Research Paper Engineering

On One Imbedding Into the Class of Continuous Mixed Derivatives

E.Zh. Aidos | KazNTU, Kazakhstan

ABSTRACT

The paper deals with obtaining necessary and sufficient conditions for the embedding $E_{\mathbf{p},d,\Gamma}(\lambda) \subset C_{(\mathbf{r}\alpha)}(\pi_d), 1 < p < \infty,$ These conditions are specified in terms of best approximations of the function by $f(\mathbf{x})$ trigonometric polynomials with harmonics from hyperbolic crosses corresponding to a mixed derivative in the metric L_p, $1 < p < \infty.$

KEYWORDS mixed derivative, trigonometric polynomial, hyperbolic cross, mixed norm, best approximation.

It was established by V.N. Temlyakovthat for the embedding $E_{\mathbf{p},d,\Gamma}(\lambda) \subset C_{(\mathbf{r}\alpha)}(\pi_d), 1 \le p < \infty,$ is enough and provided that $2^{l\left(r+\frac{1}{p}\right)} l^{\left(1-\frac{1}{p}\right)(v-1)} \lambda_l \downarrow 0,$ one must satisfy the inequality $\sum_{l=1}^{\infty} 2^{l\left(r+\frac{1}{p}\right)} l^{\left(1-\frac{1}{p}\right)(v-1)} \lambda_l < \infty,$ $(*)$ (see [1], theorem 3.1 and its corollary). We show that at $1 < p < \infty$ for the embedding $E_{\mathbf{p},d,\Gamma}(\lambda) \subset C_{(\mathbf{r}\alpha)}(\pi_d)$ inequality $(*)$ is satisfied with no additional limits on λ.

Note that for one-dimension casesthe problem of embedding into the space of continuous functions was solved by S.B.Stechkin, A.A.Konyushkov [2] and V.I.Kolyada [3], and for the multidimensional cases, it was solved by N.Temirgaliev [4] in terms of best approximations by trigonometric polynomials with harmonics from rectangles.

Let's firstcite the necessary notations and auxiliary assertions.

Let R_d - d -be a dimensional Euclidean space of points $\mathbf{x} = (x_1, x_2, ..., x_d)$ with real coordinates, $\pi_d \equiv [-\pi, \pi]^d$ - d - a dimensional cube. Let's say that $f \in L_p(\pi_d)$ if $f(\mathbf{x}) = f(x_1, ..., x_d)$ is a measurable function, 2π - periodic in each variable and such that

$$\|f\|_p = \left(\frac{1}{(2\pi)^d} \int_{\pi_d} |f(x)|^p d\mathbf{x} \right)^{\frac{1}{p}} < \infty, 1 \le p < \infty.$$

Further, let:

$\mathbf{k}=(k_1,...,k_d)$, k_j - are the integers, $\mathbf{s}=(s_1,...,s_d)$, s_j- are natural numbers, $j=1,...,d$;

$$\rho(\mathbf{s})=\left\{\mathbf{k}=(k_1,...,k_d):\ 2^{s_j-1}\le k_j<2^{s_j},\ j=1,2,...,d\right\};\quad \delta_{\mathbf{s}}(f,\mathbf{x})=\sum_{|\mathbf{k}|\in\rho(\mathbf{s})}\hat{f}(\mathbf{k})e^{i(\mathbf{k},\mathbf{x})},$$

where $|\mathbf{k}|=(|k_1|,...,|k_d|)$; $\hat{f}(\mathbf{k})=\dfrac{1}{(2\pi)^d}\int\limits_{\pi_d}f(\mathbf{x})e^{-i(\mathbf{k},\mathbf{x})}d\mathbf{x}$;

e - a set of integers of $[1;d]$ and $\boldsymbol{\gamma}=(\gamma_1,...,\gamma_d)$ is a vector, where $\gamma_j=1$ at $j\in e$ and $\gamma_j>1$ at $j\notin e$;

$\mathbf{r}=(r_1,...,r_d)$ and $r=\min\limits_{i=1,...,d}r_i\ge 0$.

