Research Paper Mathematics

Near Meanness on Armed and Double Armed Crown Of C_3 and C_4

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ABSTRACT

Let G = (V, E) be a graph with p vertices and q edges and let f: V(G) → { 0, 1, 2, ..., q-1, q+1} be an injection. The graph G is said to have a near mean labeling if for each edge, there exist an induced injective map f*: E(G) → {1, 2, ..., q} defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

The graph that admits a near mean labeling is called a near mean graph (NMG). In this paper, we proved that the graphs Special types of graphs Double armed crown of C_3 & C_4 and Armed Crown of C_3 & C4 near mean graphs.

KEYWORDS Near mean labeling, Near mean graph. 2000 Mathematics Subject Classification 05C78.

1. Introduction

By a graph, we mean a finite simple and undirected graph. The vertex set and edge set of a graph G denoted are by V(G) and E(G) respectively. Armed Crown of C_n is a graph from a cycle C_n of length n, in which each vertex v_i is replaced by a path P_m. Terms and notations not used here are as in [2].

2. Preliminaries

The mean labeling was introduced in [3]. Let G be a (p, q) graph and we define the concept of near mean labeling as follows.

Let f : V(G)→ { 0, 1, 2, …., q -1, q+1} be an injection and also for each edge e = uv, it induces a map f^*: E(G) → {1, 2, …., q} defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

A graph that admits a near mean labeling is called a near mean graph. We have proved in [4], P_n, C_n, $K_{2,n}$ are near mean graphs and K_n (n> 4) and $K_{1,n}$ (n >4) are not near mean graphs. In [5], we proved that the graphs Book B_n, Ladder L_n, Grid $P_n \times P_n$, Prism $P_m \times C_3$ and $L_n \odot K_1$ are near mean graphs. In [6], we proved Family of trees, Bi-star, Sub-division Bi-star $P_m \tilde{\odot} 2K_1$, $P_m \tilde{\odot} 3K_1$, $P_m \tilde{\odot} K_{1,4}$ and $P_m \tilde{\odot} K_{1,3}$ are near mean graphs. Special class of graphs Double armed crown of C_3 & C_4 and Armed Crown of C_3 & C4 are proved as near mean graphs.

3. Near Meanness On Special Types of Graphs

Theorem 3.1 : The graph $C_3 \tilde{\odot} P_m$ is a near mean graph

Proof :

Let $C_3 \tilde{\odot} P_m = \{V, E\}$ such that

$V = \{[u_i, v_i, w_i : 1 \le i \le m], u, v, w\}$

$E = \{(uv) \cup (vw) \cup (wu) \cup [(u_iu_{i+1}) \cup (v_iv_{i+1}) \cup (w_iw_{i+1}): 1 \le i \le m-1]\}$

with $u = u_m$, $v = v_m$, $w = w_m$

We define f : V → {0, 1, 2, …, 3m-1, 3m+1} by

$f(u_i) = i - 1 \quad 1 \le i \le m$

$f(v_i) = 2m - i \quad 1 \le i \le m$

$f(w_i) = 2m + i \quad 1 \le i \le m-1$

$f(w_m) = 3m+1$

It implies $f(u) = f(u_m)$

$f(v) = f(v_m)$

$f(w) = f(w_m)$

The induced edge labelings are

$f^*(u_iu_{i+1}) = i \quad 1 \le i \le m-1$

$f^*(uv) = m$

$f^*(vw) = 2m+1$

$f^*(uw) = 2m$

$f^*(v_iv_{i+1}) = 2m - i \quad 1 \le i \le m$

$f^*(w_iw_{i+1}) = 2m +1+ i \quad 1 \le i \le m-1$

It gives edge labeling from {1, 2, , q}

Hence, $C_3 \odot P_m$ is a near mean graph. For example the near mean labeling of $\mathbf{C_3 \odot P_5}$ is shown in figure 1.

