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Analysis of box culvert considering soil structure interaction

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ABSTRACT

Box culvert is consisting of top, bottom and two vertical side walls built monolithically which forms the square or rectangular single cell. The box structure is highly indeterminate structure which is having continues support as directly rests on soil. Hence to understand its true behaviour, soil structure interaction should taken into account. In practise approximate methods like moment distribution, force or displacement matrix were used. While using these methods certain assumption are made regarding to the boundary condition and it is assumed that bottom slab is infinitely stiff so that it will not undergo any differential settlement. But in reality bottom slab undergoes differential settlement; therefore there will be error in the results obtained by approximate methods. Hence attempt is made to study results obtained by considering soil structure interaction (S.S.I.) and without considering it. The effect of S.S.I. is incorporated by developing stiffness matrix using beam on elastic foundation concept. For this a detailed program is developed in FORTRAN 90/95.A parametric study is carried out for considering S.S.I. and without considering S.S.I. by varying number of cells in box culvert. Also study is carried out for various soil types by considering appropriate soil subgrade reaction to know the effect of type of soil on bending moment. The study reveals that the bottom slab is the element which is severely affected and variation of bending moment in bottom slab is in the range of 50% to 70%, in some other load cases the bending moment also changes the sign.

Keywords : Box culvert, Stiffness method, Beam on elastic foundation, Modulus of subgrade reaction, Soil structure interaction

1. Introduction

Analysis of box culvert is done by stiffness matrix method. Two models with respect to support conditions have been considered. In the first model box culvert is assumed as externally determinate assuming discrete boundary conditions. In the second model the bottom chord of the box culvert is assumed as resting on elastic foundation. Here the soil structure interaction is taken into account for bottom chord. Single cell box structure is assumed as rigid frame structure consisting of top slab, bottom slab and two vertical side walls which forms a closed rigid box frame. A basic assumption in analysis of the box culvert is the displacement and forces are uniform in the longitudinal direction of the culvert. This assumption holds true for certain type of loadings than others. For example soil loading applied to the surface or pavement maybe considered as uniform in the longitudinal direction. Solution therefore is independent of one of the three orthogonal axes and can be formulated in remaining two axes. Thus problem can be treated as two dimensional. The analysis is performed considering a unit wide strip along the longitudinal axes. This strip is said to be in plain strain condition signifies the fact that the out of plane deformations are zero. The loads applied through small areas of contact between wheels and pavement. Such loads for practical are considered as point loads. While analyzing the box structure in first model some basic assumptions are made to simplify the problem. It is assumed that structure is externally determinate. Also the pressure distribution at the bottom is assumed linear. In case of second mode which is highly indeterminate, it is assumed that bottom chord members are continuously supported. They undergo differential settlements and offer a resistance proportional to the transverse deflection. In both the modes axial and shear deformations are neglected. Based on above assumptions analysis is carried out for both models by displacements matrix method considering appropriate stiffness of the bottom chord members.

2. Stiffness method

The force displacement relationship for a prismatic member is shown in figures 1and 2

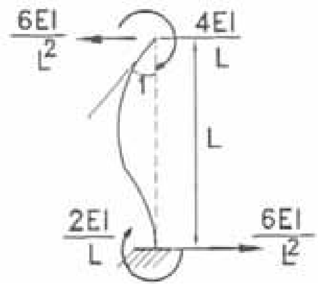


Fig. 1 Forces and moments generated ends due to unit rotation.

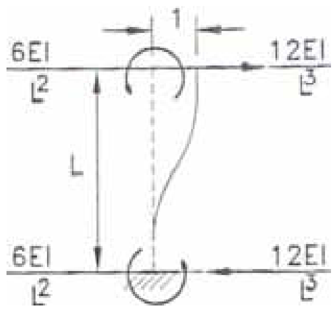


Fig. 2 Forces and moments generated at the ends due to unit displacement.

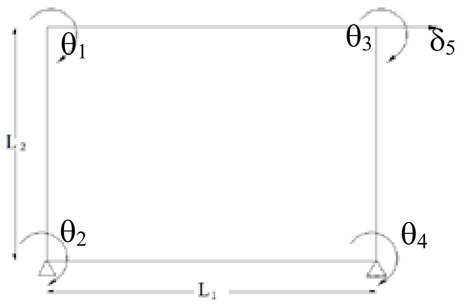


Fig. 3

Single cell box culvert showing degrees of freedom.