Let's define the following sets: $\Gamma(N,\boldsymbol{\gamma})=\left\{\mathbf{k}:\ k_j>0,\ j=1,...,d,\ \prod\limits_{j=1}^{d}k_j^{\gamma_j}\le N\right\}$.

The set of all such $\mathbf{k}$ that $|\mathbf{k}|\in\Gamma(N,\ )$ is called a hyperbolic cross, corresponding to $\boldsymbol{\gamma}=(\gamma_1,...,\gamma_d)$;

$$Q_n^{\mathbf{r}}=\bigcup_{(\boldsymbol{\gamma},\mathbf{s})\le n}\rho(\mathbf{s}),\ \mathbf{r}\boldsymbol{\gamma}=r\ =r\left(\frac{r_1}{r},...,\frac{r_d}{r}\right).$$

The set of all such $\mathbf{k}$ that $|\mathbf{k}|\in Q_n^{\mathbf{r}}$, is called a step hyperbolic cross, corresponding to $\boldsymbol{\gamma}=\left(\dfrac{r_1}{r},...,\dfrac{r_d}{r}\right)$ or to a mixed derivative $\mathbf{r}=(r_1,...,r_d)$.

Let $T(N,\boldsymbol{\gamma})$ and $\mathrm{T}(Q_n^{\mathbf{r}})$ denote the set of all polynomials $\sum\limits_{|\mathbf{k}|\in\Gamma(N,\ )}a_{\mathbf{k}}e^{i(\mathbf{k},\mathbf{x})}$ and $\sum\limits_{|\mathbf{k}|\in Q_n^{\mathbf{r}}}c_{\mathbf{k}}e^{i(\mathbf{k},\mathbf{x})}$.

Quantities defined as:

$$\mathrm{E}_{N,\boldsymbol{\gamma}}(f)_{\mathbf{p}}=\inf_{t\in\mathrm{T}(N,\boldsymbol{\gamma})}\|f-t\|_{\mathbf{p}},\quad 1\le p_i<\infty,\ \ i=1,...,d,$$
$$\mathrm{E}_{Q_n^{\mathbf{r}}}(f)_{\mathbf{p}}=\inf_{t\in T(Q_n^{\mathbf{r}})}\|f-t\|_{\mathbf{p}},\quad 1\le p_i<\infty,\quad i=1,...,d,$$

are called best approximations of the function $f(\mathbf{x})$ by trigonometric polynomials in $L_{\mathbf{p}}(\pi_d)$ metrics with harmonics from a step hyperbolic cross corresponding to the given $\mathbf{r}$.

Since at $\mathbf{r}\boldsymbol{\gamma} = r$ holds $\mathrm{T}\left(Q_n^{\mathbf{r}}\right) \subset T\left(2^n, \boldsymbol{\gamma}\right) \subset \mathrm{T}\left(Q_{n+\gamma(d)}^{\mathbf{r}}\right)$, $\gamma(d) = \sum_{j=1}^{d} \gamma_j$, it suffices to consider one of the quantities $\mathrm{E}_{Q_n^{\mathbf{r}}}\left(f\right)_{\mathbf{p}}$ or $\mathrm{E}_{N,\boldsymbol{\gamma}}\left(f\right)_{\mathbf{p}}$.

Let $\mathbf{r}\boldsymbol{\gamma} = r$, $r \geq 0$, $U_N^{\mathbf{r}}\left(\mathbf{x}, \boldsymbol{\alpha}\right) = 2^d \sum_{\mathbf{k} \in \Gamma(N,\boldsymbol{\alpha})} \prod_{j=1}^{d} k_j^{r_j} \cos\left(k_j x_j + \frac{\alpha_j \pi}{2}\right)$.