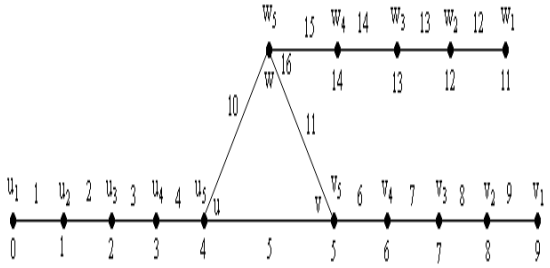


Figure 1

Theorem 3.2 : The graph $C_4 \odot P_m$ is near mean graph

Proof :

Let $C_4 \odot P_m = \{V, E\}$ such that

$V = \{ u, v, w, x, [u_i, v_i, w_i, x_i : 1 \le i \le m] \}$

$E = \{ (uv) \cup (vw) \cup (wx) \cup (xu) \cup [(u_iu_{i+1}) \cup (v_iv_{i+1}) \cup (w_iw_{i+1}) \cup (x_i x_{i+1}): 1 \le i \le m-1] \}$

with $u = u_m$, $v = v_m$, $w = w_m$, $x = x_m$

We define $f : V \to \{0, 1, 2, \ldots, 4m-1, 4m+1 \}$

Case (i) : Let m is odd

$f(u_i) = i - 1, \ 1 \le i \le m$

$f(v_{2i+1}) = 2m - 2i, 0 \le i \le \frac{m-1}{2}$

$f(v_{2i}) = 2m-1-2i, 1 \le i \le \frac{m-1}{2}$

$f(w_i) = 2m + i, \ 1 \le i \le m$

$f(x_1) = 4m+1$

$f(x_{m+1-i}) = 3m + i, 1 \le i \le m-1$

It implies $f(u) = f(u_m)$

$f(v) = f(v_m)$

$f(w) = f(w_m)$

$f(x) = f(x_m)$

The induced edge labelings are

$f^*(u_iu_{i+1}) = i, \ 1 \le i \le m-1$

$f^*(v_iv_{i+1}) = 2m - i, \ 1 \le i \le m-1$

$f^*(w_iw_{i+1}) = 2m+1+ i, \ 1 \le i \le m-1$

$f^*(x_ix_{i+1}) = 4m - (i-1), \ 1 \le i \le m-1$

$f^*(uv) = m$

$f^*(vw) = 2m+1$

$f^*(wx) = 3m+1$

$f^*(ux) = 2m$

Case (ii) : Let m is even

$f(u_i) = i - 1, \quad 1 \le i \le m-1$

$f(u_m) = m$

$f(v_{2i+1}) = 2m - 2i, \ 0 \le i < \frac{m}{2}$

$f(v_{2i}) = 2m-1-2i, 1 \le i \le \frac{m}{2}$

$f(w_i) = 2m + i, \ 1 \le i \le m$

$f(x_{m+1-i}) = 3m + i, \ 1 \le i \le m-1$

$f(x_1) = 4m + 1$

The induced edge labelings are

$f^*(u_iu_{i+1}) = i,\ 1 \le i \le m-1$

$f^*(v_iv_{i+1}) = 2m - i,\ 1 \le i \le m-1$

$f^*(w_iw_{i+1}) = 2m+1+i,\ 1 \le i \le m-1$

$f^*(x_ix_{i+1}) = 4m - (i-1),\ 1 \le i \le m-1$

$f^*(uv) = m$

$f^*(vw) = 2m$

$f^*(wx) = 3m+1$

$f^*(ux) = 2m+1$

It is clear that edges are labelled from {1, 2, …, q}.

Hence, $C_4 \odot P_m$ is a near mean graph. For example the near mean labeling of ($C_4 \odot P_7$: m - odd) and ($C_4 \odot P_6$: m - even) are shown in figure 2 and 3 respectively.

