in, defined by for all x in X and y in and q in Q. Then [A] is called a pre-image of [V] under f and is denoted by

PROPERTIES OF INTERVAL VALUED Q-FUZZY SUBHEMIRINGS

2.1 Theorem: Let (R, +,.) and (, +, .) be any two hemirings. The homomorphic image of an interval valued Q-L fuzzy subhemiring of R is an interval valued Q-L fuzzy subhemiring of

Proof: Let (R, +,.) and (, +, .) be any two hemirings. Let be a homomorphism.Then,Let [V] = f ([A]) , where [A] is an interval valued Q-L fuzzy subhemiring of R. We have to prove that [V] is an interval valued Q-L fuzzy subhemiring of .

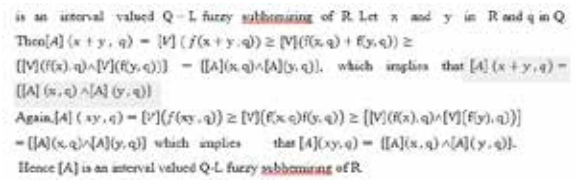
Which implies that [V]Again,

which implies that [V]Hence [V] is an interval valued Q-L fuzzy subhemiring of .

2.2 Theorem: Let (R, +,.) and (, +,.) be any two hemirings. The homomorphic preimage of an interval valued Q-L fuzzy subhemiring of is interval valued Q-L fuzzy subhemiring of R

Proof: Let (R, +, .) and (, +, .) be any two hemirings. Let be a homomorphism.

Then,Let [V] = f([A]), where [V] is an interval valued Q-L fuzzy subhemiring of We have to prove that [A] is an interval valued Q – L fuzzy subhemiring of R. Let x and y in R and q in Q. Then , which implies that


is an interval valued Q – L fuzzy subhemiring of R. Let x and y in R and q in Q
Then[A] (x + y , q) = [V] (f(x + y ,q)) ≥ [V](f(x, q) + f(y, q)) ≥
[[V](f(x). q)∧[V](f(y, q))] = [[A](x, q)∧[A](y, q)]. which implies that [A] (x + y , q) =
[[A] (x , q) ∧[A] (y , q)]
Again,[A] (xy , q) = [V](f(xy , q)) ≥ [V](f(x, q)f(y, q)) ≥ [[V](f(x), q)∧[V](f(y), q))]
= [[A](x, q)∧[A](y, q)] which implies that [A](xy, q) = [[A](x , q) ∧[A](y , q)].
Hence [A] is an interval valued Q-L fuzzy subhemiring of R

2.3 Theorem: Let (R, +, .) and (, +, .) be any two hemirings. The anti-homomorphic image of an interval valued Q-L fuzzy subhemiring of R is an interval valued Q-L fuzzy subhemiring of

Proof: Let (R, +,.) and (, +,.) be any two hemirings.Let be a ti-homomorphism. Then, Let [V]= f([A]), where [A] is an interval valued Q-L fuzzy subhemiring of R. We have to prove that [V] is an interval valued Q-L fuzzy subhemiring of .

which implies that [V]

Again, =, which implies that [V]

Hence [V] is an interval valued Q-fuzzy subhemiring of .

2.4 Theorem: Let (R, +,.) and (, +, .) be any two hemirings. The anti-homomorphic preimage of an interval valued Q-L fuzzy subhemiring of is an interval valued Q-L fuzzy subhemiring of R

Proof: Let (R, +,.) and (, +,.) be any two hemirings. Let be a anti-homomorphism. Then,Let

x where [V] is an interval valued Q-L fuzzy subhemiring of We have to prove that [A] is an interval valued Q – L fuzzy subhemiring of R. Let x and y in R and q in Q. Then =, which implies that.Again, =.which implies that Hence [A] is an interval valued Q-L fuzzy subhemiring of R.

2.5 Theorem: Let [A] be an interval valued Q-L fuzzy subhemiring of hemiring H and f is an isomorphism from a hemiring R onto H. Then [A] f is an interval valued Q-L fuzzy subhemiring of R.