2.1 Formation of a stiffness matrix for single cell

Consider a box structure as shown in the fig.3 In this case we have three unknown support reaction. i.e. two at hinge support at left and one at roller support at, right. Since three equilibrium equations are available this single cell box structure is externally statically determinate. The internal static indeterminacy is three. This structure is kinematically indeterminate to fifth degree if axial deformations are neglected. Each joint of rigid jointed plane frame has three independent displacement components viz. two linear displacements and one rotational component. Therefore, Degree of kinematic indeterminacy of the rigid jointed plane frame is given by the equation given below.

$$D_{(IK)} = 3\,N_j - (N_R + N_m) \_\_\_\_\_\_\_\_ \quad (1)$$

Where,

$D_{(IK)}$ =. degree of kinematic indeterminacy.

j = number of joints

N_R = number of reaction components.

N_m = number of constrains imposed by support condition and other factors such as inextensibility of members.

Applying the equation (1) degree of kinematic indeterminacy for the single cell box structure as shown in fig. 3 can be calculated as below.

$D_{(IK)}$ = (3x4)-(3+4) = 5

Therefore single cell box structure is kinematically indeterminate to fifth degree. In order to generate elements of stiffness matrix a unit displacement is imparted at each degree of freedom successively. The first four displacements are angular and fifth is linear. The equation 2 shows the stiffness matrix generated for single cell box culvert.

$$[K]_{5x5} = E\begin{bmatrix} (\frac{4I_1}{L_1}+\frac{4I_2}{L_2}) & \frac{2I_2}{L_2} & \frac{2I_1}{L_1} & 0 & \frac{-6I_2}{L_2^2} \\ \frac{2I_2}{L_2} & (\frac{4I_1}{L_1}+\frac{4I_2}{L_2}) & 0 & \frac{2I_1}{L_1} & \frac{-6I_2}{L_2^2} \\ \frac{2I_1}{L_1} & 0 & (\frac{4I_1}{L_1}+\frac{4I_2}{L_2}) & \frac{2I_2}{L_2} & \frac{-6I_2}{L_2^2} \\ 0 & \frac{2I_3}{L_1} & \frac{2I_2}{L_2} & (\frac{4I_1}{L_1}+\frac{4I_2}{L_2}) & \frac{-6I_2}{L_2^2} \\ \frac{-6I_2}{L_2^2} & \frac{-6I_2}{L_2^2} & \frac{-6I_2}{L_2^2} & \frac{-6I_2}{L_2^2} & \frac{-24I_2}{L_2^3} \end{bmatrix} ----(2$$

3. Beam on elastic foundation method.

Many times structural member rests on spongy material, here after called as an elastic foundation, which offers a resistance proportional to the transverse deflection. Thus there are unknown transverse forces, equal to the product of the "stiffness modulus" of the supporting material and yet unknown transverse deflection, acting on structural members on elastic foundation. In case of box structure in which bottom members are subjected to resistance proportional to the transverse deflection, the stiffness method of analysis can still be used, provided that expressions for the member stiffness matrix and for the fixed-end reactions and moments due to common types of transverse load can be found as functions of the stiffness modulus. In the following articles, these expressions will be derived.

3.1 The basic Differential Equation for beam on Elastic Foundation.

Consider a structural member AB and its elastic curves A' B' as shown in fig. 5. It is subjected to a varying down ward load of w per unit length and to an upward reactive force of KY per unit length, where k is the stiffness modulus of the elastic foundation, measured in force per unit displacement.

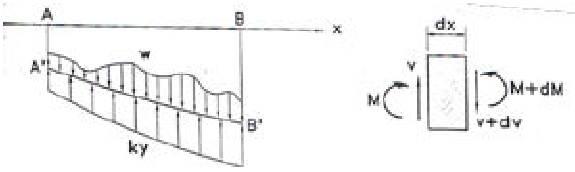


Fig. 5

Structural member on elastic foundation.