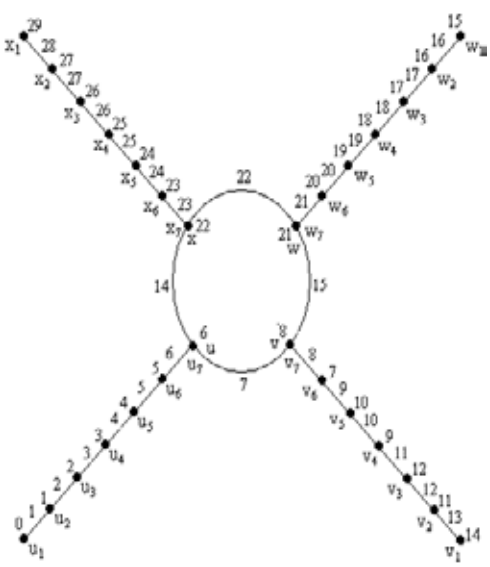


Figure 2

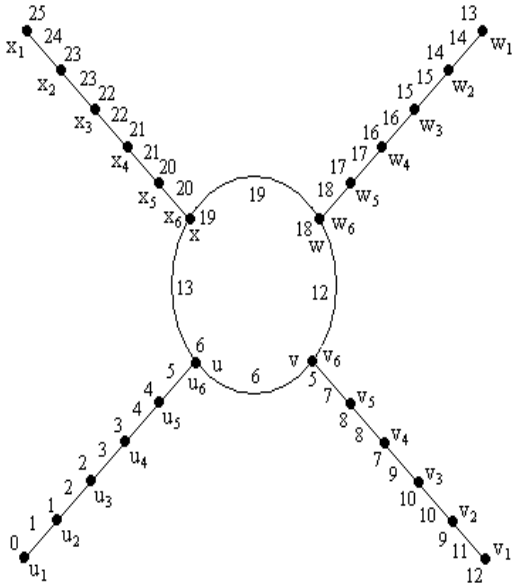


Figure 3

Theorem 3.3 : The graph $C_3 \otimes 2P_m$ is a near mean graph.

Proof :

Let $C_3 \otimes 2P_m = \{V, E\}$ such that

$V = \{[u_{ij} : 1 \le i \le 3,\ 1 \le j \le 2m-1],$

$v_i : 1 \le i \le 3\}$

with $u_{im} = v_i : 1 \le i \le 3$

$E = \{ [(v_iv_{i+1}) : i = 1,2] \cup (v_1v_3) \cup [(u_{ij}$

$u_{ij+1}) :\ 1 \le i \le 3,\ 1 \le j \le 2m-2] \}$

We define $f : V \to \{0, 1, 2, \ldots, q-1, q+1\}$ by

$f(u_{1i}) = i-1,\ 1 \le i \le 2m-1$

$$f(u_{2i}) = \begin{cases} 2m-2+i, & 1 \le i \le m-1 \\ 2m-1+i, & m \le i \le 2m-1 \end{cases}$$

$f(u_{3i}) = 4m-2+i,\ 1 \le i \le 2m-2$

$f(u_{32m-1}) = q+1 = 6m-2$

The induced edge labelings are

$f^*(u_{1j}\, u_{1j+1}) = j \qquad 1 \le j \le 2m-2$

$f^*(v_1v_2) = 2m-1$

$f^*(u_{2j}u_{2j+1}) = \begin{cases} 2m-1+j & 1 \le j \le m-1 \\ 2m+j & m \le j \le 2m-2 \end{cases}$

$f^*(v_2v_3) = 4m-1$

$f^*(u_{3j}u_{3j+1}) = 4m-1+j \quad 1 \le j \le 2m-2$

$f^*(v_1v_3) = 3m-1$

It is clear that edges are labeled distinguishly from the set {1, 2, …, q}

Hence $C_3 \otimes 2P_m$ is near mean graph. For example the near mean labeling of $C_3 \otimes 2P_5$ is shown in figure 4.

