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Lower Level Subsets Of Anti-Fuzzy Subsemiring Of A Semiring

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of lower level subsets of an anti- fuzzy subsemiring of a semiring and we introduce the some theorems in lower level subsets of an anti-fuzzy subsemiring of a semiring.

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Introduction

There are many concepts of universal algebras generalizing an associative ring (R ; + ; .). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. After the introduction of fuzzy sets by L.A.Zadeh[9], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subnearrings and ideals was introduced by S.Abou Zaid[1]. In this paper, we introduce the some theorems in lower level subsets of an anti-fuzzy subsemiring of a semiring.

Preliminaries

Definition: Let X be a nonempty set. A fuzzy subset A= x, A(x) of X is a function A : X → [0, 1].

Definition: Let (R, +, .) be a semiring. A fuzzy subset A= x, $\mu_A(x)$ of R is said to be a fuzzy subsemiring (FSSR) of R if it satisfies the following conditions:

(ι) $\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

(ιι) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R.

Definition: Let (R, +, .) be a semiring. A fuzzy subset A= x, $\mu_A(x)$ of R is said to be an anti-fuzzy subsemiring (AFSSR) of R if it satisfies the following conditions:

(i) $\mu_A(x+y) \leq \max\{\mu_A(x), \mu_A(y)\}$,

(ii) $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$, for all x and y in R.

Definition: Let (R, +, ·) and (R^1, +, ·) be any two semirings. Let f : R → R^1 be any function and A= x, $\mu_A(x)$ be an anti-fuzzy subsemiring in R, V= y, $\mu_V(y)$ be an anti-fuzzy subsemiring in f(R) = R^1, defined by $\mu_V(y) = \inf_{x \in f^{-1}(y)} \mu_A(x)$, for all x in R and y in R^1. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

Definition: Let (R, +, .) and (R^1, +, .) be any two semirings. Then the function f : R → R^1 is called a semiring homomorphism if f(x+y) = f(x) + f(y), f(xy) = f(x) f(y), for all x and y in R.

Definition: Let (R, +, .) and (R^1, +, .) be any two semirings. Then the function f : R → R^1 is called a semiring anti-homomorphism if f(x + y) = f(y) + f(x), f(xy) = f(y) f(x), for all x and y in R. α

Definition: Let A= x, $\mu_A(x)$ be a fuzzy subset of X. For α in [0, 1], the lower level subset of A is the set $A_\alpha = \{x \in X: \mu_A(x) \leq \alpha\}$.

Properties Of Lower Level Subsets Of Anti-Fuzzy Subsemiring Of A Semiring

Theorem: Let A= x, $\mu_A(x)$ be an anti-fuzzy subsemiring of a semiring (R, +, .). Then for α in [0,1], A_α is a lower level subsemiring of R.

Proof: For all x and y in A_α we have, $\mu_A(x) \leq \alpha$ and $\mu_A(y) \leq \alpha$.

Now, $\mu_A(x + y) \leq \max\{\mu_A(x), \mu_A(y)\} \leq \max\{\alpha, \alpha\} = \alpha$, which implies that $\mu_A(x+y) \leq \alpha$. And, $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\} \leq \max\{\alpha, \alpha\} = \alpha$, which implies that $\mu_A(xy) \leq \alpha$. Therefore, $\mu_A(x+y) \leq \alpha$ and $\mu_A(xy) \leq \alpha$. Therefore x + y and xy in A_α. Hence A_α is a lower level subsemiring of a semiring R.

Theorem: Let A= x, $\mu_A(x)$ be an anti-fuzzy subsemiring of a semiring (R, +, .). Then two lower level subsemiring $A_{\alpha 1}$, $A_{\alpha 2}$ and α_1, α_2 are in [0,1] with $\alpha_1 < \alpha_2$ of A are equal iff there is no x in R such that $\alpha_2 > \mu_A(x) > \alpha_1$.

Proof: Assume that $A_{\alpha 1} = A_{\alpha 2}$. Suppose there exists x in R such that $\alpha_2 > \mu_A(x) > \alpha_1$. Then $A_{\alpha 1} \subseteq A_{\alpha 2}$ implies x belongs to $A_{\alpha 2}$, but not in $A_{\alpha 1}$. This is contradiction to $A_{\alpha 1} = A_{\alpha 2}$.Therefore there is no $x \in R$ such that $\alpha_2 > \mu_A(x) > \alpha_1$. Conversely if there is no $x \in R$ such that $\alpha_2 > \mu_A(x) > \alpha_1$. Then $A_{\alpha 1} = A_{\alpha 2}$. (by the definition of lower level set).

Theorem: Let (R, +, .) be a semiring and A= x, $\mu_A(x)$ be a fuzzy subset of R such that A_α be a lower level subsemiring of R. If α in [0,1], then A is an anti-fuzzy subsemiring of R.