From the equilibrium equations of resolution and rotation of an infinitesimal segment of the structural member,

$$\frac{dv}{dx} = ky - w \quad (5)$$

$$\frac{dM}{dx} = V \quad (6)$$

Where in positive directions of shear V and bending moment M are as shown in above figure. The change in slope in between any two consecutive points at infinitesimal distance dx apart is equal to

$$d\,\frac{dy}{dx} = -\frac{M}{EI}\,dx \quad (7)$$

The negative sign is due to fact that the slope is decreasing in positive or concave bending. Combining equation a, b, c gives the basic differential equation of elastic curve

$$\left(\frac{d^4y}{dx^4}\right)+\left(\frac{k}{EI}\right)y=\left(\frac{w}{EI}\right) \quad (8)$$

The shear and bending moment become

$$M = EI\left(\frac{d^2y}{dx^2}\right) \quad (9)$$

$$V = EI\,\frac{(d^3y)}{dx^3} \quad (10)$$

3.2 General solution of the differential equation

Where there is no transverse loading on the member, the basic differential equation (8) becomes

$$\left(\frac{d^4y}{dx^4}\right)+\left(\frac{k}{EI}\right)y=0 \quad (11)$$

Of which the general solution can be given as

$$Y = A\cos\frac{\phi}{L}x\cosh\frac{\phi}{L}x + B\cos\frac{\phi}{L}x\sinh\frac{\phi}{L}x + C\sin\frac{\phi}{L}x\cosh\frac{\phi}{L}x + D\sin\frac{\phi}{L}x\sinh\frac{\phi}{L}x$$

$$\text{Where } \phi = L\sqrt[4]{\frac{k}{4EI}} \quad (12)$$

3.3 Boundary condition of an unloaded member

The general solution of the differential equation of an unloaded member resting on as elastic foundation includes four arbitrary constants; four boundary conditions are required for evaluation of these constants. Two common approaches are

1. Specifying the bending moments M_i, M_j, and the shears V_i,V_j at the points i, j

2. Specifying the slopes θ_i, θ_j and the transverse deflection Δ_i, Δ_j.

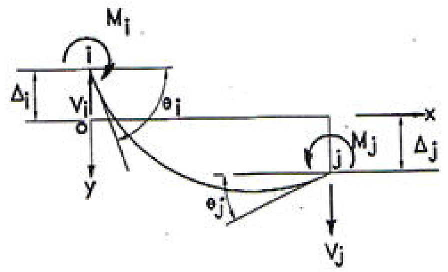


Fig. 6

Direction of forces and boundary conditions of structural member.

Using the first approach will yield a [4x4] flexibility matrix of member on elastic foundation, and the use of second approach will give the [4x4] stiffness matrix.

Hence stiffness matrix [S] of a member on elastic foundation is obtained as

	θ_i	θ_j	Δ_i	Δ_j
M_i	T1	T 2	T 5	- T 6
M_j	T2	T1	T6	-T5
V_i	T5	T6	T3	T4
V_j	-T6	-T5	T4	T3

Where,

$$T1=+\frac{2\phi(s'c'-sc)}{s'^2-s^2}\frac{EI}{L}=+\frac{4EI}{L} \quad \text{at } \phi=0 \qquad T2=+\frac{2\phi(sc'-cs')}{s'^2-s^2}\frac{EI}{L}=+\frac{2EI}{L} \quad \text{at } \phi=0$$

$$T3=+\frac{4\phi^3(sc-s'c')}{s'^2-s^2}\frac{EI}{L^3}=+\frac{12EI}{L^3} \quad \text{at } \phi=0 \qquad T4=+\frac{4\phi^3(sc'+cs')}{s'^2-s^2}\frac{EI}{L^3}=+\frac{12EI}{L^3} \quad \text{at } \phi=0$$

$$T5=+\frac{2\phi^2(s'^2+s)}{s'^2-s^2}\frac{EI}{L^2}=+\frac{6EI}{L^2} \quad \text{at } \phi=0 \qquad T6=+\frac{4\phi^2 ss'}{s'^2-s^2}\frac{EI}{L^2}=+\frac{6EI}{L^2} \quad \text{at } \phi=0$$

The degenerate values of T1 to T6 at ϕ=0 are stiffness coefficients of an ordinary prismatic member as shown in fig.5 and fig.6.