For polynomials $t\left(\mathbf{x}\right) \in \mathrm{T}\left(N, \boldsymbol{\alpha}\right)$ a notation $t^{(\mathbf{r})}\left(\mathbf{x}, \boldsymbol{\alpha}\right)$ will denote the convolution with $t\left(\mathbf{x}\right)$ c $U_N^{\mathbf{r}}\left(\mathbf{x}, \boldsymbol{\alpha}\right)$, i.e $t^{(\mathbf{r})}\left(\mathbf{x}, \boldsymbol{\alpha}\right) = \left(2\pi\right)^{-d} \int_{\pi_d} t\left(\mathbf{x} - \mathbf{y}\right) U_N^{\mathbf{r}}\left(\mathbf{y}, \boldsymbol{\alpha}\right) d\mathbf{y}$ (at $\boldsymbol{\alpha} = \mathbf{0}$, $\mathbf{r} = \mathbf{0}$ we have $t^{(\mathbf{0})}\left(\mathbf{x}, \mathbf{0}\right) \equiv t\left(\mathbf{x}\right)$).

The set of all functions $f\left(\mathbf{x}\right)$, that the derivative $f^{(\mathbf{r})}\left(\mathbf{x}, \boldsymbol{\alpha}\right)$ exists and is continuous on π_d we shall denote by $C_{(\mathbf{r}\boldsymbol{\alpha})}\left(\pi_d\right)$.

If $\lambda = \left\{\lambda_n\right\}_{n=1}^{\infty}$ – is a sequence of positive numbers, $\lambda_n \downarrow 0$ $\left(n \uparrow \infty\right)$, then through $E_{\mathbf{p},d,Q}(\lambda)$ let's denote a class of functions $f\left(\mathbf{x}\right) \in L_{\mathbf{p}}\left(\pi_d\right)$, where $\mathrm{E}_{Q_n^{\mathbf{r}}}\left(f\left(\mathbf{x}, \boldsymbol{\alpha}\right)\right)_{\mathbf{p}} = \underset{=}{0}\left(\lambda_n\right)$ $\left(n \to \infty\right)$.

By $C\left(\alpha, \beta, \ldots\right)$ let's denote some positive values that, generally speaking, are different in various formulas and depend only on parameters specified in the brackets. If A is positive and B is positive or O, then a notation $B \underset{\alpha,\beta,\ldots}{\ll} A$ will denote $\left|B\right| \leq C\left(\alpha, \beta, \ldots\right) A$. For positive A and B the notation $A \underset{\alpha,\beta,\ldots}{\asymp} B$ denotes $A \underset{\alpha,\beta,\ldots}{\ll} B \underset{\alpha,\beta,\ldots}{\ll} A$.

Now let's state the main result of this article.

Theorem. Let $r = r_1 = \ldots = r_{\nu} < r_{\nu+1} \leq \ldots \leq r_d$, $r \geq 0$, $1 \leq \nu \leq d$, $1 < p < +\infty$

and $\lambda = \left\{\lambda_n\right\}_{n=1}^{\infty}$ – be a sequence of positive numbers $\lambda_n \downarrow 0$ $\left(n \uparrow \infty\right)$.

Then, in order for the imbedding to be true

$$E_{\mathbf{p},d,\Gamma}(\lambda) \subset C_{(\mathbf{r}\boldsymbol{\alpha})}\left(\pi_d\right), (1)$$

it is necessary and sufficient to satisfy the inequality

$$\sum_{l=1}^{\infty} 2^{l\left(r+\frac{1}{p}\right) l\left(1-\frac{1}{p}\right)(\nu-1)} \lambda_l < \infty. (2)$$

Proof. As noted above, the sufficiency of (1) for the embedding (2) was established in [1], p.24. Let us prove the necessity of (1) for the embedding (2).