Proof: Let (R, +, .) be a semiring and x and y in R. Let $\mu_A(x) = \alpha_1$ and $\mu_A(y) = \alpha_2$.

Case (i): If $\alpha_1 < \alpha_2$, then x, $y \in A_{\alpha 2}$. As $A_{\alpha 2}$ is a lower level subsemiring of R, x + y and xy in $A_{\alpha 2}$. Now, $\mu_A(x + y) \leq \alpha_2 = \max\{\alpha_1, \alpha_2\} = \max\{\mu_A(x), \mu_A(y)\}$, which implies that $\mu_A(x + y) \leq \max\{\mu_A(x), \mu_A(y)\}$, all x and y in R.

Also, $\mu_A(xy) \leq \alpha_2 = \max\{\alpha_1, \alpha_2\} = \max\{\mu_A(x), \mu_A(y)\}$, implies that $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$, for all x and y in R.

Case (ii): If $\alpha_1 > \alpha_2$, then x and y in $A_{\alpha 1}$. As $A_{\alpha 1}$ is a lower level subsemiring of R, x + y and xy in $A_{\alpha 1}$. Now, $\mu_A(x + y) \leq \alpha_1 = \max\{\alpha_2, \alpha_1\} = \max\{\mu_A(y), \mu_A(x)\}$, which implies that $\mu_A(x + y) \leq \max\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. Also, $\mu_A(xy) \leq \alpha_2 = \max\{\alpha_2, \alpha_1\} = \max\{\mu_A(y), \mu_A(x)\}$, implies that $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$, for all x and y in R.

Case (iii): If $\alpha_1 = \alpha_2$ It is trivial.

In all the cases, A is an anti-fuzzy subsemiring of a semiring R.

Theorem: Let A= x, $\mu_A(x)$ be an anti-fuzzy subsemiring of a semiring (R, +, .). If any two lower level subsemirings of A belongs to R, then their intersection is also lower level subsemiring of A in R.

Proof : Let α_1 and α_2 be in [0,1].

Case (i): If $\alpha_1 < \mu_A(x) < \alpha_2$, then $A_{\alpha 1} \subseteq A_{\alpha 2}$. Therefore, $A_{\alpha 1} \cap A_{\alpha 2} = A_{\alpha 1}$, but $A_{\alpha 1}$ is a lower level subsemiring of A.

Case (ii): If $\alpha_1 > \mu_A(x) > \alpha_2$, then $A_{\alpha 2} \subseteq A_{\alpha 1}$. Therefore, $A_{\alpha 1} \cap A_{\alpha 2} = A_{\alpha 2}$, but $A_{\alpha 2}$ is a lower level subsemiring of A.

Case (iii): If $\alpha_1 = \alpha_2$, then $A_{\alpha 1} = A_{\alpha 2}$.

In all cases, intersection of any two lower level subsemirings is a lower level subsemiring of A.

Theorem: Let A= x, $\mu_A(x)$ be an anti-fuzzy subsemiring of a semiring (R, +, .). If $\alpha_i \in [0,1]$, and $A_{\alpha i}$, $i \in I$ is a collection of lower level subsemirings of A, then their intersection is also a lower level subsemiring of A.

Proof: It is trivial.

2.6 Theorem: Let A= x, $\mu_A(x)$ be an anti-fuzzy subsemiring of a semiring (R, +, .). If any two lower level subsemirings of A belongs to R, then their union is also a lower level subsemiring of A in R.

Proof: Let $\alpha_1, \alpha_2 \in [0,1]$.

Case (i): If $\alpha_1 < \mu_A(x) < \alpha_2$, then $A_{\alpha 1} \subseteq A_{\alpha 2}$. Therefore, $A_{\alpha 1} \; A_{\alpha 2} = A_{\alpha 2}$, but $A_{\alpha 2}$ is a lower level subsemiring of A.

Case (ii): If $\alpha_1 > \mu_A(x) > \alpha_2$, then $A_{\alpha 2} \subseteq A_{\alpha 1}$. Therefore , $A_{\alpha 1} \; A_{\alpha 2} = A_{\alpha 1}$, but $A_{\alpha 1}$ is a lower level subsemiring of A.

Case (iii): If $\alpha_1 = \alpha_2$, then $A_{\alpha 1} = A_{\alpha 2}$. In all cases, union of any two lower level subsemiring is also a lower level subsemiring of A.

2.7 Theorem: Let A= x, $\mu_A(x)$ be an anti-fuzzy subsemiring of a semiring (R, +, .). If $\alpha_i \in [0,1]$, and $A_{\alpha i}$, $i \in I$ is a collection of lower level subsemirings of A, then their union is also a lower level subsemiring of A.