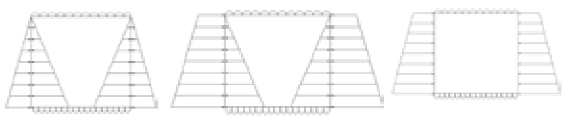
4. Parametric study

The parametric study contains variation of bending moment for with and without S.S.I. which is done For 3 types of box culvert namely single cell(4m X 4m), double cell(4m X 4m two cells) and triple cell (4m X 4m three cells) and for three load cases as shown in figure7 . Also study is carried out for various soil types by considering appropriate soil subgrade reaction to know the effect of type of soil on bending moment. For this a detailed program is developed in FORTRAN 90/95.

Case1: - Considering live load and dead load on top slab, lateral load due to live load and earth pressure.

Case2: - Considering live load and dead load on top slab, lateral load due to live load and earth pressure and water pressure from inside.

Case3: - Considering live load and dead load on top slab, lateral load due to earth pressure, water pressure from inside and lateral load due to live load.



Case 1 Case 2 Case 3

Fig. 7

Load cases for box culvert

Model values obtained by analysis of load case 1 are presented in this work. The loading magnitudes for case study are taken from the fig. 8 which obtained by solving a typical example of single cell box culvert (4m X 4m) carrying IRC class AA tracked loading.

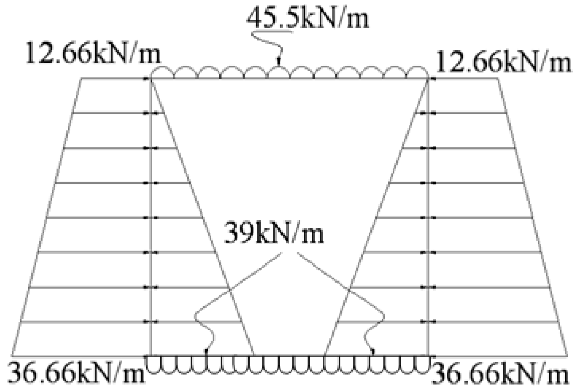


Fig. 8

Total loading diagram for box culvert

The results obtained for considering S.S.I. and without considering S.S.I. by varying number of cells in box culvert are shown in fig. 9 and fig. 10. While considering S.S.I. subgrade reaction of soil is assumed as 4800kN/m3 (loose sand).

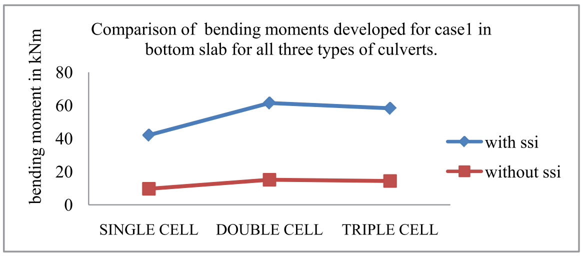


Fig.9

Comparison of bending moments developed for case1 in bottom slab for all three types of culverts.

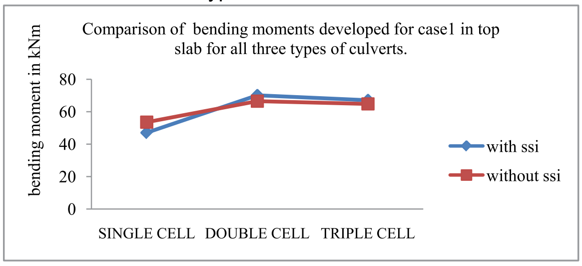


Fig. 10

Comparison of bending moments developed for case1 in top slab for all three types of culverts

The modulus of subgrade reaction is a conceptual relationship between soil pressure and deflection that is widely used in the structural analysis of foundation members like continues footings, mat or raft foundations etc. The modulus of subgrade reaction is the ratio of stress to deformation. The table no. 1 shows the values of subgrade reaction for various types of soils according to Bowels (1988). Here results are presented in fig. 11 to 13 are for variation of bending moment for various soil subgrade reactions.