Suppose that the embedding (1) is true, but the condition (2) is not satisfied, i.e

$$\sum_{l=1}^{\infty} 2^{l\left(r+\frac{1}{p}\right)} l^{\left(1-\frac{1}{p}\right)(\nu-1)} \lambda_l = \infty. \quad (3)$$

We shall demonstrate that one can construct such a function $f(x)$ of class $E_{\mathbf{p},d,\Gamma}(\lambda)$, that $f^{(\mathbf{r})}(\mathbf{x},\mathbf{0})$ is essentially discontinuous on $[0,2\pi]^d$.

Let's inductively define a sequence of numbers $\{l_k\}$ as follows.

Let $l_1 = 1$. If $l_1,...,l_k$ have already been defined, then let

$$l_{k+1} = \min\left\{\nu:\ \nu \geq l_k + 2, \quad \lambda_\nu < \frac{1}{2}\lambda_{l_k}\right\}.$$

Then, following V.I.Kolyada suppose that

$$\mu_k = \begin{cases} \lambda_{l_{k+1}}, & \text{åñëè} \quad l_{k+1} = l_k + 2, \\ \frac{1}{2}\lambda_{l_k}, & \text{åñëè} \quad l_{k+1} > l_k + 2, \end{cases} \qquad k = 1,2,....$$

Moreover, for all $k = 1,2,....$ are being satisfied the following properties:

$$\text{а)}\ \mu_{k+1} \leq \frac{1}{2}\mu_k; \quad \text{б)}\ \lambda_n \geq \mu_k \text{ для } l_k \leq n < l_{k+1}; \text{в)}\ \lambda_{l_{k+1}} \leq \mu_k.$$

Let's show that

$$\sum_{k=1}^{\infty} 2^{l_{k+1}\left(r+\frac{1}{p}\right)} l_{k+1}^{\left(1-\frac{1}{p}\right)(\nu-1)} \mu_k = +\infty. \quad (4)$$

Indeed, due to (3)

$$\infty = \sum_{l=l_2}^{\infty} 2^{l\left(r+\frac{1}{p}\right)} l^{\left(1-\frac{1}{p}\right)(\nu-1)} \lambda_l = \sum_{k=2}^{\infty} \sum_{l=l_k}^{l_{k+1}-1} 2^{l\left(r+\frac{1}{p}\right)} l^{\left(1-\frac{1}{p}\right)(\nu-1)} \lambda_l \leq \sum_{k=2}^{\infty} l_{k+1}^{\left(1-\frac{1}{p}\right)(\nu-1)} \lambda_{l_k} \sum_{l=l_k}^{l_{k+1}-1} 2^{l\left(r+\frac{1}{p}\right)} \leq$$

$$\leq C_1(r,p) \sum_{k=2}^{\infty} 2^{l_{k+1}\left(r+\frac{1}{p}\right)} l_{k+1}^{\left(1-\frac{1}{p}\right)(\nu-1)} \lambda_{l_k} = \left(\sum\right)' + \left(\sum\right)''.$$

Here the symbol $\left(\sum\ \right)'$ denotes summation over all $k=1,2,....$ for which $l_{k+1}=l_k+2,$ and

$\left(\sum\ \right)''$ - summation over all the other k.Then due to sequencing $\{\mu_k\}$ we have that

$$\left(\sum_k 2^{l_{k+1}\left(r+\frac{1}{p}\right)} l_{k+1}^{\left(1-\frac{1}{p}\right)(\nu-1)}\lambda_{l_k}\right)' = \left(\sum_k 2^{(l_k+2)\left(r+\frac{1}{p}\right)}(l_k+2)^{\left(1-\frac{1}{p}\right)(\nu-1)}\mu_{k-1}\right)' \le$$

$$\le C_2(r,p)\sum_{k=2}^{\infty} 2^{l_k\left(r+\frac{1}{p}\right)} l_k^{\left(1-\frac{1}{p}\right)(\nu-1)}\mu_{k-1} = C_2(r,p)\sum_{k=1}^{\infty} 2^{l_{k+1}\left(r+\frac{1}{p}\right)} l_{k+1}^{\left(1-\frac{1}{p}\right)(\nu-1)}\mu_k.$$