Proof: It is trivial.

Theorem: The homomorphic image of a lower level subsemiring of ananti-fuzzy subsemiring of a semiring (R, +, .) is a lower level subsemiring of an anti-fuzzy subsemiring of a semiring (R^1, +, .).

Proof: Let (R, +, .) and (R^1, +, .) be any two semirings and f : $R \rightarrow R^1$ be a homomorphism. That is, f(x+y) =f(x)+f(y) and f(xy)= f(x)f(y), for all x and y in R.

Let V=f(A), where A= x, $\mu_A(x)$ is an anti-fuzzy subsemiring of a semiring R. Clearly V is an anti- fuzzy subsemiring of a semiring R^1. Let x and y in R, implies f(x) and f(y) in R^1. Let A_α is a lower level subsemiring of A. That is, $\mu_A(x) \leq \alpha$ and $\mu_A(y) \leq \alpha$; $\mu_A(x+ y) \leq \alpha$, $\mu_A(xy) \leq \alpha$. We have to prove that f (A_α) is a lower level subsemiring of V. Now, $\mu_V(f(x)) \leq \mu_A(x) \leq \alpha$, which implies that $\mu_V(f(x)) \leq \alpha$; and $\mu_V(f(y)) \leq \mu_A(y) \leq \alpha$, which implies that $\mu_V(f(y)) \leq \alpha$ and $\mu_V(f(x) + f(y)) = \mu_V(f(x + y)) \leq \mu_A(x + y) \leq \alpha$, which implies that $\mu_V(f(x) + f(y)) \leq \alpha$. Also, $\mu_V(f(x) f(y)) = \mu_V(f(xy)) \leq \mu_A(xy) \leq \alpha$, which implies that $\mu_V(f(xy)) \leq \mu_A(xy) \leq \alpha$. Therefore, $\mu_V(f(x) + f(y)) \leq \alpha$, $\mu_V(f(x)f(y)) \leq \alpha$. Therefore, $\mu_V(f(x) + f(y)) \leq \alpha$, $\mu_V(f(x) f(y)) \leq \alpha$. Hence f ($A_\alpha$) is a lower level subsemiring of an anti-fuzzy subsemiring V of a semiring R^1.

Theorem: The homomorphic pre-image of a lower level subsemiring of an anti-fuzzy subsemiring of a semiring (R^1, +, .) is a lower level subsemiring of an anti-fuzzy subsemiring of a semiring (R, +, .).

Proof: Let (R, +, .) and (R^1, +, .) be any two semirings and f : $R \rightarrow R^1$ be a homomorphism. That is, f(x+y) =f(x)+f(y) and f(xy) = f(x)f(y) for all x and y in R.

Let V = f(A), where V is an anti-fuzzy subsemiring of a semiring R^1. Clearly A= x, $\mu_A(x)$ is an anti-fuzzy subsemiring of a semiring R. Let f(x) and f(y) in R^1, implies x and y in R. Let f(A_α) is a lower level subsemiring of V. That is, $\mu_V(f(x)) \leq \alpha$ and $\mu_V(f(y)) \leq \alpha$; $\mu_V(f(x) + f(y)) \leq \alpha$, $\mu_V(f(x) f(y)) \leq \alpha$. We have to prove that A_α is a lower level subsemiring of A.Now, $\mu_A(x)=\mu_V(f(x)) \leq \alpha$, implies that $\mu_A(x) \leq \alpha$; $\mu_A(y) = \mu_V(f(y)) \leq \alpha$, implies that $\mu_A(y) \leq \alpha$ and $\mu_A(x + y) = \mu_V(f(x + y)) = \mu_V(f(x) + f(y)) \leq \alpha$, which implies that $\mu_A(x + y) \leq \alpha$. Also, $\mu_A(xy) = \mu_V(f(xy)) = \mu_V(f(x)f(y)) \leq \alpha$. which implies that $\mu_A(xy) \leq \alpha$. Therefore, $\mu_V(f(x) +f(y)) \leq \alpha$, $\mu_V(f(x)f(y)) \leq \alpha$. Hence, A_α is a lower level subsemiring of an anti-fuzzy subsemiring A of R.

Theorem: The anti-homomorphic image of a lower level subsemiring of an anti-fuzzy subsemiring of a semiring (R, +, .) is a lower level subsemiring of an anti- fuzzy subsemiring of a semiring (R^1, +, .).