Proof:be an interval valued Q-fuzzy subhemiring of a hemiring H.

which implies that

which implies thatTherefore is an interval valued Q-fuzzy subhemiring of R.

2.6 Theorem: Let [A] be an interval valued Q-L fuzzy subhemiring of hemiring H and f is an anti- isomorphism from a hemiring R onto H. Then [A] f is an interval valued Q-L fuzzy subhemiring of R.

Proof:be an interval valued Q-L fuzzy subhemiring of a hemiring H.

which implies that which implies that Therefore is an interval valued Q-L fuzzy subhemiring of the hemiring R.

2.7 Theorem: Let (R, +,.) and (, +, .) be any two hemirings. The homomorphic image of an interval valued Q-L fuzzy normal subhemiring of R is an interval valued Q-L fuzzy subhemiring of

Proof: Let (R, +,.) and (, +, .) be any two hemirings.Let be a homomorphism.Let [A] is an interval valued Q-L fuzzy normal subhemiring of R. We have to prove that [V] is an interval valued Q-L fuzzy normal subhemiring of .

which implies that

[V]Hence [V] is an interval valued Q-L fuzzy normal subhemiring of the hemiring .

2.8 Theorem: Let (R, +,.) and (, +,.) be any two hemirings. The homomorphic preimage of an interval valued Q-L fuzzy normal subhemiring of is interval valued Q-L fuzzy normal subhemiring of R

Proof: Let (R, +,.) and (, +,.) be any two hemirings.Let be a homomorphism.Let [V] is an interval valued Q-fuzzy normal subhemiring of f(R)= We have to prove that [A] is an interval valued Q – fuzzy normal subhemiring of R. Let x and y in R and q in Q.Then

which implies that

Hence [A] is an interval valued Q-fuzzy normal subhemiring of the hemiring R.

2.9 Theorem: Let (R, +,.) and (, +, .) be any two hemirings. The antihomomorphic image of an interval valued Q-L fuzzy normal subhemiring of R is an interval valued Q-L fuzzy subhemiring of

Proof: Let (R, +,.) and (, +, .) be any two hemirings.Let be a anti homomorphism.Let [A] is an interval valued Q-L fuzzy normal subhemiring of R. We have to prove that [V] is an interval valued Q-L fuzzy normal subhemiring of . which implies that[V]Hence [V] is an interval valued Q-L fuzzy normal subhemiring of the hemiring .

2.10 Theorem: Let (R, +,.) and (, +,.) be any two hemirings. The anti homomorphic preimage of an interval valued Q-L fuzzy normal subhemiring of is interval valued Q-L fuzzy normal subhemiring of R

Proof: Let (R, +,.) and (, +,.) be any two hemirings.Let be anti homomorphism.Let [V] is an interval valued Q-L fuzzy normal subhemiring of f(R)= We have to prove that [A] is an interval valued Q – L fuzzy normal subhemiring of R. Let x and y in R and q in Q.Then since [V] is an interval valued Q-L fuzzy normal subhemiring of the hemiring . which implies thatHence [A] is an interval valued Q-L fuzzy normal subhemiring of the hemiring R.

2.11 Theorem: Let [A] be an interval valued Q-L fuzzy normal subhemiring of hemiring H and f is an isomorphism from a hemiring R onto H. Then [A] f is an interval valued Q-L fuzzy normal subhemiring of R.

Proof:be an interval valued Q-L fuzzy normal subhemiring of a hemiring H. is an interval valued Q-L fuzzy subhemiring of the hemiring R. which implies that

herefore is an interval valued Q-L fuzzy normal subhemiring of R.

2.12 Theorem: Let [A] be an interval valued Q-L fuzzy normal subhemiring of hemiring H and f is an anti- isomorphism from a hemiring R onto H. Then [A] f is an interval valued Q-L fuzzy normal subhemiring of R.

Proof:be an interval valued Q-L fuzzy normal subhemiring of a hemiring H. is an interval valued Q-L fuzzy subhemiring of the hemiring R.which implies that

ence is an interval valued Q-L fuzzy normal subhemiring of hemiring R.

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