Research Paper Engineering

Flow Equation For Elliptical Weir

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ABSTRACT

Flow measurements structures are generally designed to act as control in the channel in order to provide a relationship between upstream head and the discharge. In the present paper a discharge equation was designed for a elliptical weir. A relationship for obtaining the elliptical weir profile was developed. The theoretical discharge of flow across the elliptical weir was derived using a geometric constant based on the weir profile. An elliptical weir was developed and evaluated in laboratory hydraulic flume. The average coefficient of discharge was estimated as 0.578.

Keywords : Flow measurement, elliptical weir, Open channel flow

INTRODUCTION

Efficient operation and management of irrigation systems play important roles in the sustainability of irrigated agriculture (Mishra et al., 2001). Irrigation water use efficiency depends largely on the accuracy of flow measurement. Information on relationships of water, soil, and plants cannot be utilized without water measurement in irrigation practice (Israelsen & Hansen, 1962). Flow measurement structure is generally designed to act as a control in the channel in order to provide a unique relationship between the upstream head and the discharge (Boiten, 2002). Thin plate weirs are the simplest of irrigation measurement devices that can be used for a wide range of discharges in field channels and lined canals; however, the discharge equation is relatively difficult to apply by hand. Several standard sources provide discharge relationships of various shaped sharp crested weirs employed in irrigation water measurement (Israelsen & Hansen, 1962; James, 1988; Troeh et al., 2004). Discharge equations of sharp crested flow measurement weirs, of different shapes in general, are functions of length of weir crest l and flow depth h with an exponent n. Values of the flow depth exponent in discharge equations are invariably some fraction for weirs of different shapes (e.g. the flow depth with an exponent for rectangular and trapezoidal weirs is h3/2, and for triangular weirs is h5/2). These fractional values of n make the manual calculation difficult, although possible, to achieve accurate results quickly. Keshava murthy and Giridhar (1990) find the average Cd value of especially sharp crested weirs, such as for the inverted V- notch or chimney weirs with weir angles of 25° -45°(measured from vertical) ranged from 0.60-0.61. Michale Brown, Larry Froney and Jude Sommerfeld (1997) developed analytical expression for the shapes rectangular, circular, parabolic and triangular. Generalized comparison charts were developed for the cases of equal flow areas of overflow devices and equal maximum flow capacities.

Michale Brown, Larry Froney and Jude Sommerfeld (1997) developed analytical expression for the shapes rectangular, circular, parabolic and triangular. Generalized comparison charts were developed for the cases of equal flow areas of overflow devices and equal maximum flow capacities. Kadlubowski (1997) applied the value of coefficient of discharge for circular and parabolic weirs were same around 0.60. C.Chatterjee, R. Singh, S.K.Kar, S.N.Panda and S.L.Bohara (1998) developed a theoretical submerged flow equation for chimney weir is developed. Based on experiments, it is found that the theoretical submerged flow equation, along with parabolic equation relating the coefficient of discharge and the ratio of upstream head to crest height, predict the discharge accurately. K.Keshava Murthy, H.S. Ramesh, M.N. Shesha Prakash (1998) discusses the design and experimental verification of a geometrically simple logarithmic weir. The discharge through this weir is proportional to the logarithm of head measured above a fixed reference plane for all heads in the range 0.23R<h<3.65R within a maximum deviation of ± 2% from the theoretical discharge. Srikanth, Prakash, and Ramesh (2007) developed a parabolic weir profile gave the simplified discharge equation with two as a flow depth exponent. Parabolic weirs with 45° weir angle was developed and evaluated in laboratory hydraulic flume.

The present study intends to (1) to find coefficient of discharge, (2) to study variation of Cd with different heads, (3) to develop mathematical equation in terms of head and coefficient of discharge.

MATERIALS AND METHODS

Weir profile determination for simplified discharge measurement

In open channel flow across an elliptical weir is depicted in figure: 1. in elliptical weir a and b denotes the lengths of the semi-major and semi-minor axes, respectively. Apply the Bernoulli equation to points 1 (liquid crest at elevation of h) and 2 (arbitrary liquid depth of z) in Figure results in equation 2 and the total theoretical volumetric flow rate Qt remains given by equation 1. In this case, the expression for the elliptical cross-section is:

$$Q_t = \int_0^h v_2 dA \tag{1}$$

v2=√2g (h-z) (2)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{3}$$

After insertion of the points x = C/2 and y = b- z, the chord length at point 2 is given by:

$$C = \frac{2a}{b}\sqrt{z(2b-z)} \tag{4}$$

For elliptical weir equation of Qt becomes

$$Q_t = \frac{2a}{b}\sqrt{2g}\int_0^h \sqrt{z(2b-z)(h-z)}dz \tag{5}$$

This expression reduces to the equation obtained by Stevens for the special case of a circular weir with a diameter of D, where a = b = D/2.