Proof: Let (R, +, .) and (R^1, +, .) be any two semirings and f: $R \rightarrow R^1$ be an anti-homomorphism. That is, f(x + y) = f(y)+f(x) and f(xy) = f(y)f(x), for all x and y in R. Let V = f(A), where A= x, $\mu_A(x)$ is an anti-fuzzy subsemiring of R. Clearly V is an anti-fuzzy subsemiring of R^1. Let x and y in R, implies f(x) and f(y) in R^1. Let A_α is a lower level subsemiring of A.That is,$\mu_A(x) \leq \alpha$ and $\mu_A(y) \leq \alpha$.$\mu_A(y + x) \leq \alpha$,$\mu_A(yx) \leq \alpha$. We have to prove that f (A_α) is a lower level subsemiring of V. Now, $\mu_V(f(x)) \leq \mu_A(x) \leq \alpha$, which implies that $\mu_V(f(x)) \leq \alpha$; $\mu_V(f(y)) \leq \mu_A(y) \leq \alpha$, which implies that $\mu_V(f(y)) \leq \alpha$. Now, $\mu_V(f(x) + f(y)) = \mu_V(f(x) + f(y)) = \mu_V(f(y + x)) \leq \mu_A(y + x) \leq \alpha$, which implies that, $\mu_V(f(x) + f(y)) \leq \alpha$. Also, $\mu_V(f(x)f(y)) = \mu_V(f(yx)) \leq \mu_A(yx) \leq \alpha$, which implies that $\mu_V(f(x)f(y)) \leq \alpha$. Therefore, $\mu_V(f(x) + f(y)) \leq \alpha$ and $\mu_V(f(x)f(y)) \leq \alpha$. Hence f($A_\alpha$) is a lower level subsemiring of an anti-fuzzy subsemiring V of R^1.

2.11 Theorem: The anti-homomorphic pre-image of a lower level subsemiring of an anti-fuzzy subsemiring of a semiring (R^1 , +, .) is a lower level subsemiring of an anti-fuzzy subsemiring of a semiring (R , +, .).

Proof: Let (R, +, .) and (R^1, +, .) be any two semirings and f : $R \rightarrow R^1$ be an anti-homomorphism. That is, f(x + y) = f(y) + f(x) and f(xy) = f(y)f(x), for all x and y in R. Let V = f(A), where V is an anti-fuzzy subsemiring of a semiring R^1. Clearly A= x, $\mu_A(x)$ is an anti-fuzzy subsemiring of a semiring R. Let f(x) and f(y) in R^1, implies x and y in R. Let f(A_α) is a lower level subsemiring of V. That is, $\mu_V(f(x)) \leq \alpha$ and $\mu_V(f(y)) \leq \alpha$; $\mu_V(f(y) + f(x)) \leq \alpha$, $\mu_V(f(y) f(x)) \leq \alpha$. We have to prove that A_α is a lower level subsemiring of A. Now, $\mu_A(x) = \mu_V(f(x)) \leq \alpha$, which implies that $\mu_A(x) \leq \alpha$; $\mu_A(y) = \mu_V(f(y)) \leq \alpha$, which implies that $\mu_A(y) \leq \alpha$. Now, $\mu_A(x + y) = \mu_V(f(x + y)) = \mu_V(f(y) + f(x)) \leq \alpha$, which implies that $\mu_A(x + y) \leq \alpha$. Also, $\mu_A(xy) = \mu_V(f(xy)) = \mu_V(f(y)f(x)) \leq \alpha$, which implies that $\mu_A(xy) \leq \alpha$.Therefore, $\mu_V(f(x) + f(y)) \leq \alpha$ and $\mu_V(f(x) f(y)) \leq \alpha$. Hence A_α is a lower level subsemiring of an anti-fuzzy subsemiring A of R.

REFERENCES

Abou Zaid. S, On fuzzy subnear rings and ideals, fuzzy sets and systems, 44(1991), 139-146. | Akram.M and Dar.K.H, On fuzzy d-algebras, Punjab university journal of mathematics, 37(2005), 61-76. | Akram.M and Dar.K.H, Fuzzy left h-ideals in hemirings with respect to a s-norm, International Journal of Computational and Applied Mathematics, Volume 2 Number 1 (2007), pp. 714 | Asok Kumer Ray, On product of fuzzy subgroups, fuzzy sets and systems, 105, 181-183 (1999). | Davvaz.B and Wieslaw.A.Dudek, Fuzzy n-ary groups as a generalization of rosenfeld fuzzy groups, ARXIV-0710.3884VI(MATH.RA)20 OCT 2007,1-16.| Dixit.V.N., Rajesh Kumar, Naseem Ajmal., Level subgroups and union of fuzzy subgroups, Fuzzy sets and systems, 37, 359-371 (1990). | Rajesh Kumar, Fuzzy Algebra, Volume 1, University of Delhi Publication Division, July -1993. | Sivaramakrishna das.P, Fuzzy groups and level subgroups, Journal of mathematical analysis and applications, 84, 264-269 (1981). | Zadeh.L.A., Fuzzy sets , Information and control ,Vol.8, 338-353 (1965).p