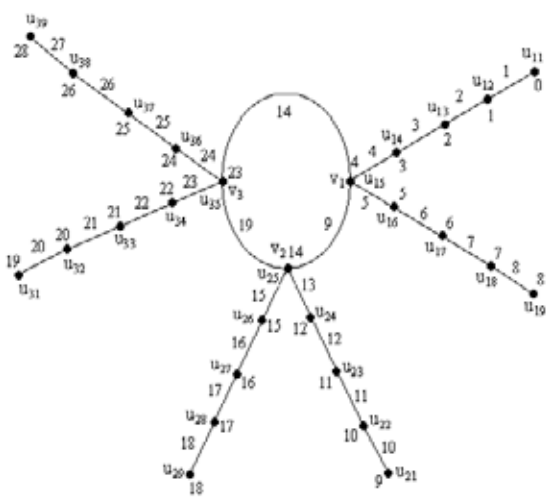


Figure 4

Theorem 3.4: The graph $C_4 \otimes 2P_m$ is a near mean graph.

Proof :

Let $C_4 \otimes 2P_m = \{V, E\}$ such that

$V = \{[u_{ij} : 1 \le i \le 4, 1 \le j \le 2m-1],$

$v_i : 1 \le i \le 4\}$

with $u_{im} = v_i : 1 \le i \le 4$

$E = \{ [(u_{ij}\, u_{ij+1}) : 1 \le i \le 4, 1 \le j \le 2m-2] \cup [(v_iv_{i+1}) : 1 \le i \le 3] \cup (v_1v_4) \}$

We define $f : V \to \{0, 1, 2, \ldots, 8m-5, 8m-3\}$ by

$f(u_{1j}) = j-1, \quad 1 \le j \le 2m-1$

$f(u_{2j}) = 2m-2+i, \; 1 \le i \le 2m-1$

$f(u_{3j}) = \begin{cases} 4m-2+i & 1 \le i \le m-1 \\ 5m-1 & i = m \end{cases}$

$f(u_{4\,2m-1}) = q+1 = 8m-3$

$f(u_{4i}) = 6m-3+i \qquad 1 \le i \le 2m-2$

The induced edge labeling are

$f^*(u_{1j}\, u_{1j+1}) = j \; 1 \le j \le 2m-2$

$f^*(v_1v_2) = 2m-1$

$f^*(u_{2j}u_{2j+1}) = 2m-1+j \quad 1 \le j \le 2m-2$

$f^*(v_2v_3) = 4m-1$

$f^*(u_{3j}u_{3j+1}) = 4m-1+j \quad 1 \le j \le m$

$f^*(v_3v_4) = 6m-2$

$f^*(v_1v_4) = 4m-2$

$f^*(u_{4\,2m-j}\, u_{4\,2m-1-j}) = 8m-3-i, \quad 1 \le j \le 2m-2$

Case (i) : When m is odd

$f(u_{3\,2m-1}) = \begin{cases} 5m & \text{if } m \ge 5 \\ 4m+1 & \text{if } m = 3 \end{cases}$

$f(u_{3\,2m-2}) = 5m \qquad \text{if } m = 3$

$f(u_{3m+i}) = 6m-2-i \qquad 1 \le i \le \dfrac{m-1}{2}$

$$f\left(u_{3,\frac{3m+1}{2}}\right) = 5m-2$$

$$f\left(u_{3,\frac{3m+1}{2}+i}\right) = 5m-1+i,\ 1 \le i \le \frac{m-3}{2}$$

The induced edge labelings are

$$f^*(u_{3m}\, u_{3m+1}) = \frac{11m-3}{2}$$

$$f^*(u_{3\,m+1}\, u_{3\,m+2}) = 5m-1 \text{ if } m=3$$

$$f^*(u_{3\,m+i}\, u_{3\,m+1+i}) = 6m-2-i,\quad 1 \le i \le \frac{m-3}{2}$$

$$f^*\left(u_{3,\frac{3m-1}{2}} u_{3,\frac{3m+1}{2}}\right) = 5m \quad \text{if } m>3$$

$$f^*\left(u_{3,\frac{3m+1}{2}} u_{3,\frac{3m+3}{2}}\right) = 5m-1$$

$$f^*\left(u_{3,\frac{3m+1}{2}+i} u_{3,\frac{3m+3}{2}+i}\right) = 5m+i,\quad 1 \le i \le \frac{m-5}{2}$$