The integration of equation (5) in terms of elliptical integrals

$$Q_t = \frac{32}{15}\sqrt{g}ba^{\frac{3}{2}}[2(1-k^2+k^4)E(k)-(2-k^2)(1-k^2)K(k)] \quad (6)$$

Where E (k) and K (k) are complete elliptic integrals of the second and first kinds, respectively, and k is the modulus of these integrals, given by:

$$K(k) = \frac{\pi}{4}\left(\frac{1}{\sqrt{1-0.106k^2}} + \frac{1}{\sqrt{1-0.89k^2}}\right) \quad (7)$$

$$E(k) = \frac{\pi}{4}\left(\sqrt{1-0.106k^2} + \sqrt{1-0.894k^2}\right) \quad (8)$$

$$k = \sqrt{h/} \quad (9)$$

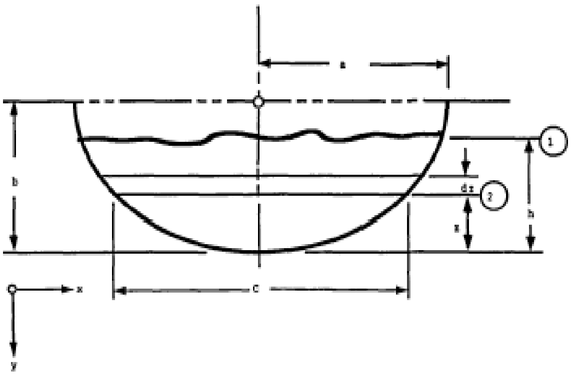


Figure 1: sketch of elliptical weir

Evaluation of the developed parabolic weirs

A holding frame with a base held the weir, and packing sheet strips made the assembly water-tight. The frame with the weir was then fixed in the channel in a perfect vertical position perpendicular to channel bed and flow direction and the elevation of weir bottom was noted (fig.2). Wax sealant prevented water leakage between channel walls and frame edges. A point gauge on a travelling carriage measured the steady-state flow depth h well upstream of the weir (46 H). The volumetric method, wherein the time required for collecting a particular volume of water in a measuring tank set at the spout of flume, was used to determine the actual discharge measurements. Discharge measurements were replicated thrice and the mean was used in the analysis. The measurements were carried out by varying discharge as shown in table 1.

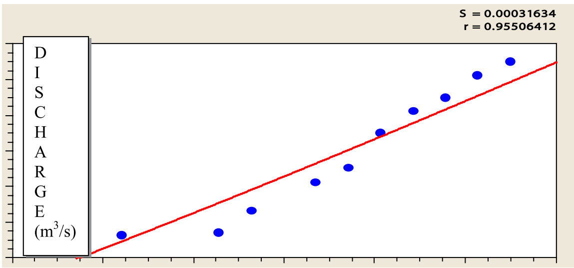
Figure 2. Weir evaluation experiment in hydraulic flume

RESULT AND DISCUSSION

Theoretical discharge was proportional to flow depth and was observed to be always grater than the actual discharge (Table 1). Theoretical discharge was greater than actual discharge values because of simplifying assumptions that do not consider friction, velocity of approach, and end contraction. The coefficient of discharge Cd is a characteristic value based on various flow parameters including the shape of weir profile. Considering the discharge data of weir the values for Cd varies from 0.48 to 1.03. Mean value of discharge coefficient Cd is 0.578.

Table 1: Discharge measurements for parabolic weir

Weir vertex reading (m)	Channel water surface reading	Head (H)	Difference in Discharge Tank	Time sec	Discharge Qa	Discharge Qt	Co efficient of Discharge Cd
0.100	0.160	0.060	0.110	60	0.00238	0.00238	1.0316
0.100	0.170	0.070	0.113	60	0.00245	0.00245	0.7826
0.100	0.185	0.085	0.115	60	0.00249	0.00249	0.4808
0.100	0.190	0.090	0.130	60	0.00282	0.00282	0.5596
0.100	0.200	0.100	0.150	60	0.00325	0.00325	0.5444
0.100	0.205	0.105	0.160	60	0.00347	0.00347	0.5503
0.100	0.210	0.110	0.185	60	0.00401	0.00401	0.5171
0.100	0.215	0.115	0.200	60	0.00433	0.00433	0.5016
0.100	0.220	0.120	0.210	60	0.00455	0.00455	0.5299
0.100	0.225	0.125	0.225	60	0.00488	0.00488	0.5256
0.100	0.230	0.130	0.235	60	0.00509	0.00509	0.5082



PARAMETER	VALUE
Bt	14.1cm
H	26.3cm
P	10cm
K	0.50
M	1.09

Discharge is found to be proportional to 1.09th power of the head.

The value of multiplying constant in the discharge equation is found to be 0.50

CONCLUSION

A relationship for obtaining a elliptical weir profile for fabrication similar to specification of the triangular weir based on include angle, was developed. Using the weir profile relationship, elliptical weir having the simplified discharge equation was designed, fabricated and evaluated successfully in laboratory. Weirs with discharges proportional to flow depth having a whole number of the depth exponent will have applications in automated control systems, beside ready applications in irrigation and industrial flow measurements. The equation of elliptical weir profile is Q = 0.50H1.09

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