For the second sum $\left(\sum\ \right)''$ i.e when $l_{k+1} > l_k+2,$ we have (see the definition $\{\mu_k\}$)

$$\left(\sum_k 2^{l_{k+1}\left(r+\frac{1}{p}\right)} l_{k+1}^{\left(1-\frac{1}{p}\right)(\nu-1)}\lambda_{l_k}\right)'' = 2\left(\sum_k 2^{l_{k+1}\left(r+\frac{1}{p}\right)} l_{k+1}^{\left(1-\frac{1}{p}\right)(\nu-1)}\mu_k\right)'' \le$$

$$\le \sum_{k=1}^{\infty} 2^{l_{k+1}\left(r+\frac{1}{p}\right)} l_{k+1}^{\left(1-\frac{1}{p}\right)(\nu-1)}\mu_k.$$

These estimates imply (4).

Now let's consider the series $(\boldsymbol{\alpha}=\mathbf{0})$

$$\sum_{k=1}^{\infty} 2^{l_{k+1}\left(\frac{1}{p}-1\right)} l_{k+1}^{-\frac{1}{p}(\nu-1)}\mu_k G_{l_{k+1}-1}(\mathbf{x},\mathbf{0}), \qquad (5)$$

where $G_l(\mathbf{x},\mathbf{0}) = \sum_{l<\sum_{i=1}^{d} s_i\gamma_i \le l+1} \sum_{|k_1|=2^{s_1-1}}^{2^{s_1}-1} \dots \sum_{|k_d|=2^{s_d-1}}^{2^{s_d}-1} \prod_{j=1}^{d} \cos k_j x_j.$

It is known that (see [3], p.25)

$$\left\|G_l(\mathbf{x},\mathbf{0})\right\|_{L_p(\pi_d)} \le C_3(p)2^{l\left(1-\frac{1}{p}\right)} l^{\frac{1}{p}(\nu-1)}, \quad 1<p<\infty, \qquad (6)$$

and

$$G_l^{(\mathbf{r})}(\mathbf{0},\mathbf{0}) \asymp 2^{l(r+1)} l^{(\nu-1)}. \qquad (7)$$

As (see. (6) and the property a))

$$\left\| \sum_{k=1}^{\infty} 2^{l_{k+1}\left(\frac{1}{p}-1\right)} l_{k+1}^{-\frac{1}{p}(\nu-1)} \mu_k G_{l_{k+1}-1}(\mathbf{x},\mathbf{0}) \right\|_{L_p(\pi_d)} \le \sum_{k=1}^{\infty} 2^{l_{k+1}\left(\frac{1}{p}-1\right)} l_{k+1}^{-\frac{1}{p}(\nu-1)} \mu_k \left\| G_{l_{k+1}-1}(\mathbf{x},\mathbf{0}) \right\|_{L_p(\pi_d)} \le$$

$$\le C_4(p) \sum_{k=1}^{\infty} 2^{l_{k+1}\left(\frac{1}{p}-1\right)} l_{k+1}^{-\frac{1}{p}(\nu-1)} \mu_k \cdot 2^{l_{k+1}\left(1-\frac{1}{p}\right)} l_{k+1}^{\frac{1}{p}(\nu-1)} = \sum_{k=1}^{\infty} \mu_k < \infty,$$

then the series (5) defines a function $f(\mathbf{x})$ of $L_p(\pi_d)$, i.e

$$f(\mathbf{x}) \overset{L_p(\pi_d)}{=} \sum_{k=1}^{\infty} 2^{l_{k+1}\left(\frac{1}{p}-1\right)} l_{k+1}^{-\frac{1}{p}(\nu-1)} \mu_k G_{l_{k+1}-1}(\mathbf{x},\mathbf{0}). \quad (8)$$

Now let's show that $f(\mathbf{x},\mathbf{0}) \in E_{\mathbf{p},d,\Gamma}(\lambda)$, but $f(\mathbf{x},\mathbf{0}) \notin C_{(\mathbf{r}\alpha)}(\pi_d)$.