Table no. 1

Values of modulus of subgrade reaction [Bowles (1988)]

Type of soil	Modulus of subgrade reaction (k) in kN/m3
Loose sand	4800- 16000
Medium dense sand	9600- 80000
clayey medium dense sand	32000- 80000
Dense sand	64000-128000

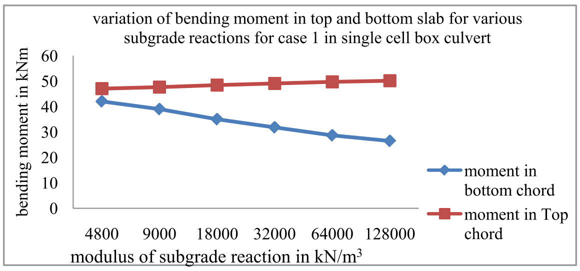


Fig. 11

Variation of bending moment in top and bottom for various subgrade reactions slab for case 1 in single cell box culvert

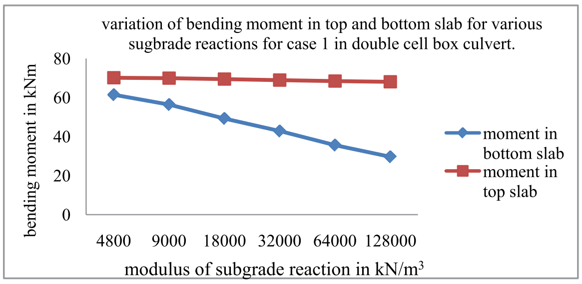


Fig. 12

Variation of bending moment in top and bottom slab for various subgrade reactions for case 1 in double cell box culvert.

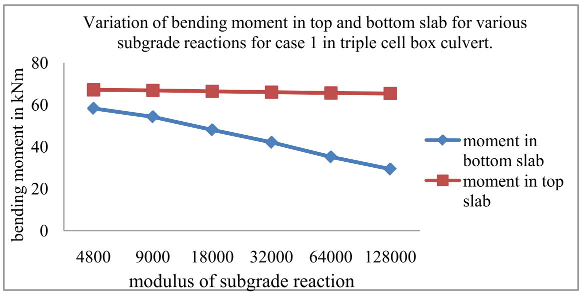


Fig. 13

Variation of bending moment in top and bottom slab for various subgrade reactions for case 1 in triple cell box culvert.

5. Observations and Conclusion

5.1 Observations

In previous chapter the box structure is analyzed for various conditions by changing number of cells, for different load cases and for various types of soil subgrade reaction. From that we can observe that in

1. Single cell box culvert.

- When values of modulus of soil subgrade reaction are changed from loose soil to dense soil bending moment varies substantially (45%) in bottom slab and marginal (6%) in top slab.
- In all three cases bending moment generation is higher in case of supports having lower values of soil subgrade (i.e. loose soils) when compared with nonyielding supports.

2. Double cell box culvert.

- When values of modulus of soil subgrade reaction are changed from loose soil to dense soil bending moment varies substantially (55%) in bottom slab and marginal (3%) in top slab.
- In case of structure with S.S.I. maximum moments are developed at central nodes of the bottom slab, where as in case of structure without S.S.I. maximum moments are developed at edge nodes of bottom slab.

3. Triple cell box culvert

- When values of modulus of soil subgrade reaction are changed from loose soil to dense soil bending moment varies substantially (50%) in bottom slab and marginal (3%) in top slab.
- In case of structure with S.S.I. maximum moments are developed at just right of the second node or just left of the third node in bottom slab (i.e. in middle span), where as In case of structure without S.S.I. where discrete nonyielding supports are considered maximum moments are developed at edges of the bottom slab.

4.2 Conclusion

From the above observations it can be stated that for bottom slab the positive bending moment (tension at bottom) starts reducing as modulus of subgrade reaction is varied from lower values of soil subgrade (loose sand) to higher values of soil subgrade (dense sand) and for further higher values of subgrade reactions i.e. for rock the values of bending moment may be equal or nearer to the values obtained for the nonyielding supports.

Considering S.S.I. variation of bending moment is marginal in top slab and substantial in bottom slab. In the vertical members also variation of bending moment is marginal. Thus most critical element of box culvert is bottom slab and moment in this slab is to be calculated by appropriate S.S.I. effect.

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