Case (ii) : when m is even

$$f(u_{3m+i}) = 6m-2-i \qquad 1 \le i \le \frac{m}{2}$$

$$f\left(u_{3,\frac{3m}{2}+1}\right) = 5m-2$$

$$f\left(u_{3,\frac{3m+2}{2}+i}\right) = 5m-1+i,\ 1 \le i \le \frac{m-4}{2}$$

The induced edge labelings are

$$f^*(u_{3m}\, u_{3m+1}) = \frac{11m-4}{2}$$

$$f^*(u_{3\,m+i}\, u_{3\,m+i+1}) = 6m-2-i,\quad 1 \le i \le \frac{m-2}{2}$$

$$f^*(u_{3\,2m-2}\, u_{3\,2m-1}) = 5m-1 \quad \text{if } m=4$$

$$f^*\left(u_{3,\frac{3m}{2}} u_{3,\frac{3m}{2}+1}\right) = 5m \quad \text{if } m>4$$

$$f^*\left(u_{3,\frac{3m+2}{2}} u_{3,\frac{3m+4}{2}}\right) = 5m-1 \quad \text{if } m>4$$

$$f^*\left(u_{3,\frac{3m+4}{2}+i} u_{3,\frac{3m+4}{2}+i+1}\right) = 5m+I,\quad 1 \le i \le \frac{m-6}{2}$$

It is clear that edges are labeled distinguishly from the set {1, 2, ..., q}

Hence $C_4 \otimes 2P_m$ is near mean graph. For example the near mean labeling of ($C_4 \otimes 2P_5$: m - odd) and ($C_4 \otimes 2P_6$: m - even) are shown in figure 5 and 6 respectively.

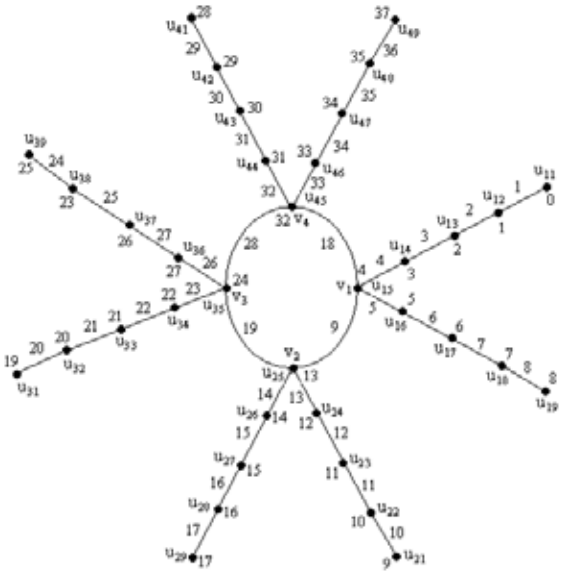


Figure 5

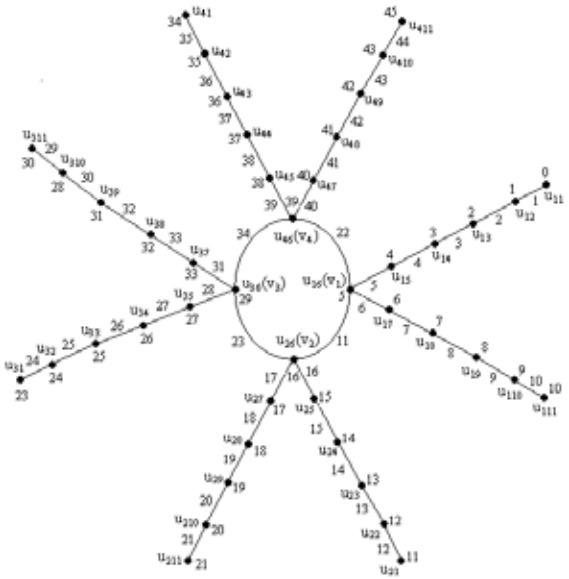


Figure 6

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