Given n, $n = 1,2,...$ and let the given number m is such that $l_m \le n < l_{m+1}$.

Then due to inequality (6) and properties a) and b) of the sequence μ_k, we have

$$\mathrm{E}_{2^n,\gamma}(f)_{\mathbf{p}} \le \mathrm{E}_{2^{l_m},\gamma}(f)_{\mathbf{p}} \le \mathrm{E}_{Q_{l_m}^{\mathbf{r}}}(f)_{\mathbf{p}} \le \left\| \sum_{i=m}^{\infty} 2^{l_{i+1}\left(\frac{1}{p}-1\right)} l_{i+1}^{-\frac{1}{p}(\nu-1)} \mu_i G_{l_{i+1}-1}(\mathbf{x},\mathbf{0}) \right\|_{L_p(\pi_d)} \le$$

$$\le \sum_{i=m}^{\infty} 2^{l_{i+1}\left(\frac{1}{p}-1\right)} l_{i+1}^{-\frac{1}{p}(\nu-1)} \mu_i \left\| G_{l_{i+1}-1}(\mathbf{x},\mathbf{0}) \right\|_{L_p(\pi_d)} \le C_5(p) \sum_{i=m}^{\infty} 2^{l_{i+1}\left(\frac{1}{p}-1\right)} l_{i+1}^{-\frac{1}{p}(\nu-1)} \mu_i 2^{l_{i+1}\left(1-\frac{1}{p}\right)} l_{i+1}^{\frac{1}{p}(\nu-1}$$

$$= C_5(p) \sum_{i=m}^{\infty} \mu_i \le C_6(p) \mu_m \le C_6(p) \cdot \lambda_n,$$

i.e $f(\mathbf{x},\mathbf{0}) \in E_{\mathbf{p},d,\Gamma}(\lambda)$.

It remains to show that $f^{(\mathbf{r})}(\mathbf{x},\mathbf{0})$ is a π_d essentially unbounded function.

In fact, if $f^{(\mathbf{r})}(\mathbf{x},\mathbf{0})$ is not essentially unlimited, then changing $f^{(\mathbf{r})}(\mathbf{x},\mathbf{0})$ on the empty set o Lebesgue, we obtain the function $f^{*(\mathbf{r})}(\mathbf{x},\mathbf{0})$. bounded on π_d.

Let $S_{n_1,\ldots,n_d}\left(\mathbf{x}, f^{*(\mathbf{r})}\right)$ - be rectangular partial sums of functions $f^{*(\mathbf{r})}(\mathbf{x},\mathbf{0})$.

Then, their arithmetic mean, i.e.

$$\sigma_{n_1,\ldots,n_d}\left(\mathbf{x}, f^{*(\mathbf{r})}\right) = \frac{1}{n_1+1} \cdots \frac{1}{n_d+1} \sum_{0}^{n_1,\ldots,n_d} S_{k_1,\ldots,k_d}\left(\mathbf{x}, f^{*(\mathbf{r})}\right) (9)$$

can be represented in the form (see [5], p.455)

$$\sigma_{n_1,\ldots,n_d}\left(\mathbf{x}, f^{*(\mathbf{r})}\right)=\frac{1}{\pi_d}\int\limits_{\pi_d} f^{*(\mathbf{r})}\left(x_1+t_1,\ldots,x_d+t_d\right)\prod_{i=1}^{d}K_{n_i}\left(t_i\right)dt_1..dt_d, \quad (10)$$

where $K_n\left(t\right)=\frac{1}{2^{n+1}}\cdot\frac{\sin\left(n+1\right)\frac{t}{2}}{\sin\frac{t}{2}}$ - Fejer kernel.

It is known (see [6], p.138-139), that the Fejer kernel $K_n\left(t\right)$ has, in particular, the following properties:

1) $K_n\left(t\right)\geq 0$ for all $t\in\left(-\infty;+\infty\right)$;

2) $\frac{1}{\pi}\int\limits_{-\pi}^{\pi}K_n\left(t\right)dt=1, \quad n=1,2,\ldots$.

From these properties, and by virtue of $\left\|f^{*(\mathbf{r})}\right\|_{L^\infty(\pi_d)}\leq M$ in any $\left(x_1,\ldots,x_d\right)\in\left[-\pi,\pi\right]^d$ and $n_1,\ldots,n_d=0,1,\ldots$ we obtain (see(10))

$$\left|\sigma_{n_1,\ldots,n_d}\left(\mathbf{x}, f^{*(\mathbf{r})}\right)\right|\leq M. \quad (11)$$

On the other hand, a number (8) for $\mathbf{x}=\mathbf{0}$ is a numeric d –multiple series with non-negative terms.

Further, if $S_n\left(\mathbf{x}, f^{*(\mathbf{r})}\right)$ - are hyperbolic partial sums of functions $f^{*(\mathbf{r})}$,

Then $S_n\left(\mathbf{0}, f^{*(\mathbf{r})}\right)\rightarrow\infty, \quad n\rightarrow\infty.$ Indeed, from (7) and (3) it follows that

$$\sum_{k=1}^{\infty}2^{l_{k+1}\left(\frac{1}{p}-1\right)}l_{k+1}{}^{-\frac{1}{p}(\nu-1)}\mu_k G^{(\mathbf{r})}_{l_{k+1}-1}\left(\mathbf{0},\mathbf{0}\right) \quad \square \quad \sum_{k=1}^{\infty}2^{l_{k+1}\left(\frac{1}{p}-1\right)}l_{k+1}{}^{-\frac{1}{p}(\nu-1)}\mu_k\cdot 2^{l_{k+1}(r+1)}l_{k+1}{}^{(\nu-1)}=$$

$$=\sum_{k=1}^{\infty}2^{l_{k+1}\left(r+\frac{1}{p}\right)}l_{k+1}{}^{(1-\frac{1}{p})(\nu-1)}\mu_k=+\infty.$$

Consequently, $S_{n_1,\ldots,n_d}\left(\mathbf{0}, f^{*(\mathbf{r})}\right)\geq 0$ and $S_{n_1,\ldots,n_d}\left(\mathbf{x}, f^{*(\mathbf{r})}\right)\rightarrow\infty$ at $\min\left(n_1,\ldots,n_d\right)\rightarrow\infty,$

because numerical series with non-negative terms (8) has the same sum regardless of the method of formation of partial sums.Therefore (see(9)).

$\sigma_{n_1,\ldots,n_d}\left(\mathbf{0}, f^{*(\mathbf{r})}\right) \to \infty$, at $\min\left(n_1,\ldots,n_d\right) \to \infty$. It led to a contradiction with (11), and, thus, there have been proved the essential unlimitedness of the function $f^{(\mathbf{r})}(\mathbf{x},\mathbf{0})$.

The theorem is proved.

REFERENCES

1. Temlyakov V.N. Approximation of functions with bounded mixed derivative / / Trudy Mat. - 1986. T.178- p.3-112. | 2. Konyushkov A.A. Best approximations by trigonometric polynomials and Fourier coefficients / / Mathematical collection - 1958. - 44, 1.p. 53-84. | 3. Kolyada V.I. Embedding theorems and inequalities in various metrics for best approximations / / Mathematical collection - 1977. –v. 102, №2 - p.195-215. | 4. Temirgaliev N. About the imbedding of certain classes of functions in / / Proceedings of universities.Mathematics. - 1978. - №8 (195). –p.88 - 90. | 5. Zygmund A., Trigonometric series.V.2. - New York: MIR, 1965. P. 537 p. | 6. Bari N.K. Trigonometric series. - Moscow: Fizmatgiz, 1961. P.- 936.